

PERIASTRON PRECESSION OF SPINNING BINARY BLACKHOLES

Lingfeng Li 李凌风

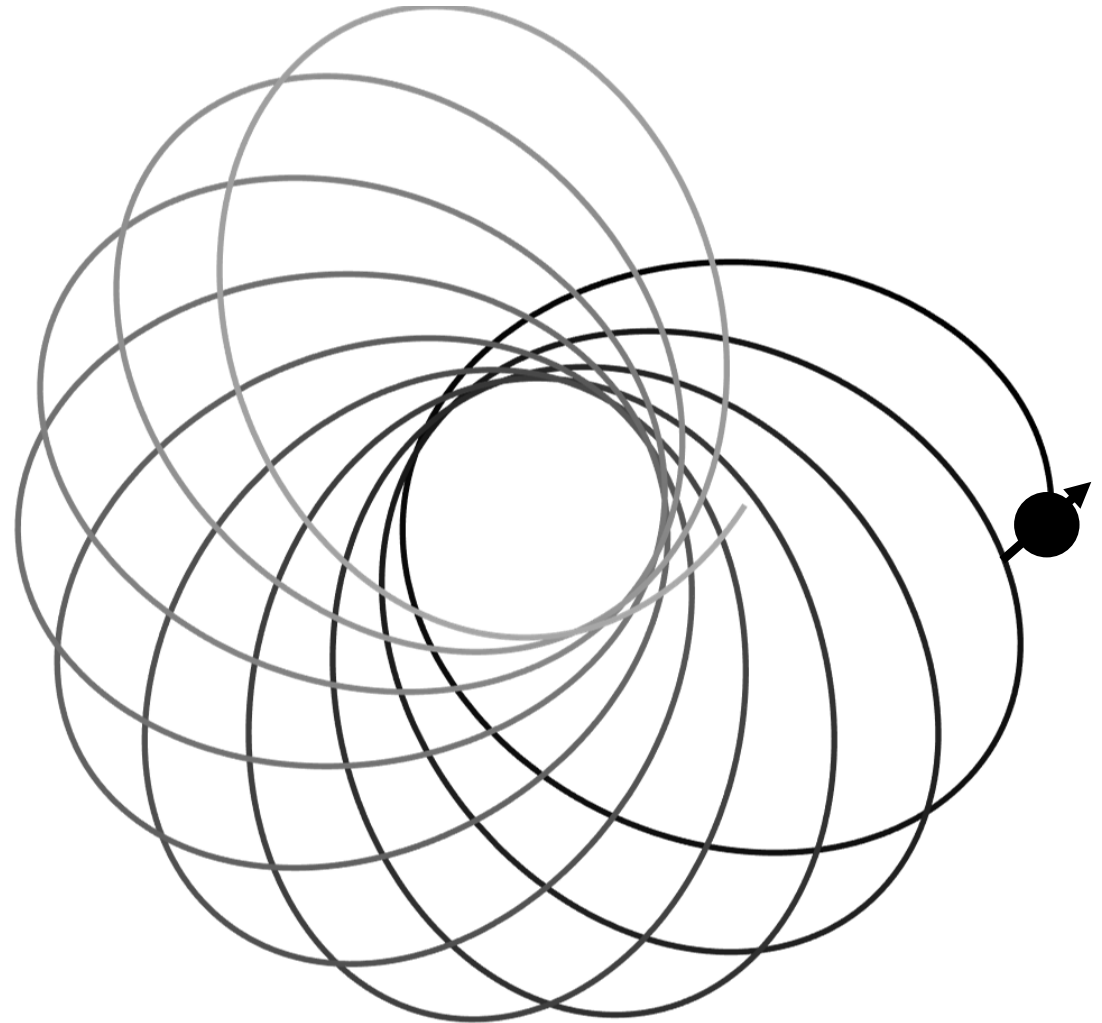
Nanjing Normal U.

Aug 24. 2026

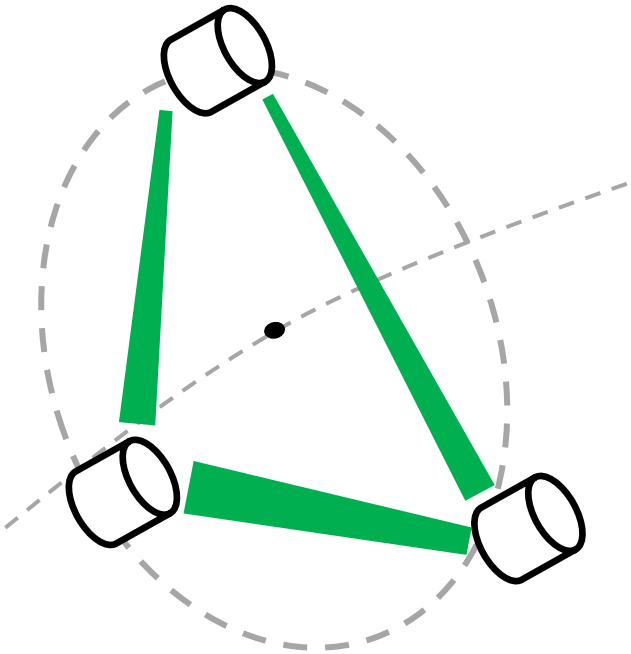


2nd New Horizons for Psi Workshop
Black Holes and Fundamental Fields

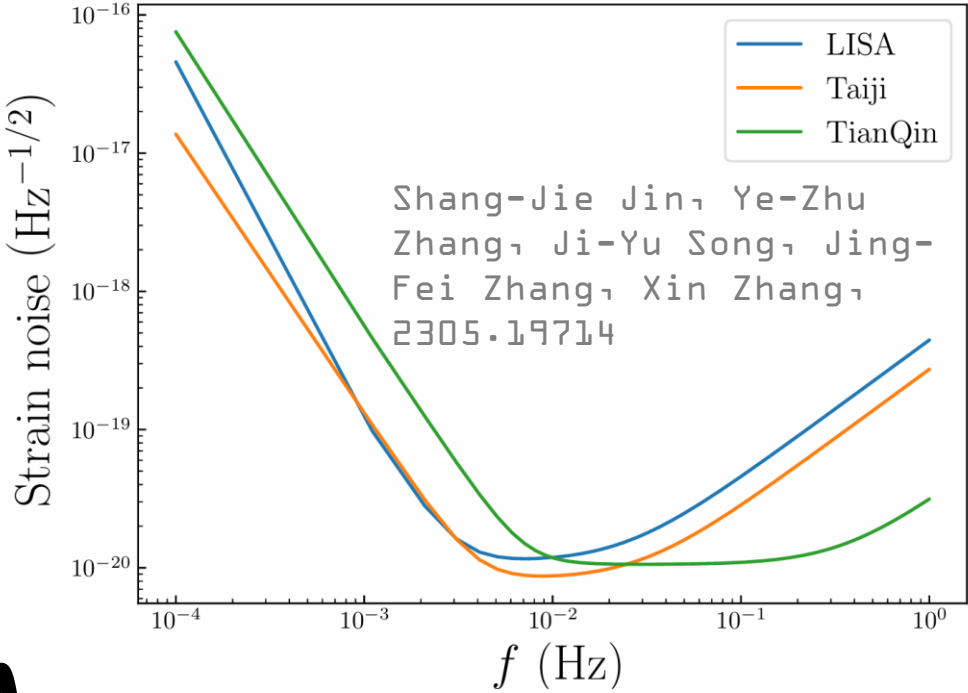
w/ I.J. Allali, P. Singh
et al., In prep.



Motivation: Spaceborne GW

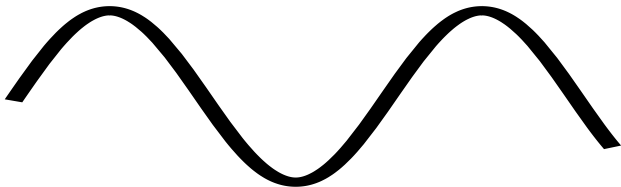
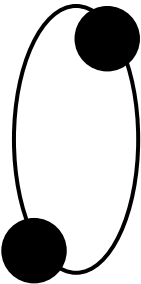


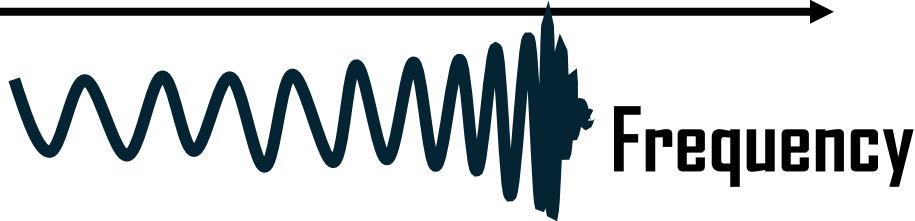
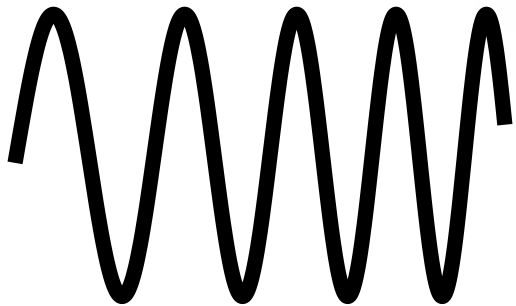
- Taiji
- LISA
- TianQin
-



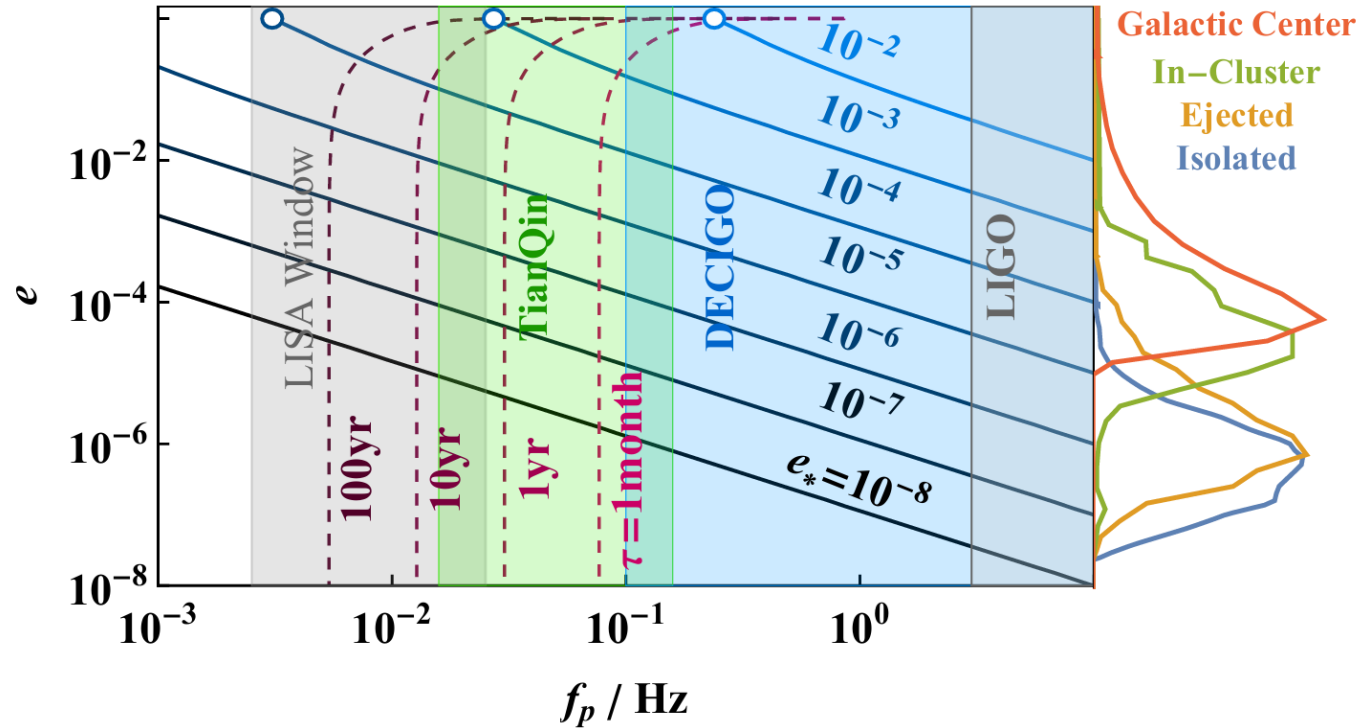


Taiji Program in Space
Gravitational wave physics
空间太极计划

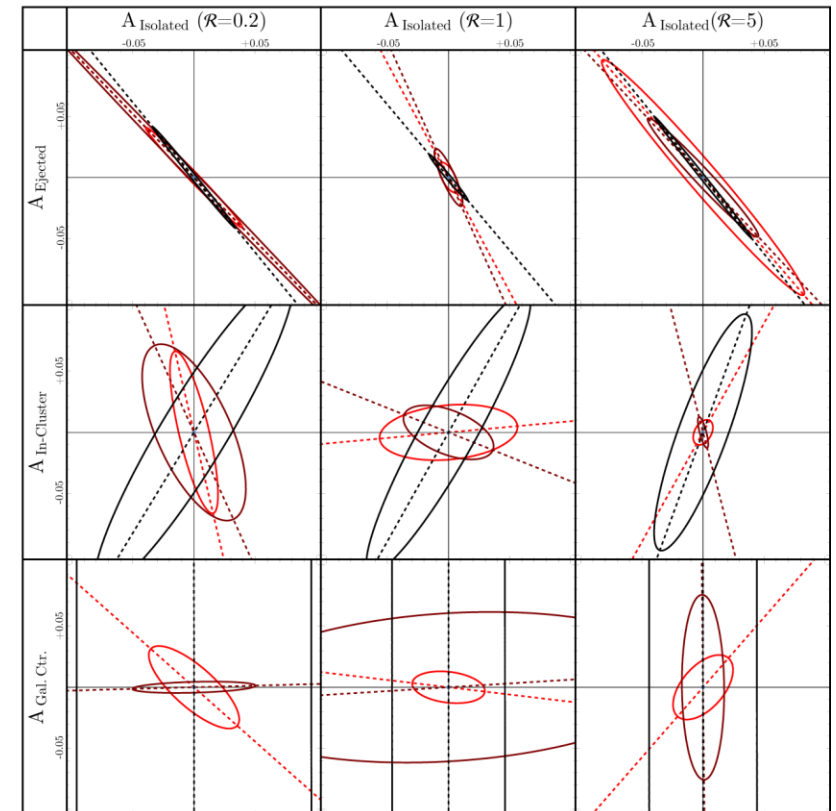




The Case with Eccentricity



Constraining different populations better by tracing eccentricity evolution



L. Randall and Z.-Z. Xianyu, 1907.02283; N. DePorzio, L. Randall, Z.-Z. Xianyu, 2402.09513

Spin Precession (Spin-Orbit/Spin)

$$\dot{\mathbf{S}}_1 = (\delta_1 \mathbf{L} + \delta_{12} \mathbf{S}_2) \times \mathbf{S}_1$$

$$\dot{\mathbf{S}}_2 = (\delta_2 \mathbf{L} + \delta_{12} \mathbf{S}_1) \times \mathbf{S}_2$$

$$\dot{\mathbf{L}} = (\delta_1 \mathbf{S}_1 + \delta_2 \mathbf{S}_2) \times \mathbf{L}$$

where

$$\delta_1 := \frac{1}{2a^3 (1 - e_t^2)^{3/2}} \left(4 + 3q - \frac{3 (\mathbf{S}_2 + q \mathbf{S}_1) \cdot \mathbf{L}}{L^2} \right)$$

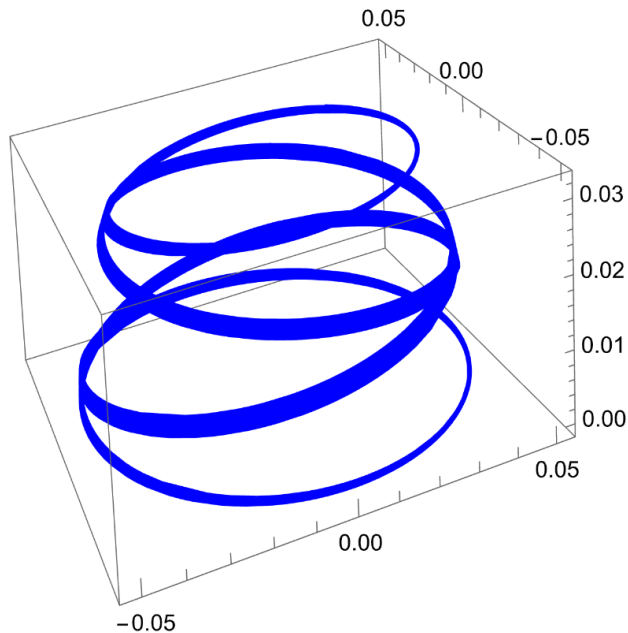
$$\delta_2 := \frac{1}{2a^3 (1 - e_t^2)^{3/2}} \left(4 + 3q^{-1} - \frac{3 (\mathbf{S}_1 + q^{-1} \mathbf{S}_2) \cdot \mathbf{L}}{L^2} \right)$$

$$\delta_{12} := \frac{1}{2a^3 (1 - e_t^2)^{3/2}}$$

Lead to many interesting works!

G. Faye, L. Blanchet and A. Buonanno, 0605139; K. G. Arun, A. Buonanno, G. Faye and E. Ochsner, 0810.5336; R. A. Porto, 1005.5730, A. Bohe, S. Marsat and L. Blanchet, 1303.7412; N. T. Maia, C. R. Galley, A. K. Leibovich and R. A. Porto, 1705.07934

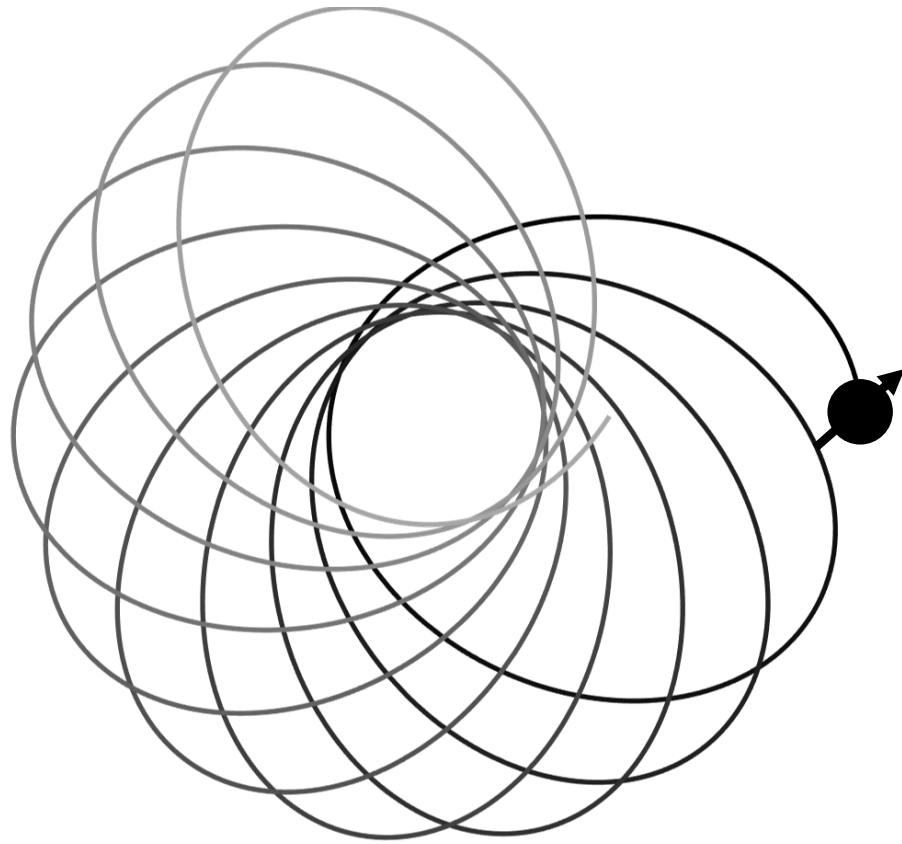
.....



Periastron Precession

Leading GR correction to the
Newtonian potential

$$V_{\text{eff}}(r) = -\frac{GM}{r} + \frac{L^2}{2r^2} - \frac{GML^2}{c^2 r^3}$$



Why consider both spin and eccentricity?

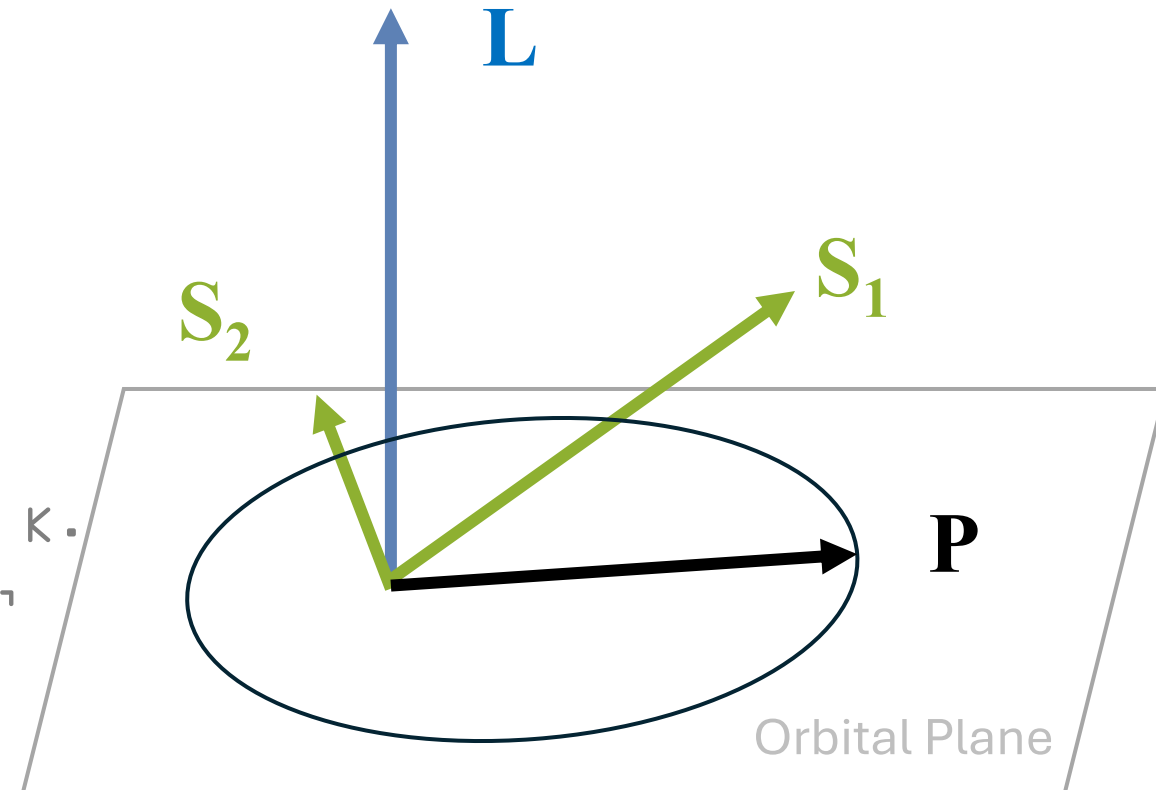
- ❑ At early stages of binary formation there could be significant eccentricity
- ❑ Black holes have spin in general
- ❑ Practical/numerical interest

(cannot differentiate the high-spin GW231123 upto $e \approx 0.6$) LVK . 2507.08219

Post Newtonian Expansion

$$\begin{aligned}\dot{\omega} &= \eta \bar{x}^{11/2} (1 - e_t^2)^2 [\mathcal{O}_N + \mathcal{O}_{1\text{PN}} \bar{x} + \mathcal{O}_{1.5\text{PN}}^{\text{hered, SO}} \bar{x}^{3/2} + \mathcal{O}_{2\text{PN}}^{\text{incl. SS}} \bar{x}^2 + \mathcal{O}_{2.5\text{PN}}^{\text{hered}} \bar{x}^{5/2} + \mathcal{O}_{3\text{PN}}^{\text{incl. hered}} \bar{x}^3], \\ \dot{e}_t^2 &= -2\eta \bar{x}^4 (1 - e_t^2)^{3/2} [\mathcal{E}_N + \mathcal{E}_{1\text{PN}} \bar{x} + \mathcal{E}_{1.5\text{PN}}^{\text{hered, SO}} \bar{x}^{3/2} + \mathcal{E}_{2\text{PN}}^{\text{incl. SS}} \bar{x}^2 + \mathcal{E}_{2.5\text{PN}}^{\text{hered}} \bar{x}^{5/2} + \mathcal{E}_{3\text{PN}}^{\text{incl. hered}} \bar{x}^3], \\ \dot{k} &= \eta \bar{x}^4 (1 - e_t^2)^{3/2} [\mathcal{K}_{1\text{PN}} \bar{x} + \mathcal{K}_{1.5\text{PN}}^{\text{SO}} \bar{x}^{3/2} + \mathcal{K}_{2\text{PN}}^{\text{incl. SS}} \bar{x}^2 + \mathcal{K}_{2.5\text{PN}}^{\text{hered}} \bar{x}^{5/2} + \mathcal{K}_{3\text{PN}} \bar{x}^3],\end{aligned}$$

K. S. Phukon, N. K. Johnson-McDaniel, A. Singh, A. Gupta, 2504.20543; A. Singh, N. K. Johnson-McDaniel, A. Gupta, K. S. Phukon, 2505.07238

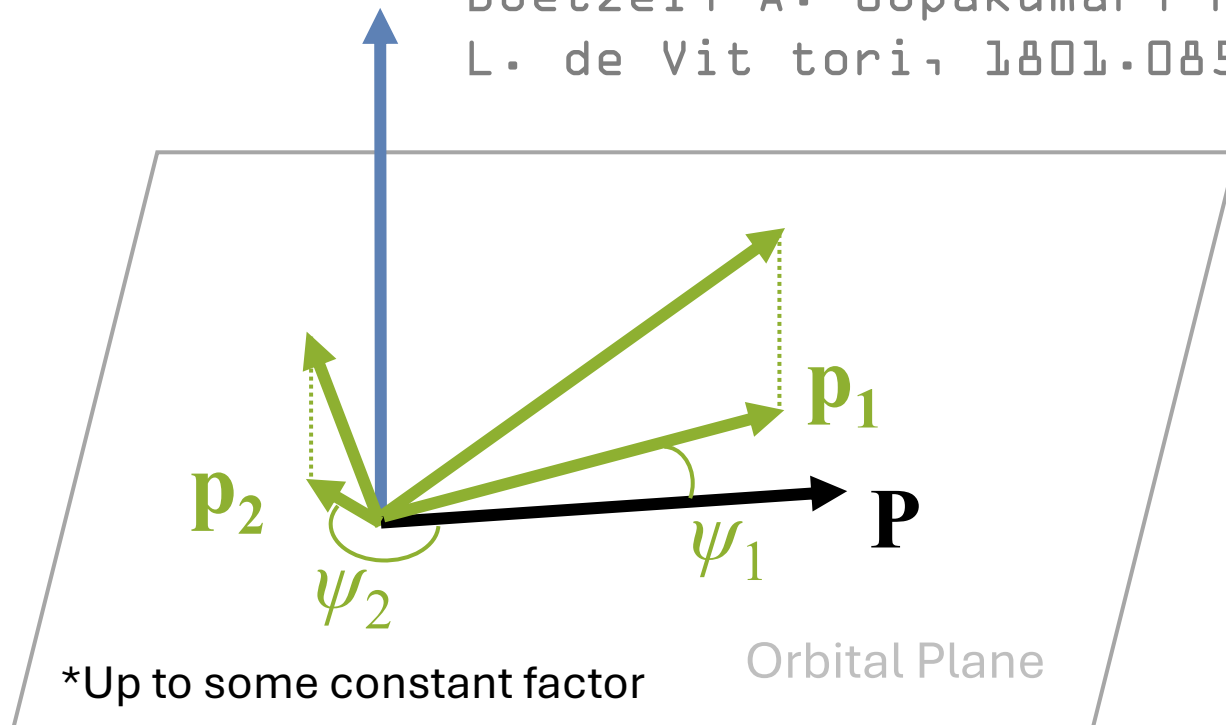


$$\begin{aligned}\dot{\omega} &= \eta \bar{x}^{11/2} (1 - e_t^2)^2 [\mathcal{O}_N + \mathcal{O}_{1\text{PN}} \bar{x} + \mathcal{O}_{1.5\text{PN}}^{\text{hered, SO}} \bar{x}^{3/2} + \mathcal{O}_{2\text{PN}}^{\text{incl. SS}} \bar{x}^2 + \mathcal{O}_{2.5\text{PN}}^{\text{hered}} \bar{x}^{5/2} + \mathcal{O}_{3\text{PN}}^{\text{incl. hered}} \bar{x}^3], \\ \dot{e}_t^2 &= -2\eta \bar{x}^4 (1 - e_t^2)^{3/2} [\mathcal{E}_N + \mathcal{E}_{1\text{PN}} \bar{x} + \mathcal{E}_{1.5\text{PN}}^{\text{hered, SO}} \bar{x}^{3/2} + \mathcal{E}_{2\text{PN}}^{\text{incl. SS}} \bar{x}^2 + \mathcal{E}_{2.5\text{PN}}^{\text{hered}} \bar{x}^{5/2} + \mathcal{E}_{3\text{PN}}^{\text{incl. hered}} \bar{x}^3], \\ \dot{k} &= \eta \bar{x}^4 (1 - e_t^2)^{3/2} [\mathcal{K}_{1\text{PN}} \bar{x} + \mathcal{K}_{1.5\text{PN}}^{\text{SO}} \bar{x}^{3/2} + \mathcal{K}_{2\text{PN}}^{\text{incl. SS}} \bar{x}^2 + \mathcal{K}_{2.5\text{PN}}^{\text{hered}} \bar{x}^{5/2} + \mathcal{K}_{3\text{PN}} \bar{x}^3],\end{aligned}$$

$$\sigma(a, b, c, d, e, f) := a \|\mathbf{s}\|^2 + b (\mathbf{s} \cdot \hat{\mathbf{L}})^2 + c \|\mathbf{s} \times \hat{\mathbf{L}}\|^2 \cos 2\psi$$

Quad-Monopole A. Klein, Y.
Boetzel, A. Gopakumar, P. Jetzer,
L. de Vit tori, 1801.08542

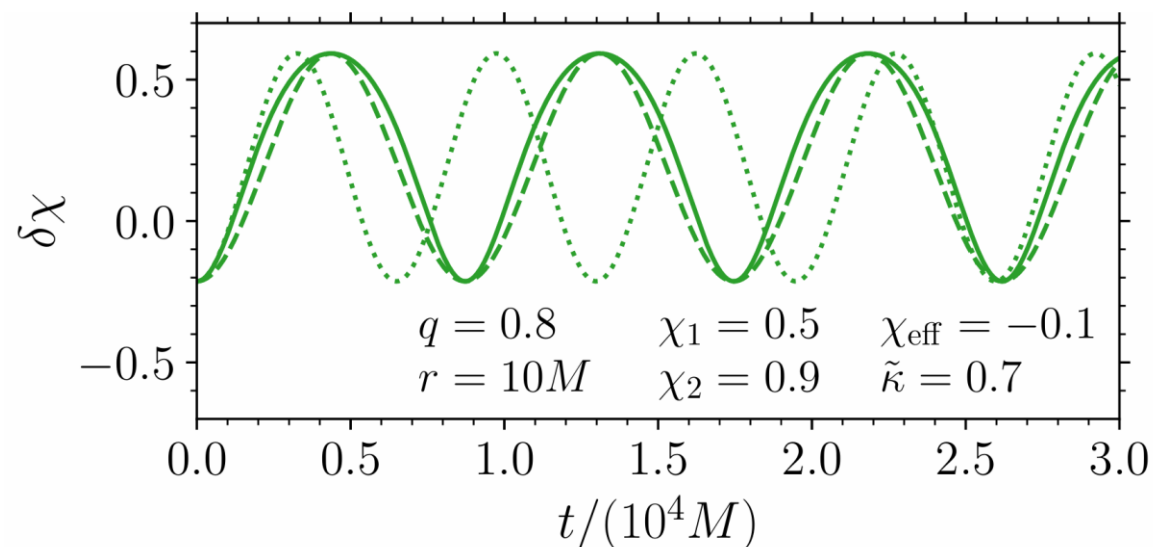
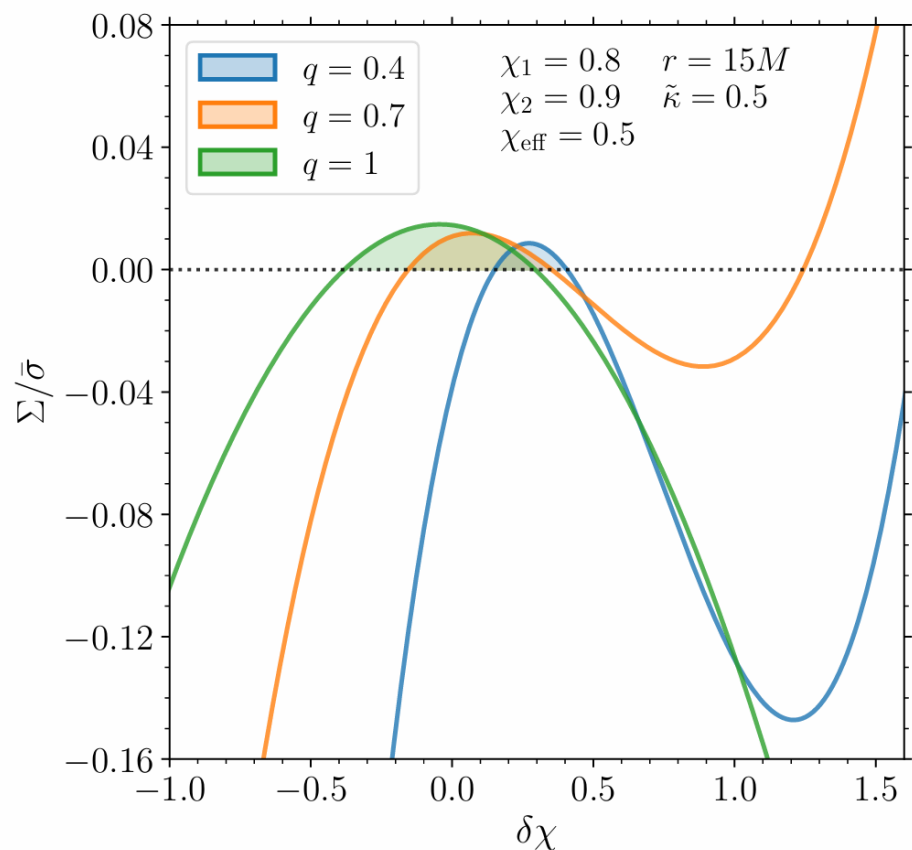
$$\begin{aligned}&+ d(\|\mathbf{s}_1\|^2 + \|\mathbf{s}_2\|^2) \\ &+ e \left[(\mathbf{s}_1 \cdot \hat{\mathbf{L}})^2 + (\mathbf{s}_2 \cdot \hat{\mathbf{L}})^2 \right] \\ &+ f \left(\|\mathbf{s}_1 \times \hat{\mathbf{L}}\|^2 \cos 2\psi_1 \right. \\ &\quad \left. + \|\mathbf{s}_2 \times \hat{\mathbf{L}}\|^2 \cos 2\psi_2 \right),\end{aligned}$$



**Spin-induced quadrupole-monopole
term after orbit averaging**

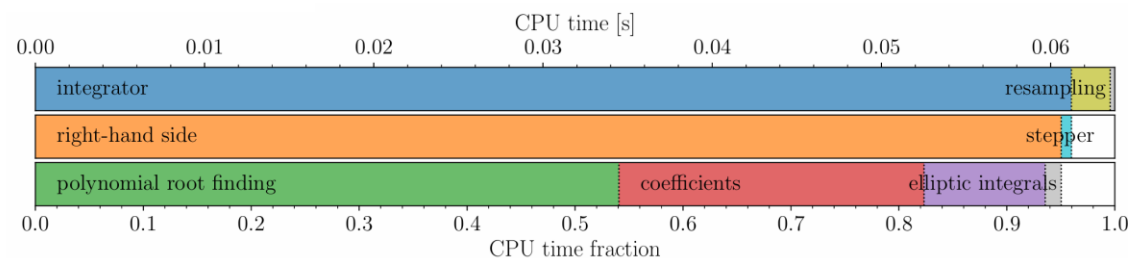
Integrated/Secular Effect of Spin Precession

D. Gerosa and M. Kesden, 1605.01067; D. Gerosa, G. Fumagalli, M. Mould, G. Cavallotto, D. P. Monroy, D. Gangardt et al., 2304.04801



$$\chi_{\text{eff}} \equiv \frac{\chi_1 \hat{\mathbf{S}}_1 \cdot \hat{\mathbf{L}} + q \chi_2 \hat{\mathbf{S}}_2 \cdot \hat{\mathbf{L}}}{1 + q} \quad \text{almost a constant}$$

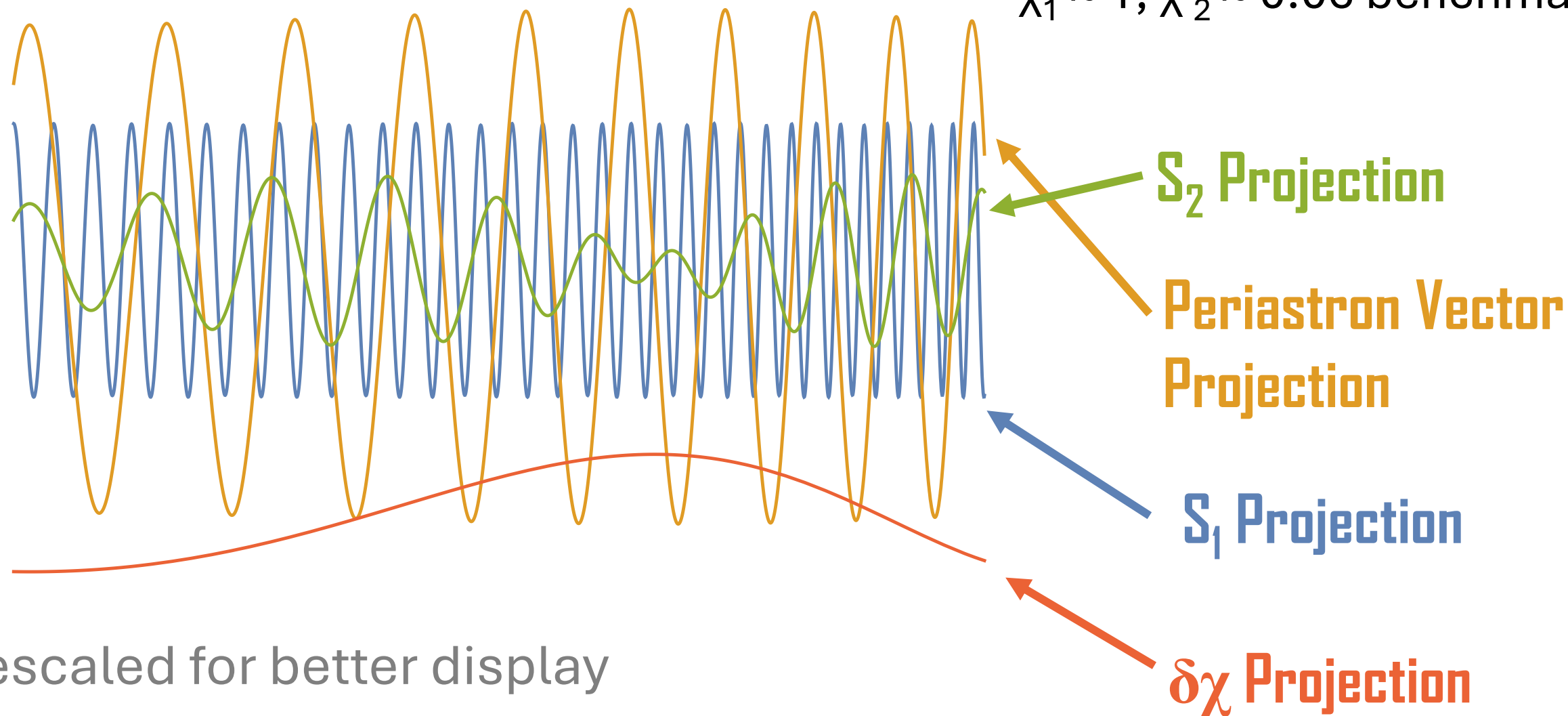
$$\delta\chi \equiv \frac{\chi_1 \hat{\mathbf{S}}_1 \cdot \hat{\mathbf{L}} - q \chi_2 \hat{\mathbf{S}}_2 \cdot \hat{\mathbf{L}}}{1 + q} \quad \text{bouncing quasi-harmonically}$$



“Integrating out” greatly reduces the cost

High Frequency Modes

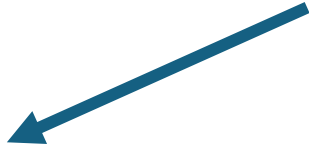
$\chi_1 \approx 1, \chi_2 \approx 0.06$ benchmark



* Rescaled for better display

Suppression of Oscillatory Terms

$$\begin{aligned}\int_0^T A(t) \cos(2\Psi(t)) dt &= \frac{A(t)}{2\Omega(t)} \sin(2\Psi(t)) \Big|_0^T - \int_0^T \frac{d}{dt} \left(\frac{A(t)}{2\Omega(t)} \right) \sin(2\Psi(t)) dt \\ \Omega(t) &\equiv \dot{\Psi}(t) &= \frac{A}{2\Omega} \sin(2\Psi) \Big|_0^T - \int_0^T \left(\frac{\dot{A}}{2\Omega} - \frac{A\dot{\Omega}}{2\Omega^2} \right) \sin(2\Psi) dt.\end{aligned}$$


$$\epsilon_A(t) \equiv \frac{|\dot{A}(t)|}{|A(t)| |\Omega(t)|},$$

Envelope Modulation Term


$$\epsilon_\Omega(t) \equiv \frac{|\dot{\Omega}(t)|}{\Omega(t)^2}$$

Frequency Drift Term

Breakdown of Terms

Since we have two spins, for each define

$$S \equiv |\mathbf{S}_i|, \quad S' \equiv |\mathbf{S}_j|, \quad j \neq i$$

Get basic scaling blocks:

$$L \sim \bar{x}^{-1/2}, \quad \delta_{12} \sim \bar{x}^3, \quad \delta_i \sim \bar{x}^3 + \bar{x}^{7/2}(S + S')$$

And higher-order ones:

$$\begin{aligned} \frac{d}{dt}(\mathbf{S}_i \cdot \hat{\mathbf{L}}) &= (\delta_{12} - \delta_j) \mathbf{S}_j \cdot (\mathbf{S}_i \times \hat{\mathbf{L}}) \\ &\sim SS' \bar{x}^3 + SS'(S + S') \bar{x}^{7/2} + \mathcal{O}(SS' \bar{x}^4) \end{aligned}$$

.....

component	generic scaling
$\mathbf{S}_i, \mathbf{p}_i, s_i$	S
$\mathbf{S}_j$	S'
L	$\bar{x}^{-1/2}$
δ_{12}	$\bar{x}^3$
δ_i	$\bar{x}^3 + \bar{x}^{7/2}(S + S')$
$\dot{\delta}_{12}$	$\eta \bar{x}^7$
$\dot{\delta}_i$	$\eta \bar{x}^7 + SS' \bar{x}^{13/2}$
$\dot{\mathbf{S}}_i$	$S \bar{x}^{5/2} + S(S + S') \bar{x}^3$
$\dot{\mathbf{L}}$	$(S + S') \bar{x}^3$
$\dot{s}_i$	$SS' \bar{x}^3 + SS'(S + S') \bar{x}^{7/2} + \mathcal{O}(SS' \bar{x}^4)$
$\dot{p}_i^2/p_i^2$	$S' \bar{x}^3 + S'(S + S') \bar{x}^{7/2} + \mathcal{O}(S' \bar{x}^4)$
$\ddot{\mathbf{S}}_i \text{ raw}$	$S \bar{x}^5 + S(S + S') \bar{x}^{11/2} + S(S + S')^2 \bar{x}^6 + \eta S \bar{x}^{13/2}$
$[\ddot{\mathbf{S}}_i]_{\hat{\mathbf{L}} \times \mathbf{p}_i}$	$SS' \bar{x}^{11/2} + SS'(S + S') \bar{x}^6 + \eta S \bar{x}^{13/2}$
$\ddot{\mathbf{L}} \text{ raw}$	$(S + S') \bar{x}^{11/2} + (S + S')^2 \bar{x}^6 + \eta (S + S') \bar{x}^7$
$[\ddot{\mathbf{L}}]_{\hat{\mathbf{L}} \times \mathbf{p}_i}$	$S' \bar{x}^{11/2} + S'(S + S') \bar{x}^6 + \eta (S + S') \bar{x}^7$

The Building Blocks are all You Need

Scaling of Frequency Drift Terms

$$\Omega_i \equiv \dot{\Psi}_i = \Omega[\mathbf{p}_i] - \Omega[\hat{\mathbf{P}}] \quad \left\{ \begin{array}{l} \Omega_{\text{PP}} \equiv \Omega[\hat{\mathbf{P}}] = \hat{\mathbf{L}} \cdot (\hat{\mathbf{P}} \times \dot{\hat{\mathbf{P}}}) \simeq kn \\ \Omega_{\text{SP}}^{(i)} \equiv \Omega[\mathbf{p}_i] = \frac{\hat{\mathbf{L}} \cdot (\mathbf{p}_i \times \dot{\mathbf{p}}_i)}{\|\mathbf{p}_i\|^2} \end{array} \right.$$

$$\begin{aligned} \Omega_i &= \Omega_{\text{SP}}^{(i)} - \Omega_{\text{PP}} = \frac{\hat{\mathbf{L}} \cdot (\mathbf{p}_i \times \dot{\mathbf{S}}_i)}{\|\mathbf{p}_i\|^2} - (\mathbf{S}_i \cdot \hat{\mathbf{L}}) \frac{\hat{\mathbf{L}} \cdot (\mathbf{p}_i \times \dot{\hat{\mathbf{L}}})}{\|\mathbf{p}_i\|^2} - \hat{\mathbf{L}} \cdot (\hat{\mathbf{P}} \times \dot{\hat{\mathbf{P}}}) \\ \dot{\Omega}_{\text{SP}}^{(i)} &= \frac{\langle \dot{\hat{\mathbf{L}}} \mathbf{p}_i \dot{\mathbf{S}}_i \rangle + \langle \hat{\mathbf{L}} \mathbf{p}_i \ddot{\mathbf{S}}_i \rangle - \dot{s}_i \langle \hat{\mathbf{L}} \mathbf{p}_i \dot{\hat{\mathbf{L}}} \rangle - s_i \langle \hat{\mathbf{L}} \mathbf{p}_i \ddot{\hat{\mathbf{L}}} \rangle}{p_i^2} + 2 \Omega_{\text{SP}}^{(i)} \frac{s_i \dot{s}_i}{p_i^2} \end{aligned}$$

$$\epsilon_{\Omega, \text{SP}}^{(i)} \sim \bar{x}^{3/2} + S' \bar{x}^{1/2} + S'(S + S') \bar{x} + (S + S') \bar{x}^2$$

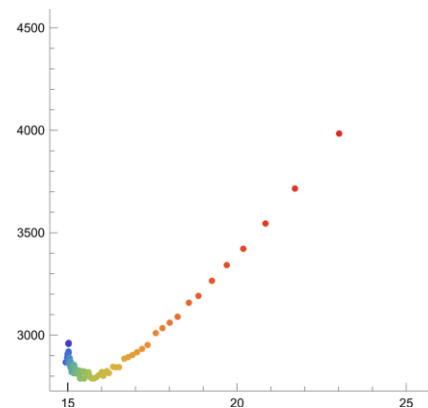
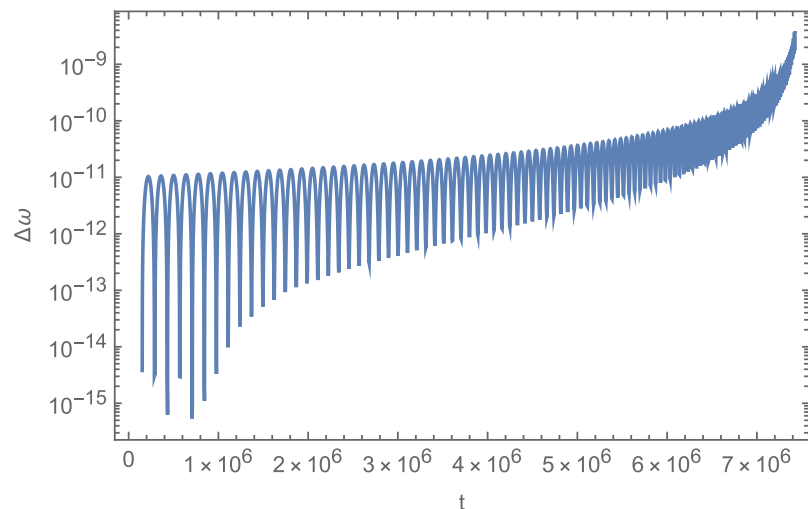
GW Emission Leading Spin-Spin Higher Order Spin-Spin ...

Summary

- ❑ BBH systems could have non-trivial spin and eccentricity
- ❑ In this case periastron precession couples with spin precession
- ❑ The timescale/frequency of different phenomena diverse
- ❑ The leading effect of PP vanishes after integration
- ❑ The remaining contribution shall follow specific scaling laws

Things Remain to be Done

- ❑ Thorough numerical tests for scaling laws and exceptional cases (ongoing)
- ❑ Closed form or numerical approximations ready to use (long-term goal)
- ❑ Impacts on BBH morphology (long-term goal)



M. Kesden, D.
Gerosa, R.
Ó'Shaughnessy,
E. Berti and
U. Sperhake,
1411.0674