

FoPTs, PBHs, SIGWs, and SWs from early Universe

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Chinese Academy of Sciences**

**The 2nd New Horizons for Psi Workshop
2026-08-26 10:00-10:30**

Collaborators



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APCTP



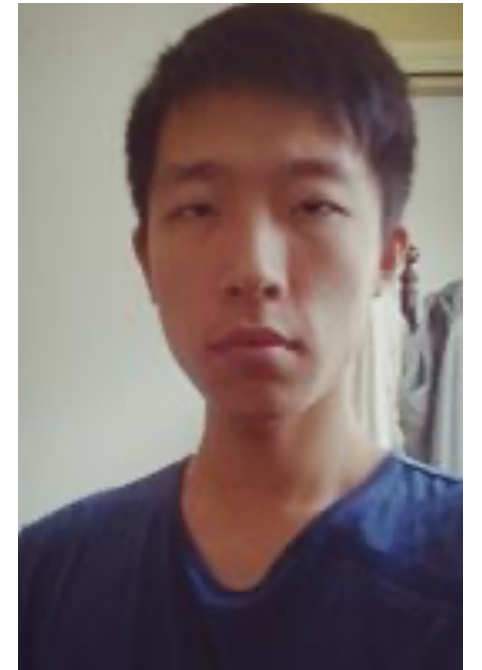
Rong-Gen Cai
Ningbo U.



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Jing Liu
ICTP-AP, UCAS



Yu-Shi Hao
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APCTP



Xiang-Xi Zeng
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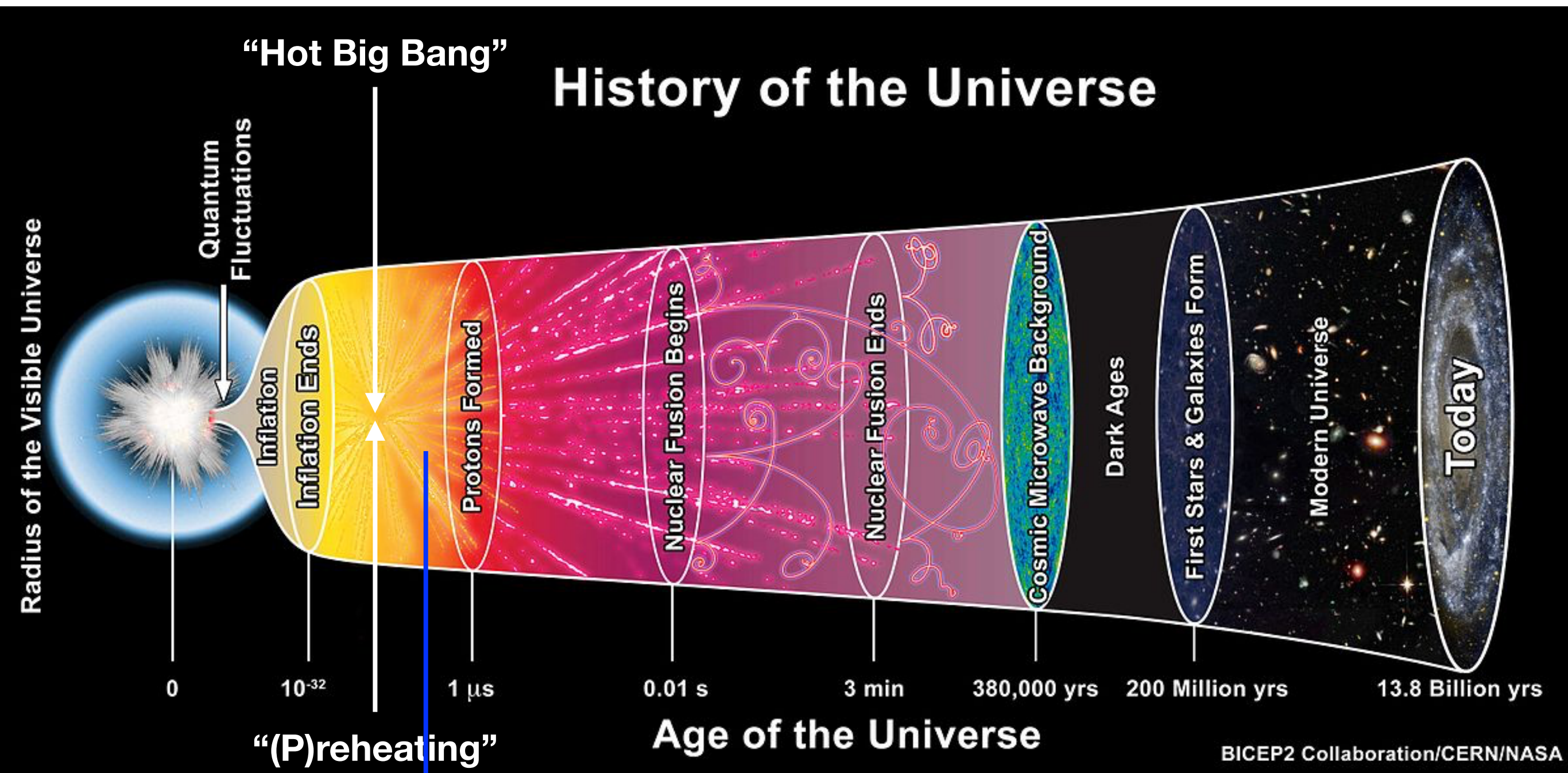


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HIAS-UCAS



Jun-Chen Wang
PKU->HKUST

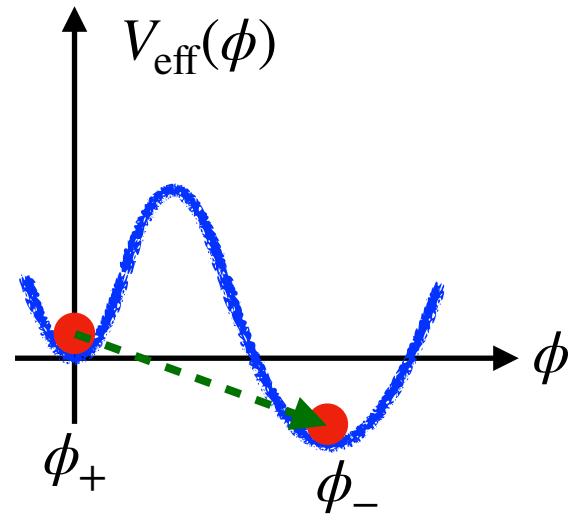
GWs from FOPTs



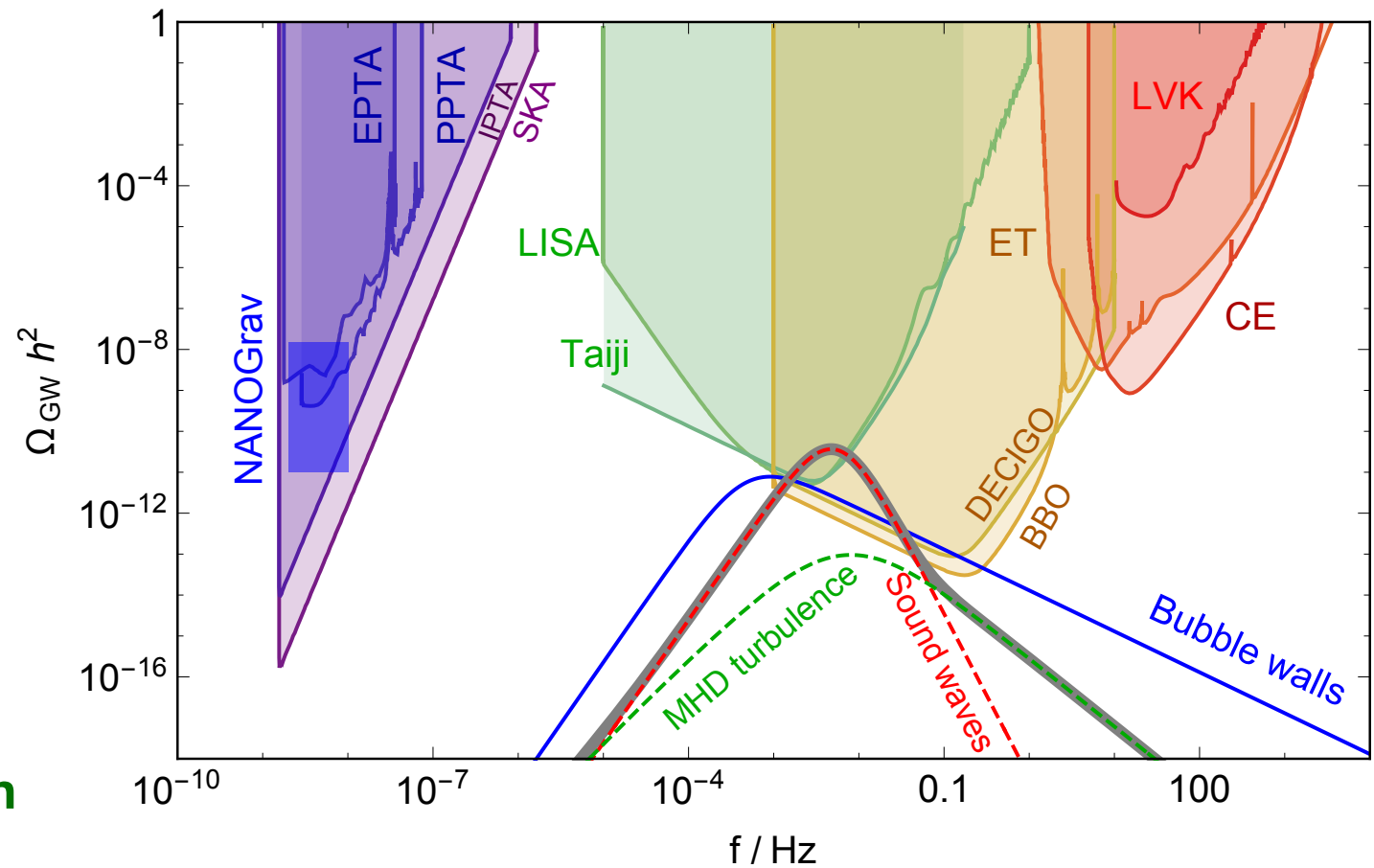
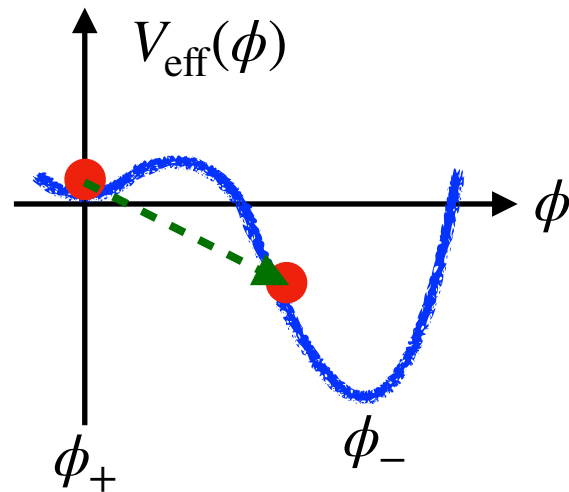
GWs from FOPTs

FOPTs

Thermal barrier



Vacuum barrier



Exponential nucleation

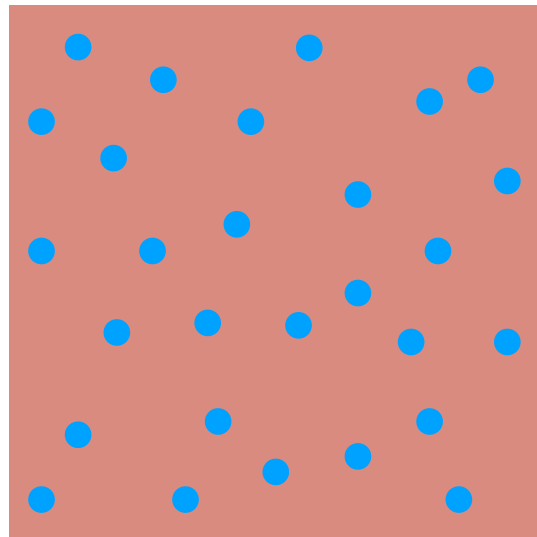
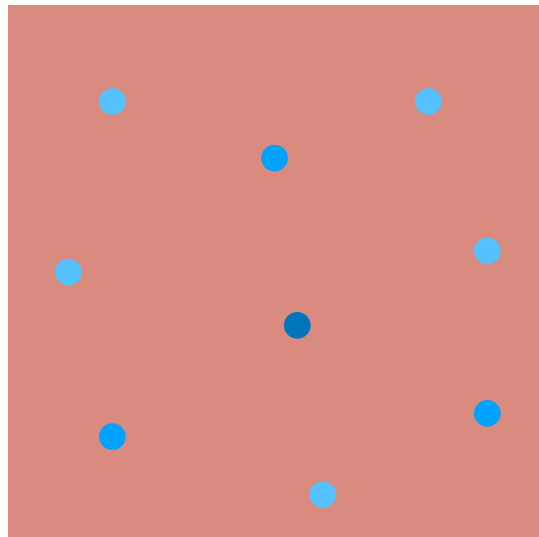
Simultaneous nucleation

$$\Gamma(t) = \Gamma_0 e^{\beta_1(t-t_f)}$$

$$\Gamma(t) = \Gamma_0 e^{-\frac{1}{2}\beta_2^2(t-t_m)^2}$$

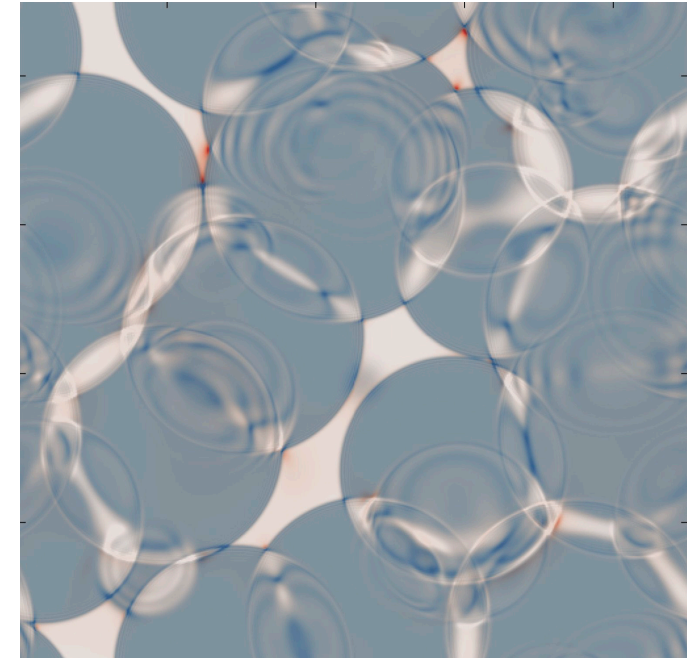
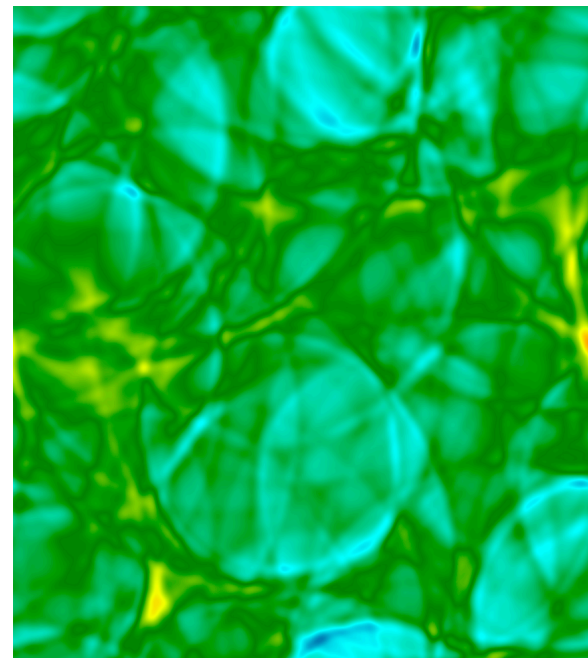
Fluid motions

Wall collisions



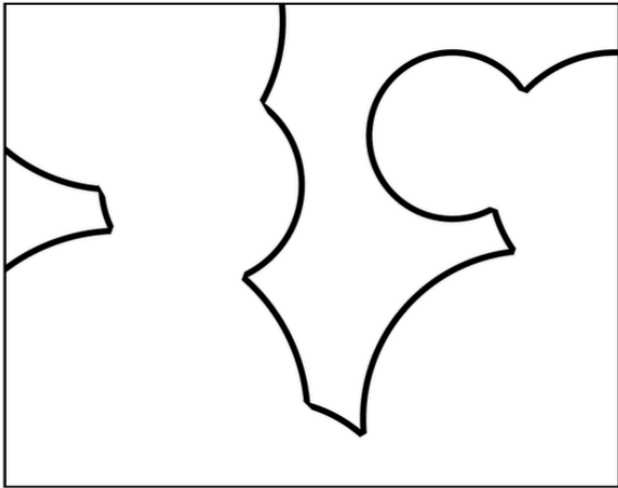
Fluid motions

Wall collisions

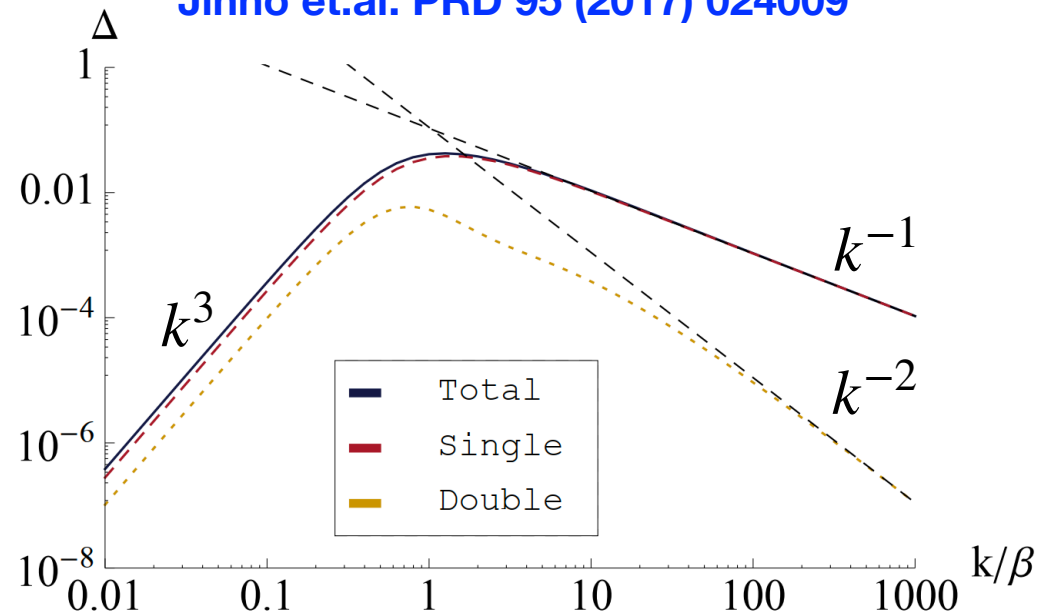


GWs from wall collisions

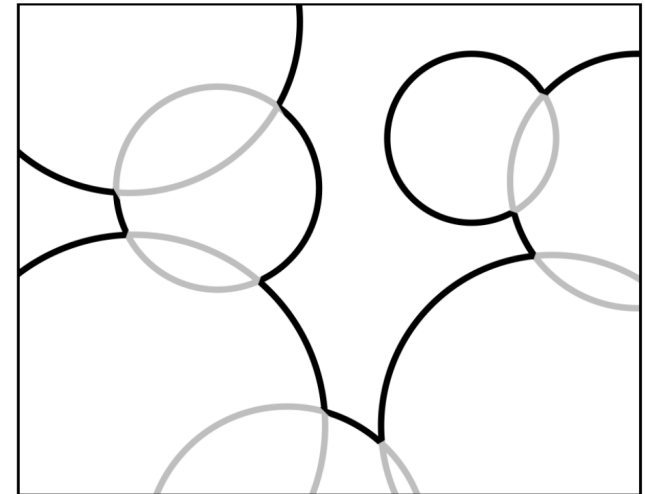
Envelope approximation



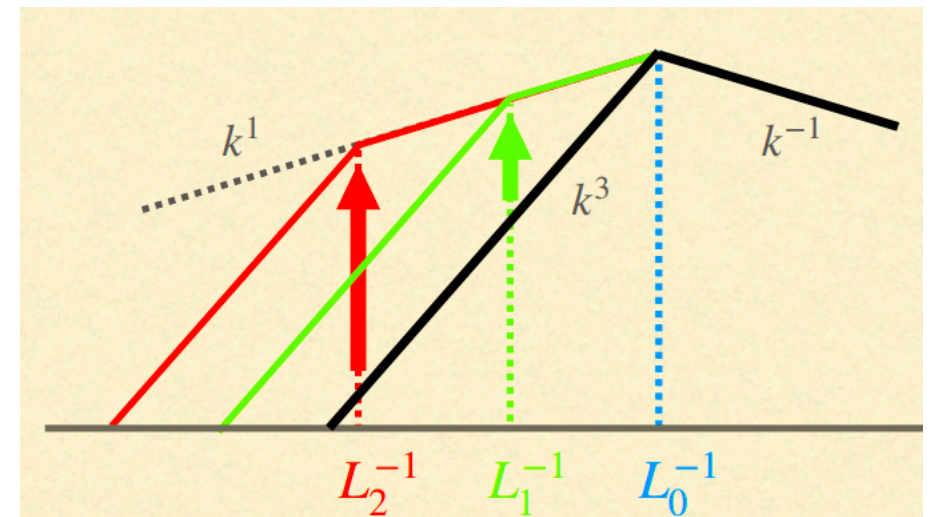
Jinno et.al. PRD 95 (2017) 024009



Beyond envelope approximation



Jinno et.al. JCAP 01 (2019) 060



Analytic template of wall collisions

$$h^2 \Omega_{\text{env}} = 1.67 \times 10^{-5} \left(\frac{100}{g_{\text{dof}}} \right)^{\frac{1}{3}} \left(\frac{H_*}{\beta} \right)^2 \left(\frac{\kappa_\phi \alpha}{1 + \alpha} \right)^2 \frac{0.48 v_w^3}{1 + 5.3 v_w^2 + 5 v_w^4} S_{\text{env}}(f)$$

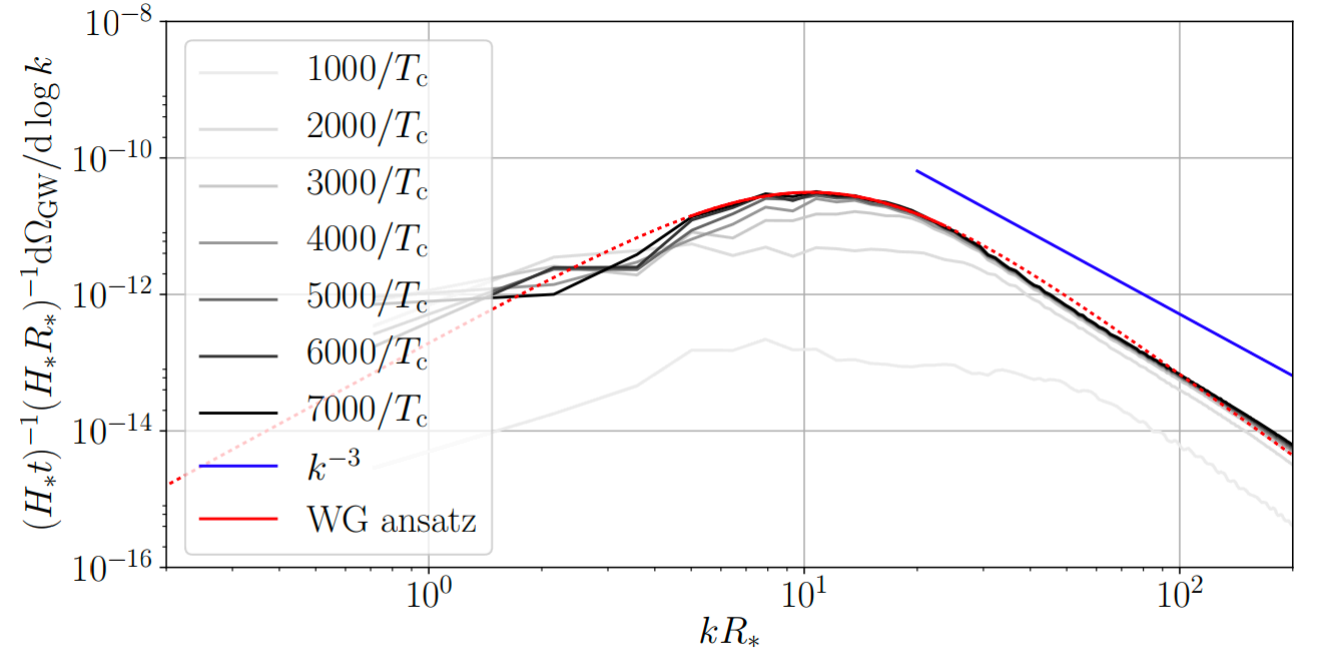
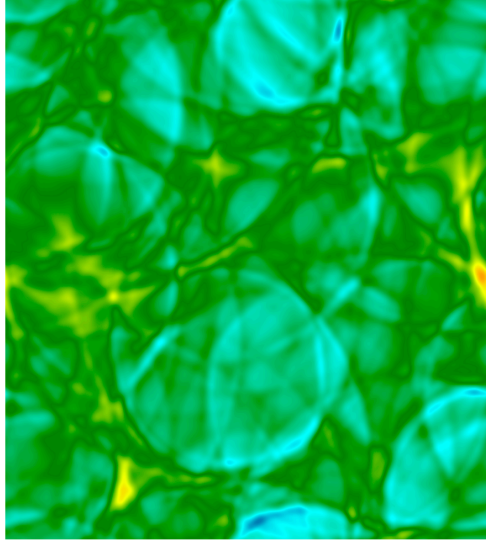
$$\frac{f_{\text{env}}}{\text{Hz}} = 1.65 \times 10^{-5} \left(\frac{g_{\text{dof}}}{100} \right)^{\frac{1}{6}} \left(\frac{T_*}{100 \text{ GeV}} \right) \frac{0.35(\beta/H_*)}{1 + 0.069 v_w + 0.69 v_w^4}$$

$$S_{\text{env}}(f) = \left[c_l \left(\frac{f}{f_{\text{env}}} \right)^{-3} + (1 - c_l - c_h) \left(\frac{f}{f_{\text{env}}} \right)^{-1} + c_h \left(\frac{f}{f_{\text{env}}} \right) \right]^{-1}$$

GWs from sound waves

Numerical simulations

Sound waves Helsinki-Sussex: 2013, 2015, 2017, 2019



Scalar-plasma EoMs: friction ansatz $\frac{\partial p_{\delta f}}{\partial \phi} \propto u^\mu \partial_\mu \phi$

$$\nabla_\mu \nabla^\mu \phi - \frac{\partial V_{\text{eff}}}{\partial \phi} = -\frac{\partial p_{\delta f}}{\partial \phi}$$

$$\nabla_\mu T_f^{\mu\nu} + \nabla^\nu \phi \frac{\partial V_T}{\partial \phi} = \nabla^\nu \phi \frac{\partial p_{\delta f}}{\partial \phi}$$

$$-\ddot{\phi} + \nabla^2 \phi - \frac{\partial V}{\partial \phi} = \eta \gamma (\dot{\phi} + v^i \partial_i \phi)$$

$$E = \gamma \epsilon : \quad \dot{E} + \partial_i (E v^i) + p[\dot{\gamma} + \partial_i (\gamma v^i)] - \frac{\partial V}{\partial \phi} \gamma (\dot{\phi} + v^i \partial_i \phi) = \eta \gamma (\dot{\phi} + v^i \partial_i \phi)$$

$$P_i = \gamma(\epsilon + p)U_i : \quad \dot{P}_i + \partial_j (P_i v^j) + \partial_i p + \frac{\partial V}{\partial \phi} \partial_i \phi = -\eta \gamma (\dot{\phi} + v^j \partial_j \phi) \partial_i \phi$$

$$h^2 \Omega_{\text{sw}} = 2.65 \times 10^{-6} \left(\frac{100}{g_{\text{dof}}} \right)^{\frac{1}{3}} \left(\frac{H_*}{\beta} \right) \left(\frac{\kappa_{\text{sw}} \alpha}{1 + \alpha} \right)^2 v_w \frac{7^{7/2} (f/f_{\text{sw}})^3}{(4 + 3(f/f_{\text{sw}})^2)^{7/2}} \Upsilon$$

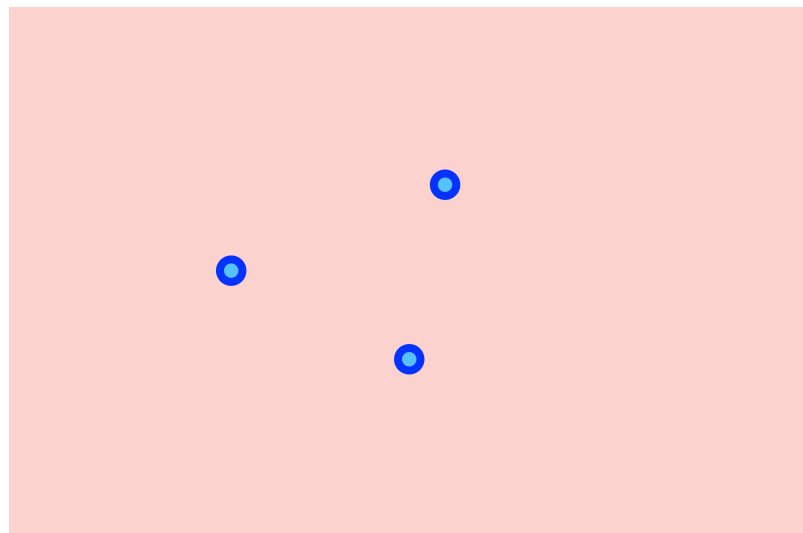
$$\frac{f_{\text{sw}}}{\text{Hz}} = 1.9 \times 10^{-5} \left(\frac{g_{\text{dof}}}{100} \right)^{\frac{1}{6}} \left(\frac{T_*}{100 \text{ GeV}} \right) \left(\frac{1}{v_w} \right) \left(\frac{\beta}{H_*} \right)$$

$$\Upsilon \equiv 1 - (1 + 2\tau_{\text{sw}} H_*)^{-1/2}$$

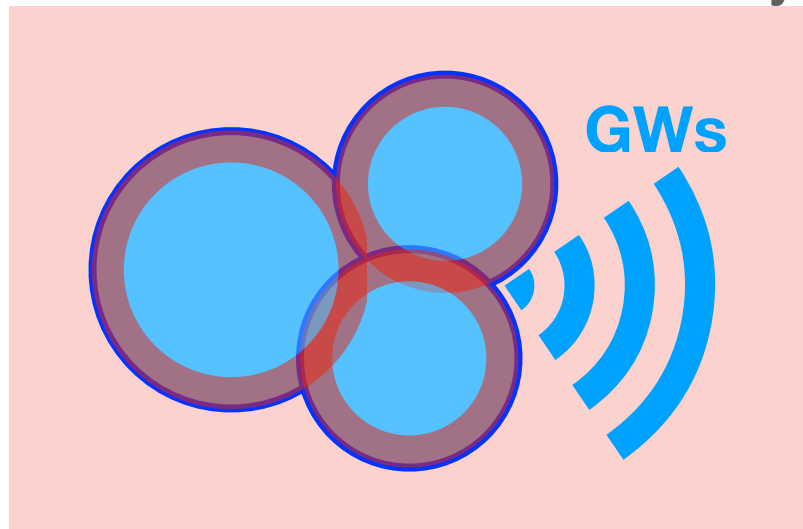
$$\tau_{\text{sw}} H_* \approx (8\pi)^{1/3} v_w / (\beta/H_*) / \bar{U}_f$$

$$\bar{U}_f^2 = 3\kappa_{\text{sw}} \alpha / [4(1 + \alpha)] \quad \text{Huai-Ke Guo et al. 20}$$

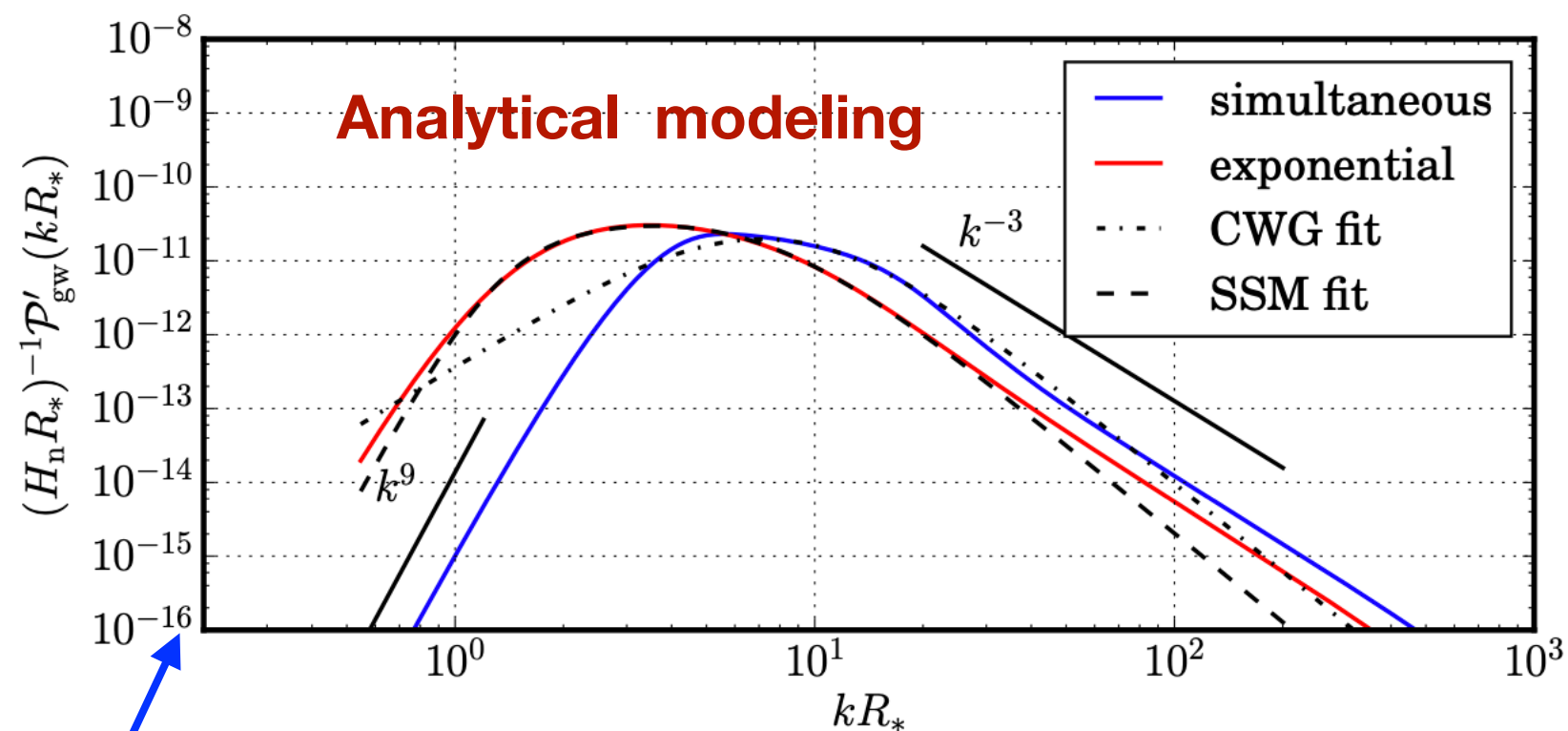
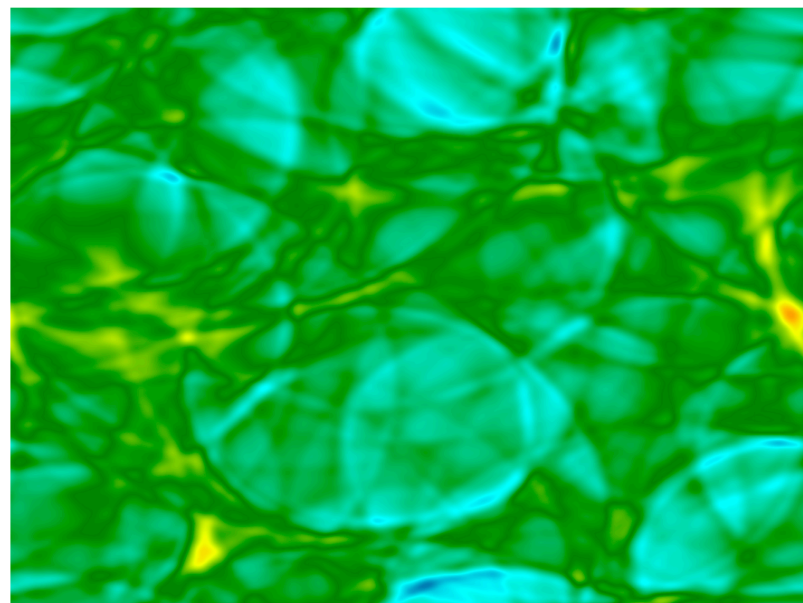
GWs from sound-shell superpositions



Sound-shell forced-collisions at early time

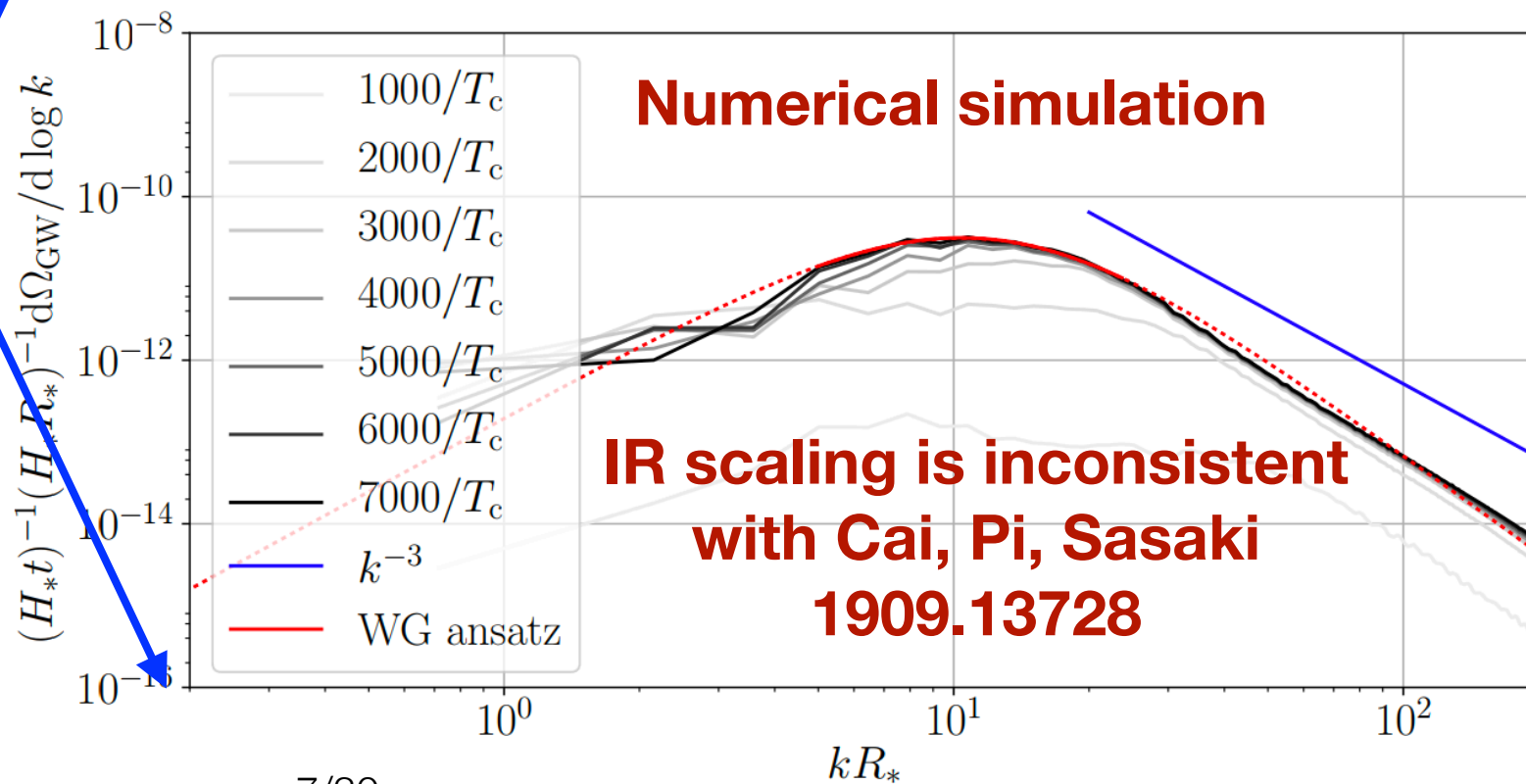


Sound-shell free-collisions at late time

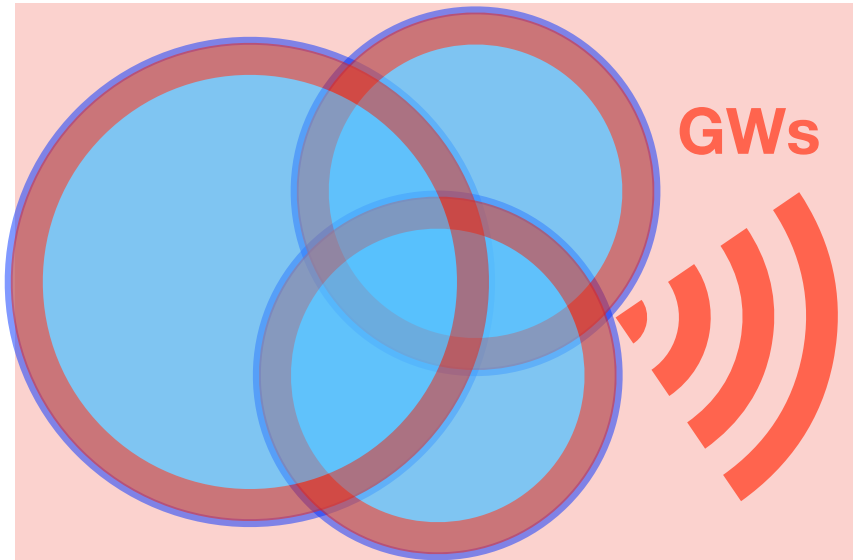


Sound shell model Hindmarsh 18 PRL, Hindmarsh & Hijazi 19

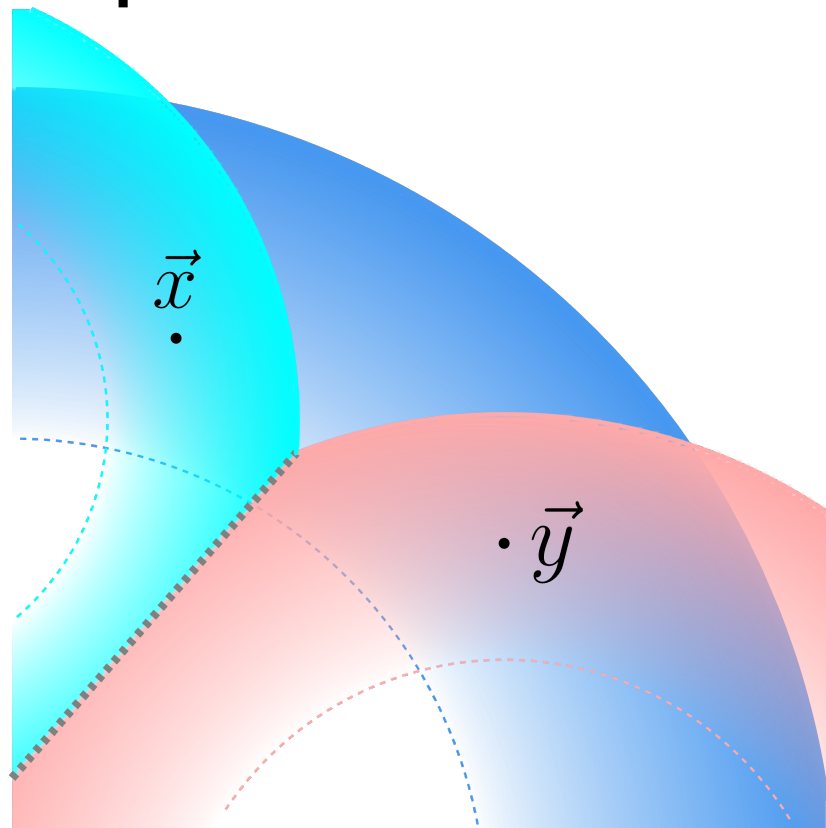
(a) Weak, $v_w = 0.92$



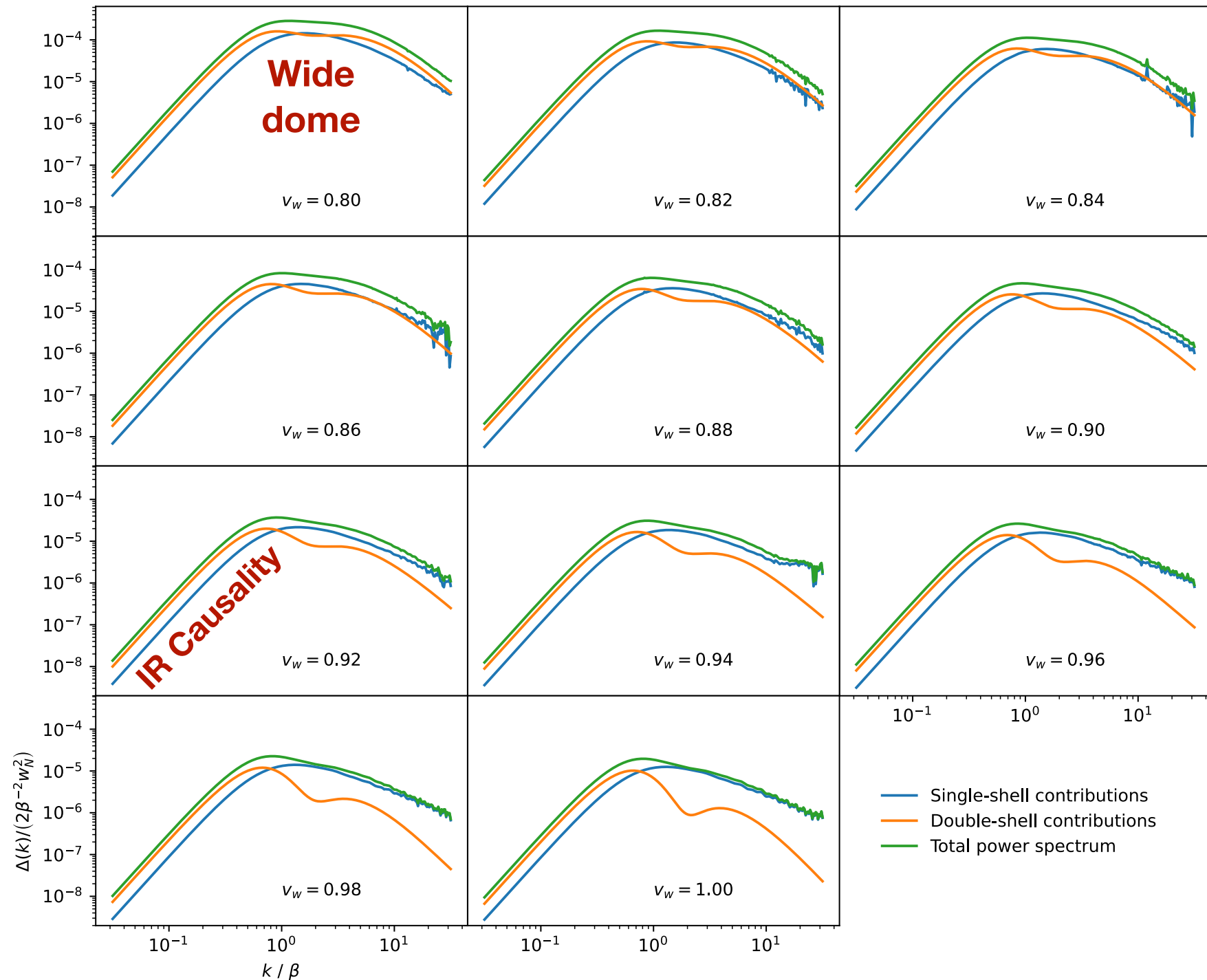
GWs from sound-shell envelopes



GWs from sound-shell envelopes before fully percolations of walls

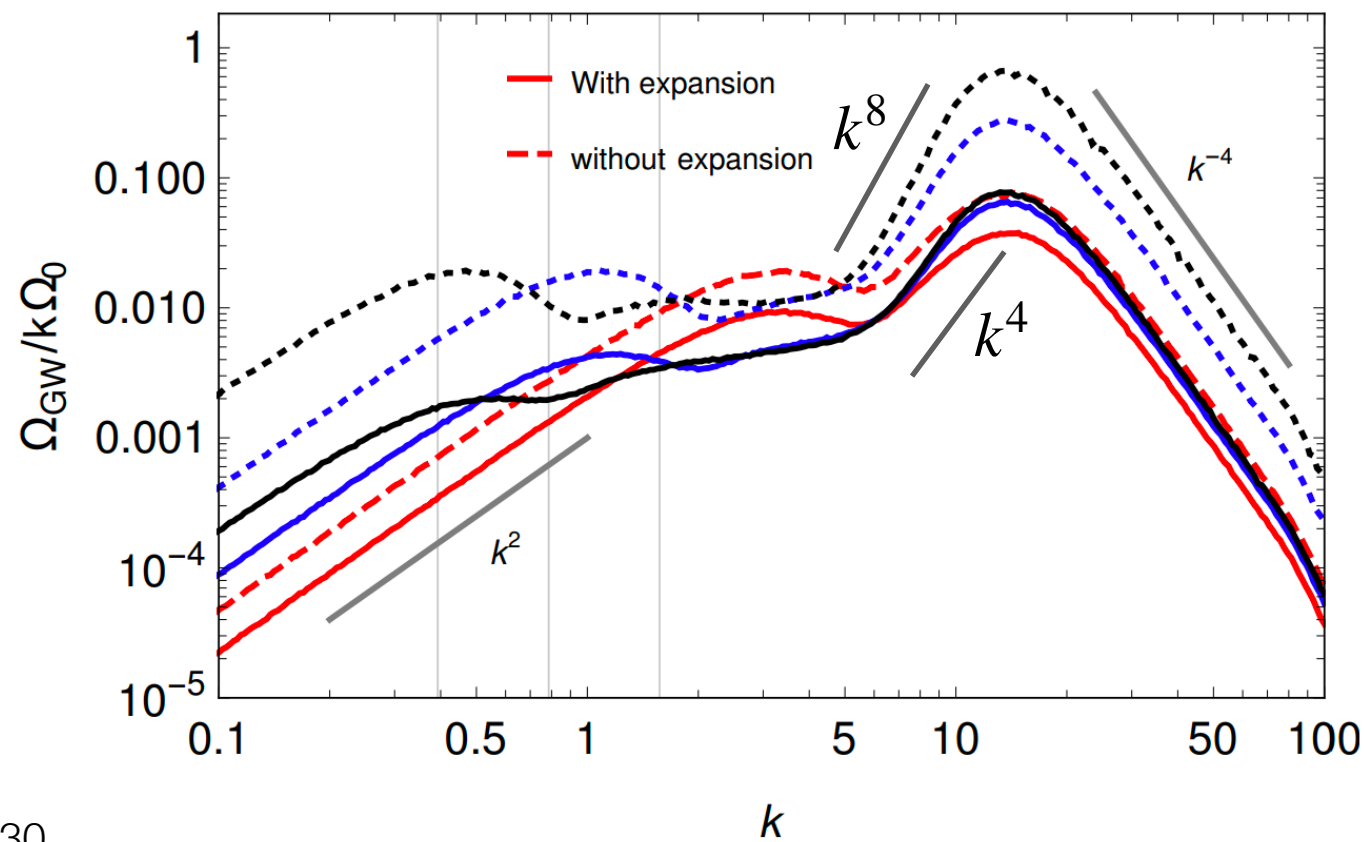
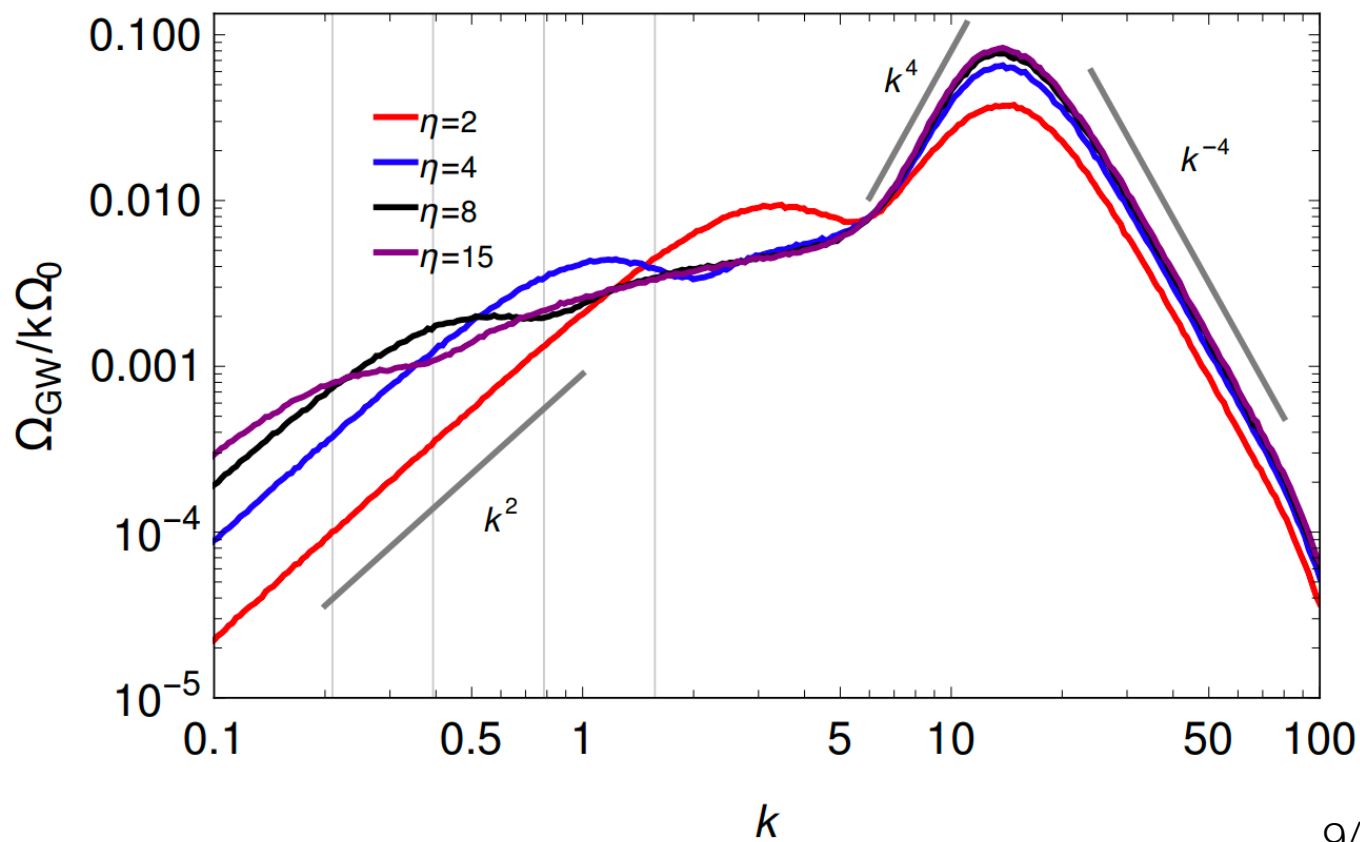
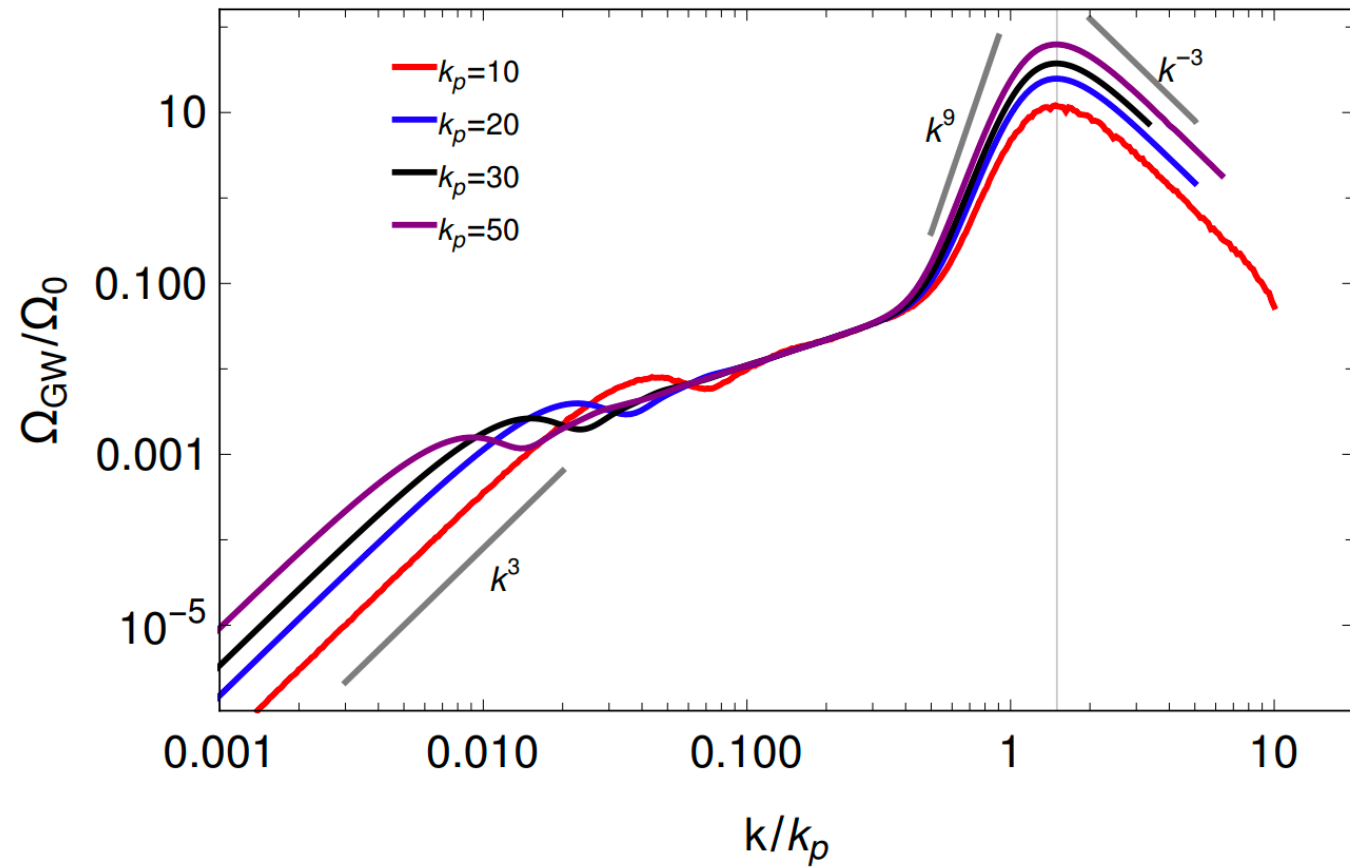
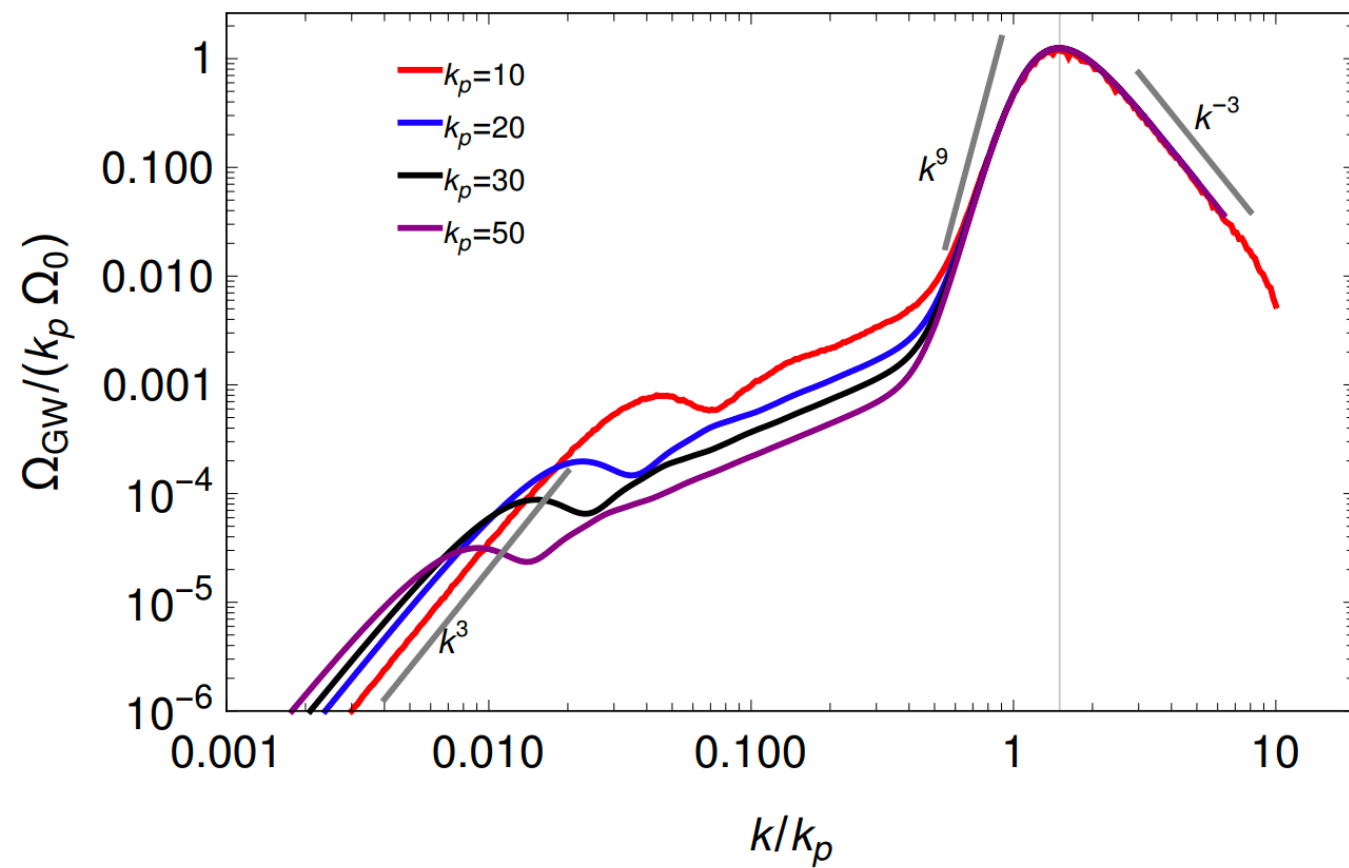


2305.00074 Rong-Gen Cai, SJW, Zi-Yan Yuwen
“Hydrodynamic sound shell model”
 Phys. Rev. D 108 (2023) 2, L021502



LISA's response

Simulation: **2308.12916** ...Hindmarsh “Shallow relic gravitational wave spectrum with acoustic peak”

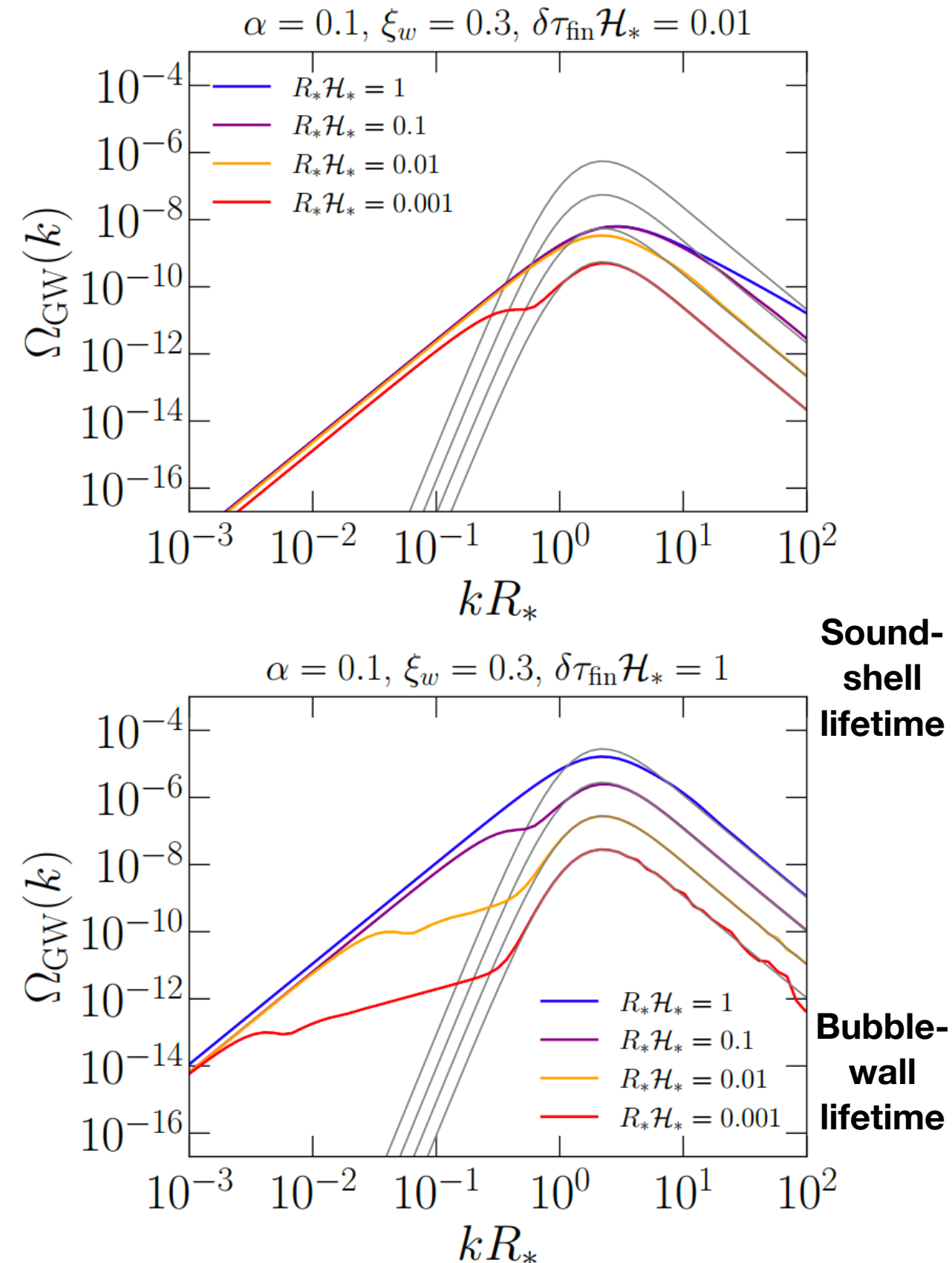


LISA's response

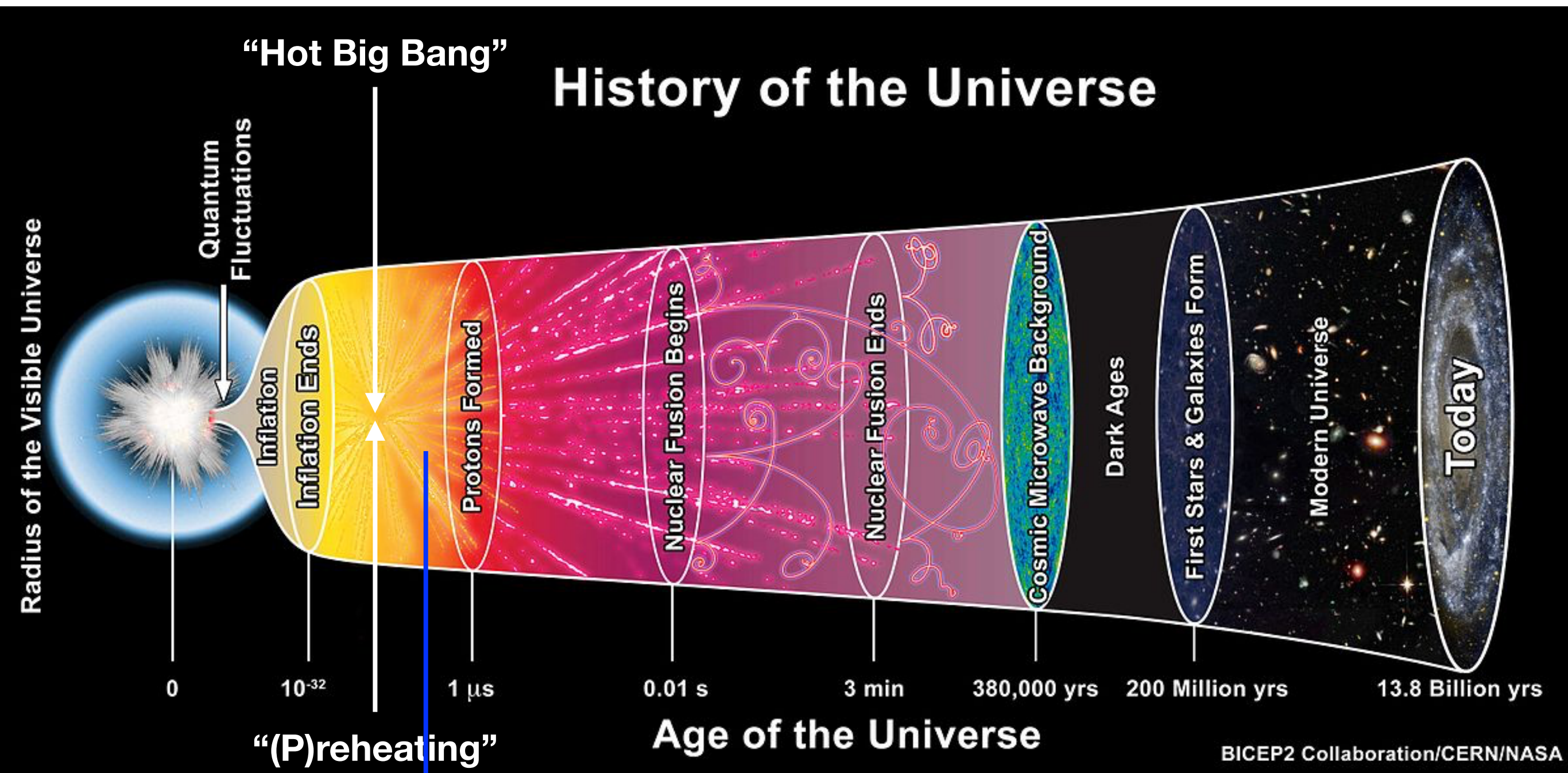
Analytical modeling: 2308.12943 ...Caprini “Characterization of the gravitational wave spectrum from sound waves within the sound shell model”

In Sec. (IV A), we start by collecting the results of Sec. (III) to evaluate the GW spectrum, and comment on the consequences of the expansion of the Universe. We find in Sec. (IV B) that the GW spectrum follows a k^3 scaling at low k : this is expected from previous analyses, both analytical [60] and numerical [30, 89], but it is in contradiction with the findings of the original sound-shell model of ref. [28], which obtains instead that, at scales larger than the peak, the GW spectrum goes as k^9 . In Sec. (IV C), we reproduce the calculation of ref. [28] and show that the k^9 behavior is recovered only when one makes an assumption for the UETC that is, however, only justified under certain conditions that do not hold in the $k \rightarrow 0$ limit. We therefore claim that the k^3 scaling is the correct one in the low- k limit.

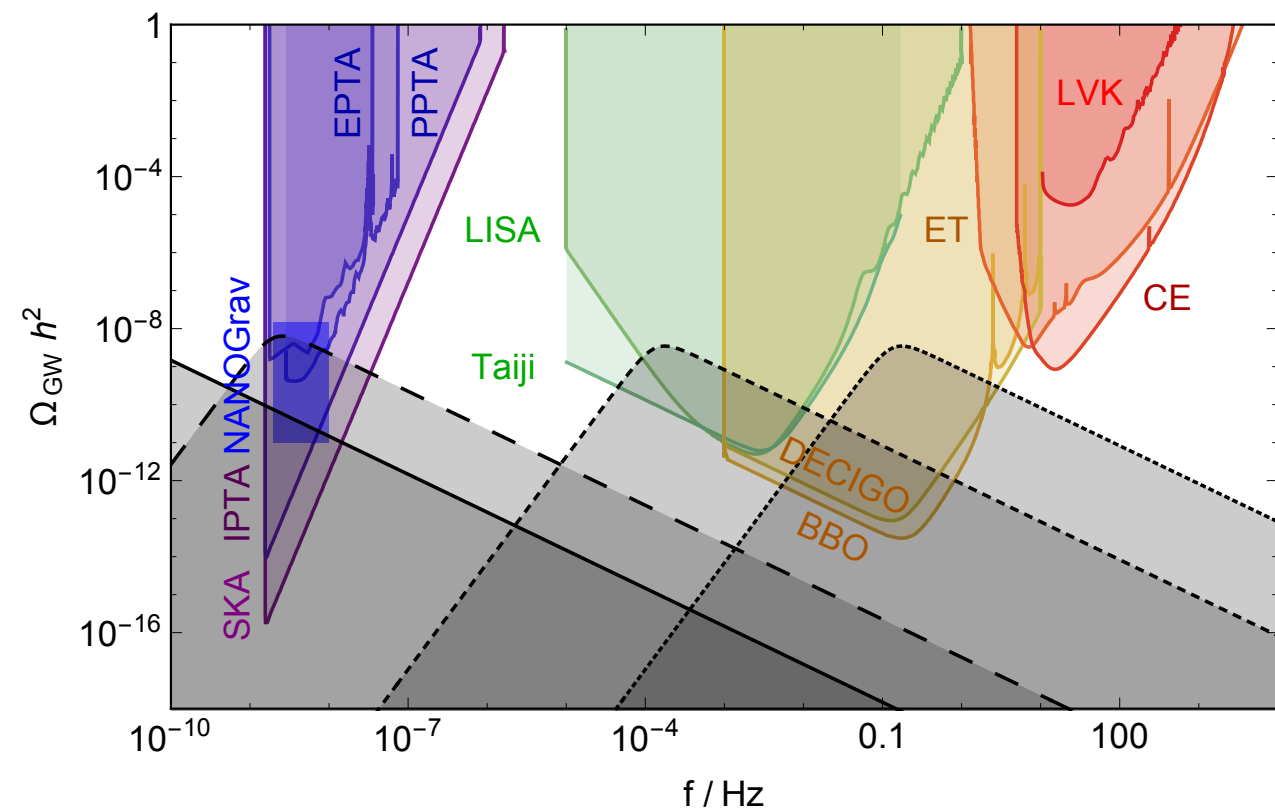
IR causal scaling law: When bubble-wall lifetime is close to sound-shell lifetime



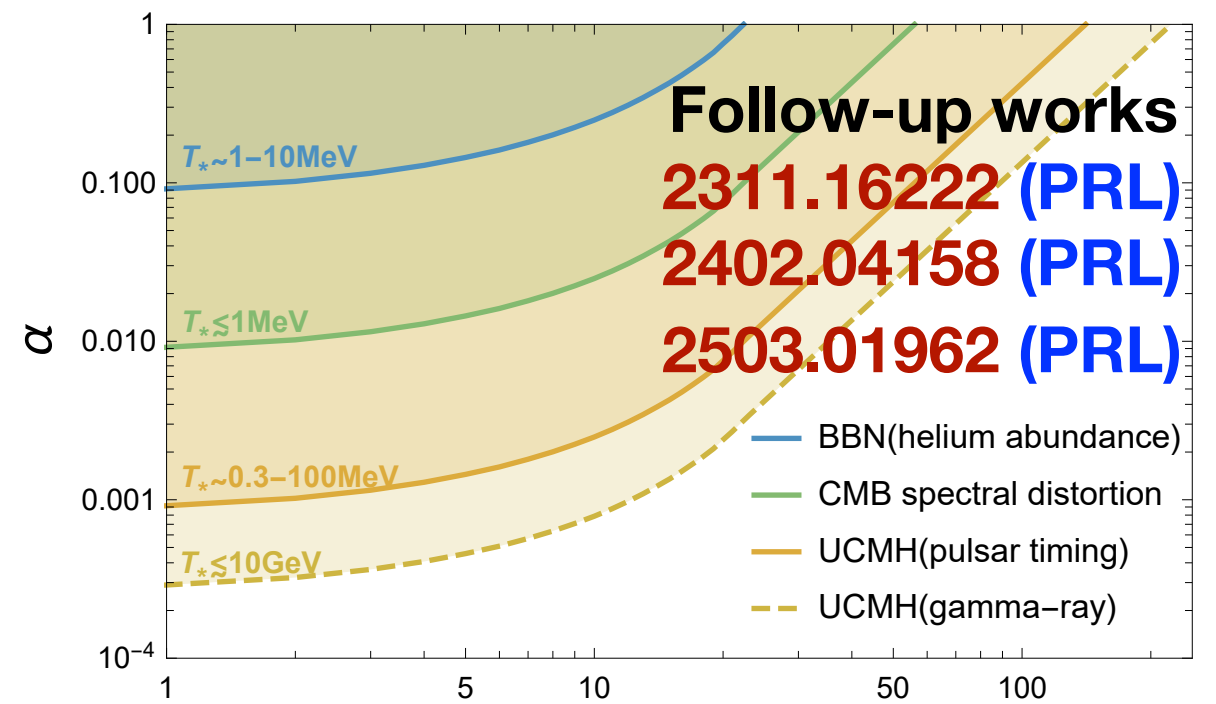
PBHs and SIGWs from FOPTs



PBHs/Curvature perturbations from FOPT



2208.14086 Liu, Bian, Cai, Guo, Wang (PRL)



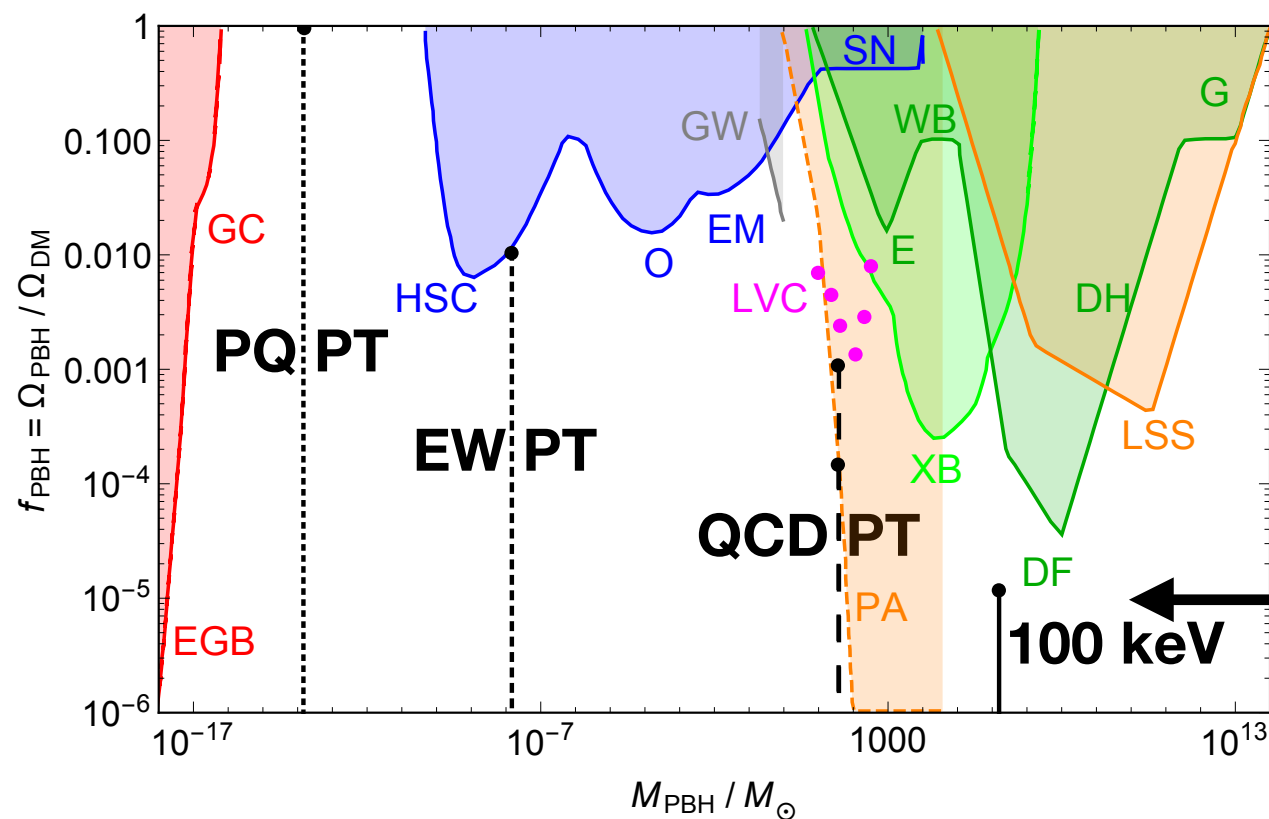
Follow-up works

2311.16222 (PRL)

2402.04158 (PRL)

2503.01962 (PRL)

2106.05637 Liu, Bian, Cai, Guo, Wang (PRD Letter)



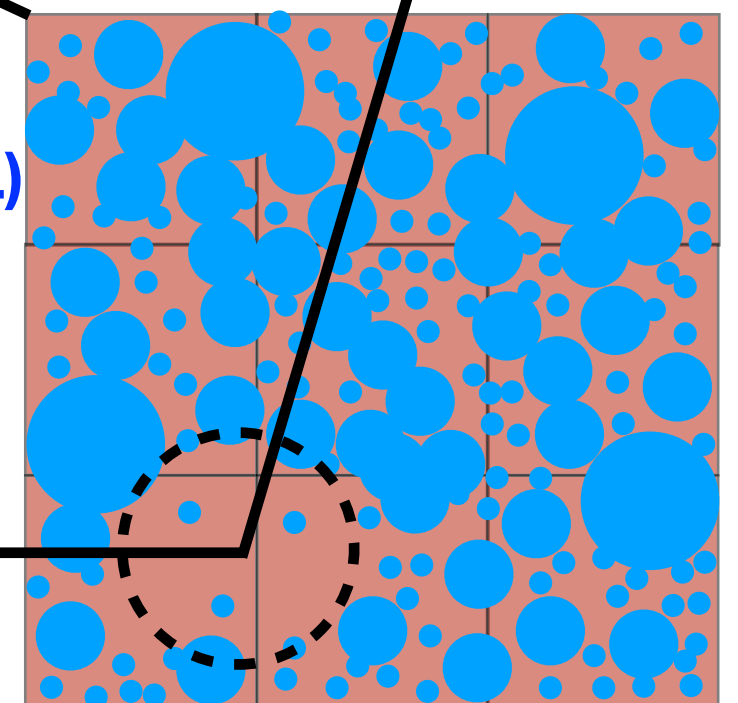
GWs

Follow-up work
2307.04239 (PRL)

PBHs

Sato, Sasaki,
Kodama, Maeda
1981 & 1982

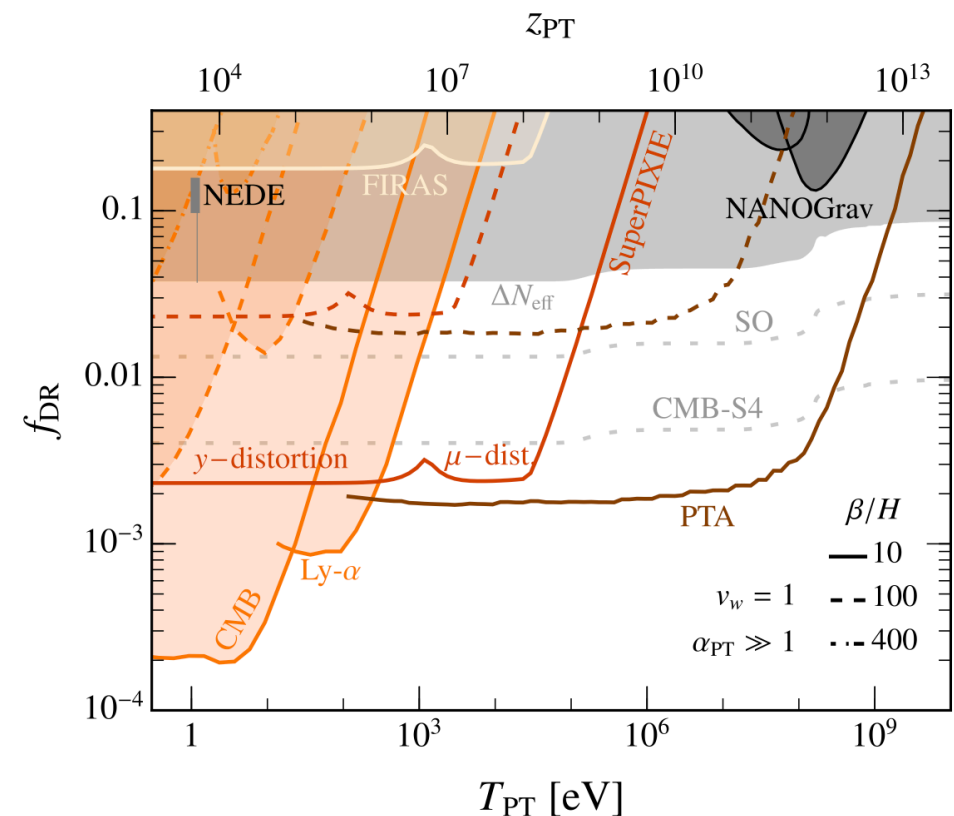
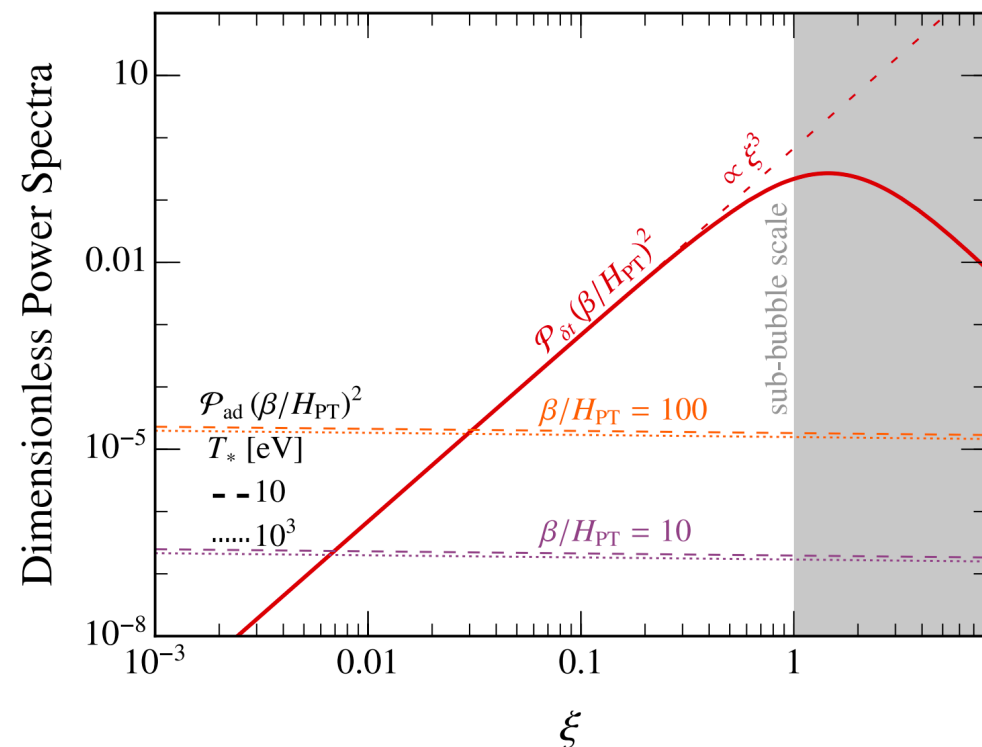
Curvature perturbations



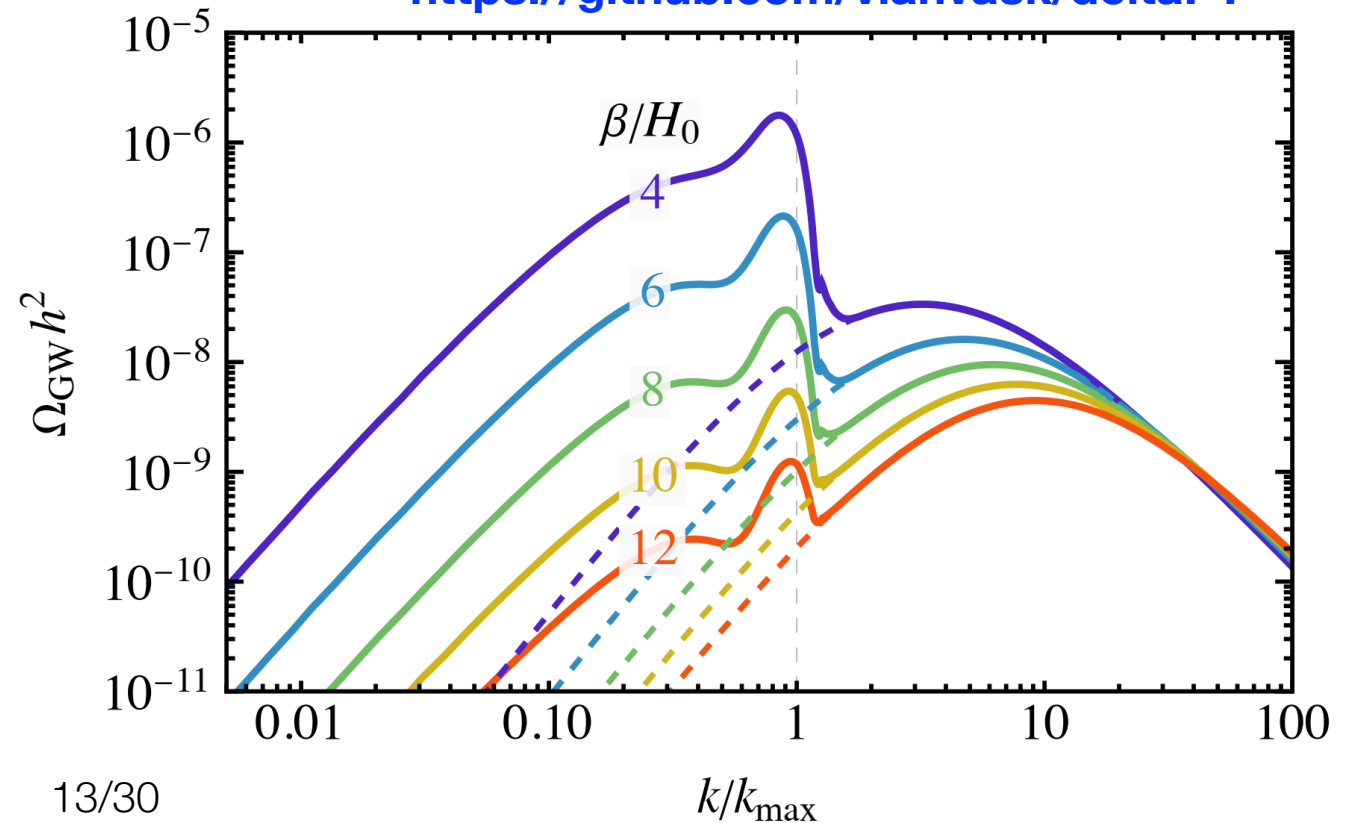
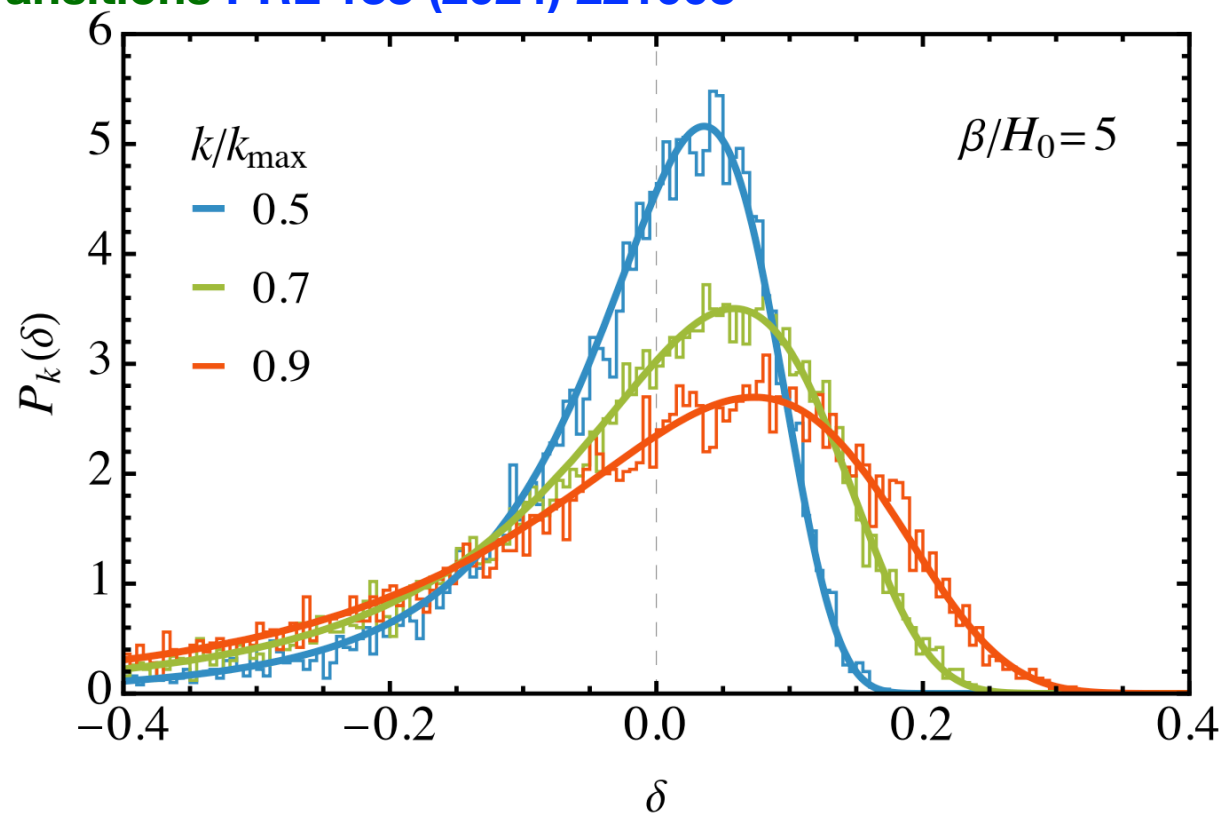
2404.06506 Cai, Hao, Wang (SCPMA)

Curvature perturbations/SIGWs from FOPT

2311.16222 Gilly Elor, Ryusuke Jinno, Soubhik Kumar, Robert McGehee, Yuhsin Tsai **Finite Bubble Statistics Constrain Late Cosmological Phase Transitions** **PRL 133 (2024) 211003**

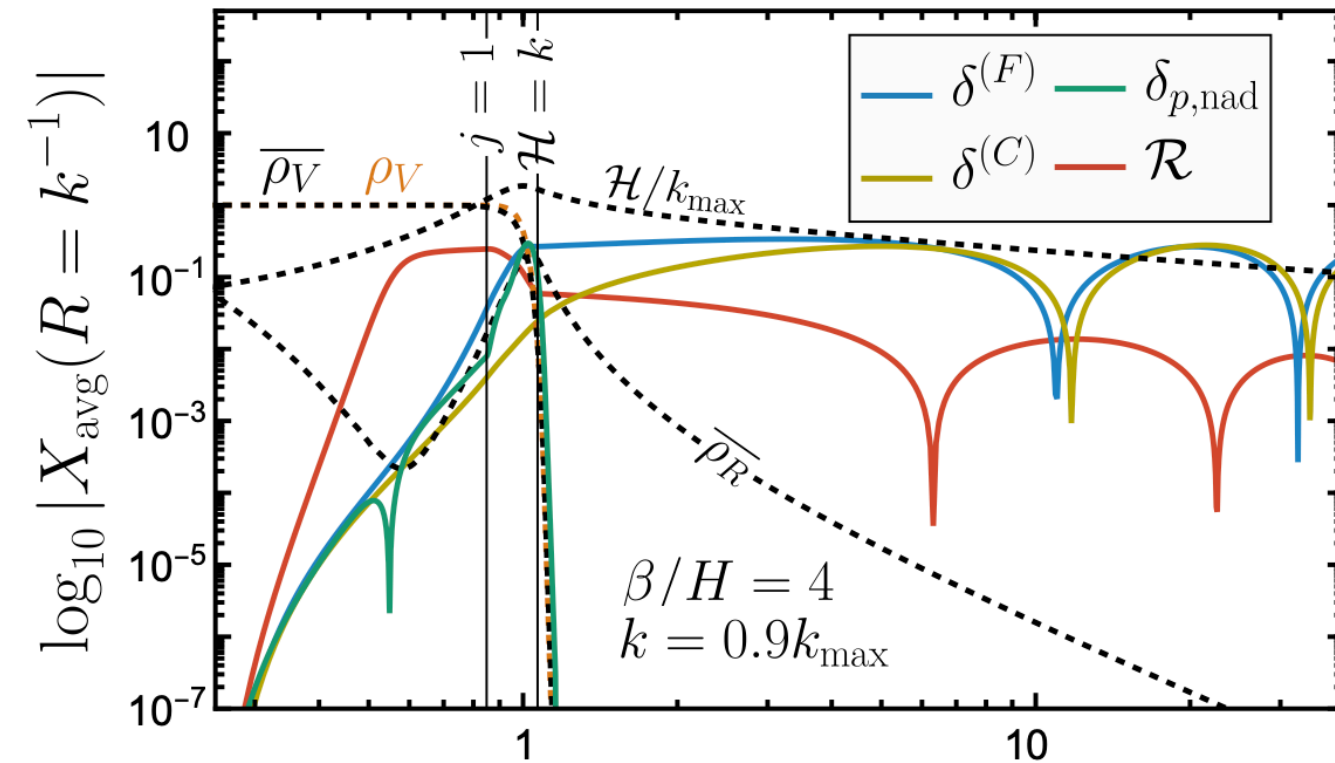


2402.04158 Marek Lewicki, Piotr Toczek, Ville Vaskonen **Black holes and gravitational waves from slow phase transitions** **PRL 133 (2024) 221003**
<https://github.com/vianvask/deltaPT>

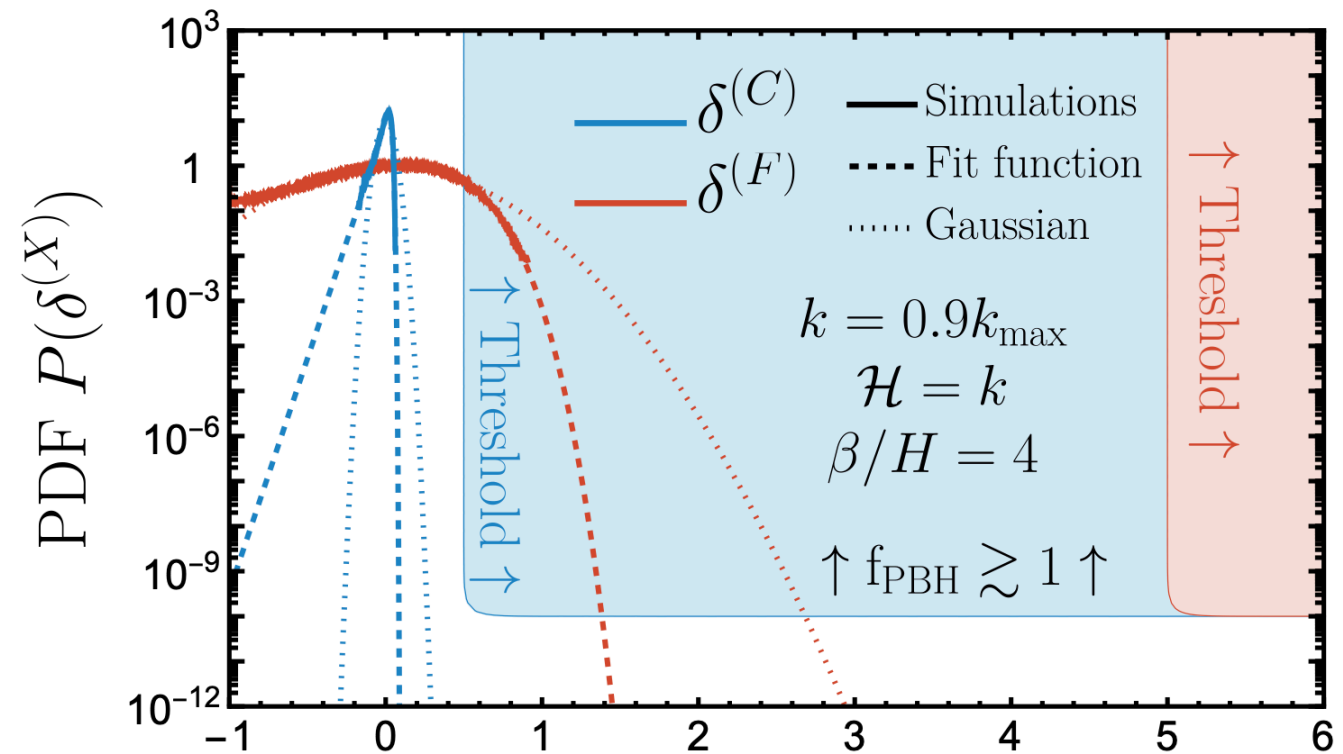


PBHs from FOPT

2503.01962 Gabriele Franciolini, Yann Gouttenoire, Ryusuke Jinno **Curvature Perturbations from First-Order Phase Transitions: Implications to Black Holes and Gravitational Waves** *PRL* 136 (2026) 171404



$$\delta^{(F)} \simeq 10\delta^{(C)} \quad t/t_{\text{max}}$$

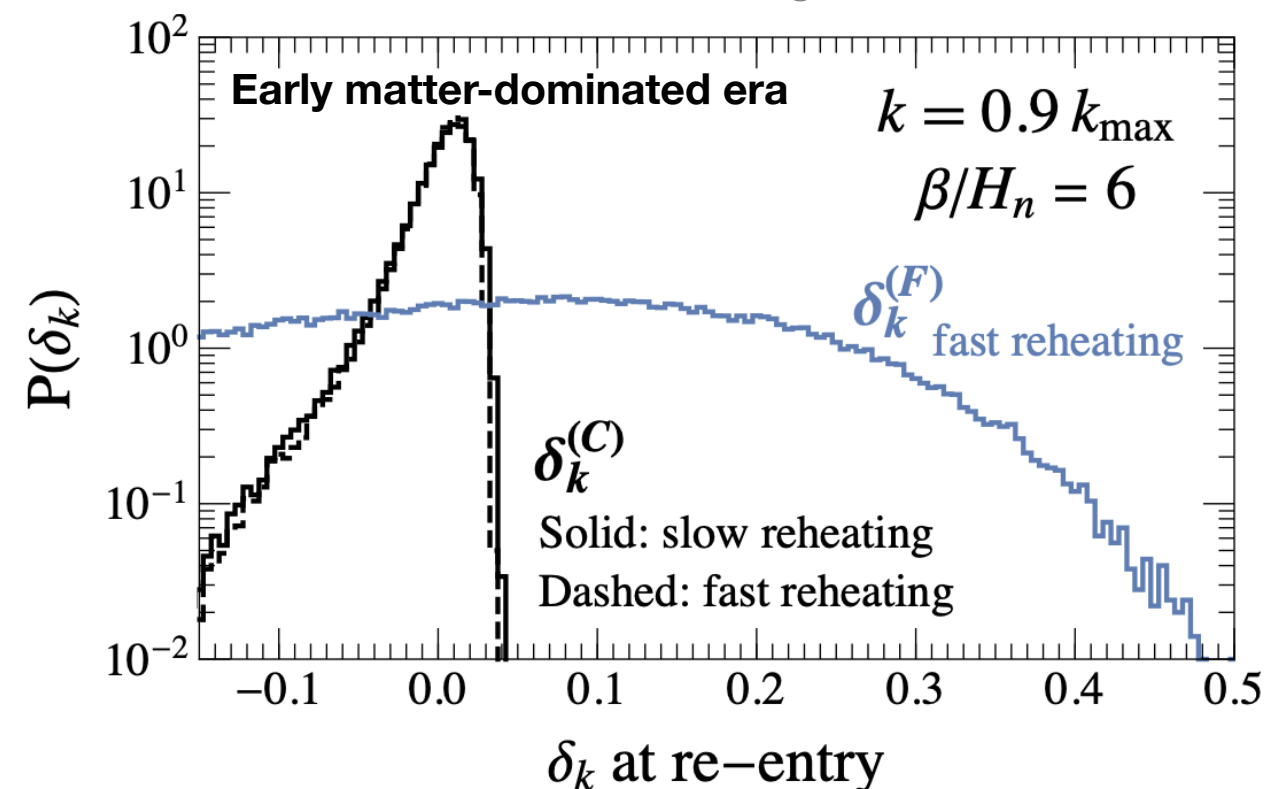


Density contrast $\delta_{\text{avg}}^{(X)}(R = k^{-1})$

2601.14412 Xiao Wang, Csaba Balázs, Ran Ding, Chi Tian, **How large are curvature perturbations from slow first-order phase transitions? A gauge-invariant analysis**

2605.11332 Wen-Yuan Ai, Ke-Pan Xie, **Reviving primordial black hole formation in slow first-order phase transitions**

2607.19469 Henda Mansour, Yann Gouttenoire, Felix Kahlhoefer, **Can the universe be matter-dominated after a supercooled first-order phase transition?**



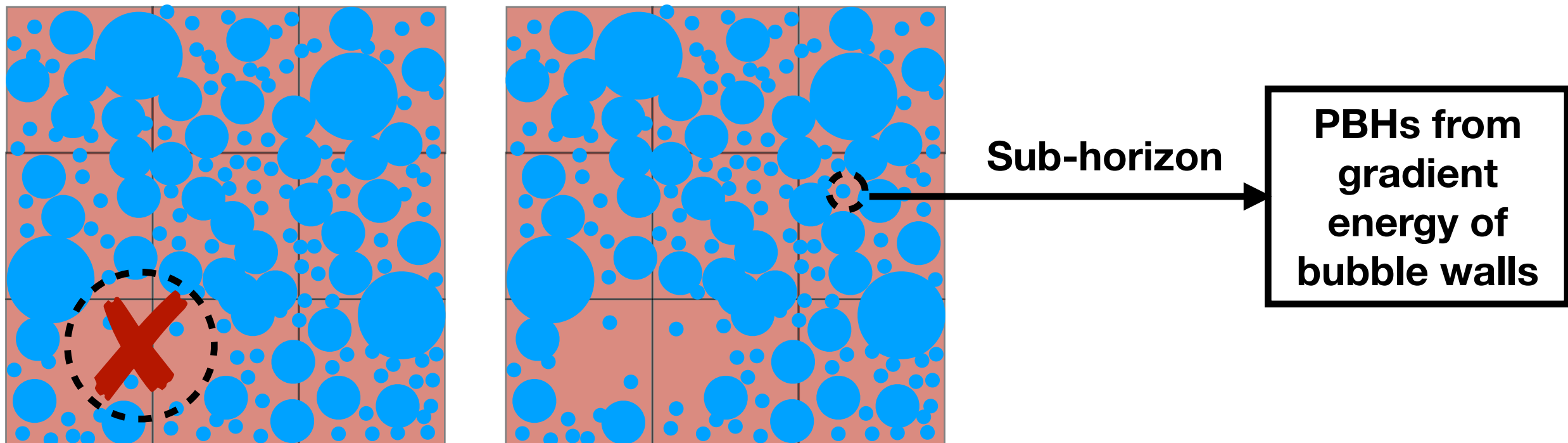
PBHs from FOPT

2601.06024 Bernard Carr, Antonio J. Iovino, Gabriele Perna, Ville Vaskonen, Hardi Veermäe,
Primordial black holes: constraints, potential evidence and prospects

Recent attention has focused on slow and strongly supercooled first-order phase transitions, including a period of thermal inflation during which the Universe remains trapped in a metastable vacuum until well below the critical temperature [119, 132–138]. In such transitions, the formation of only a few large bubbles per Hubble volume gives rise to considerable stochastic density fluctuations and the PBH formation is based on critical collapse. A subsequent work [137] found that, once the coupling between density perturbations and metric perturbations is properly accounted for, PBH formation is highly suppressed. However, accounting for the gradient energy of the bubble walls may change this result [139].

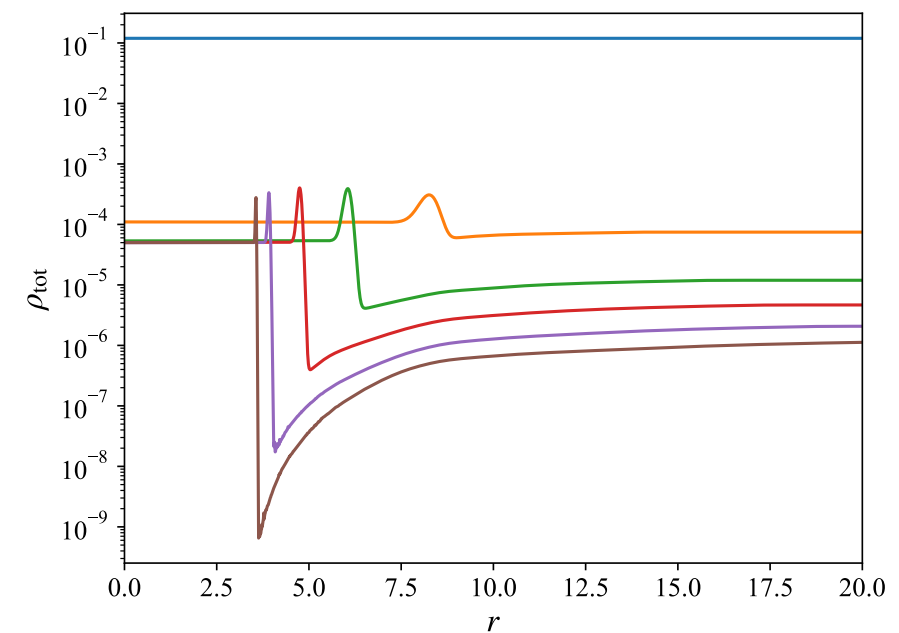
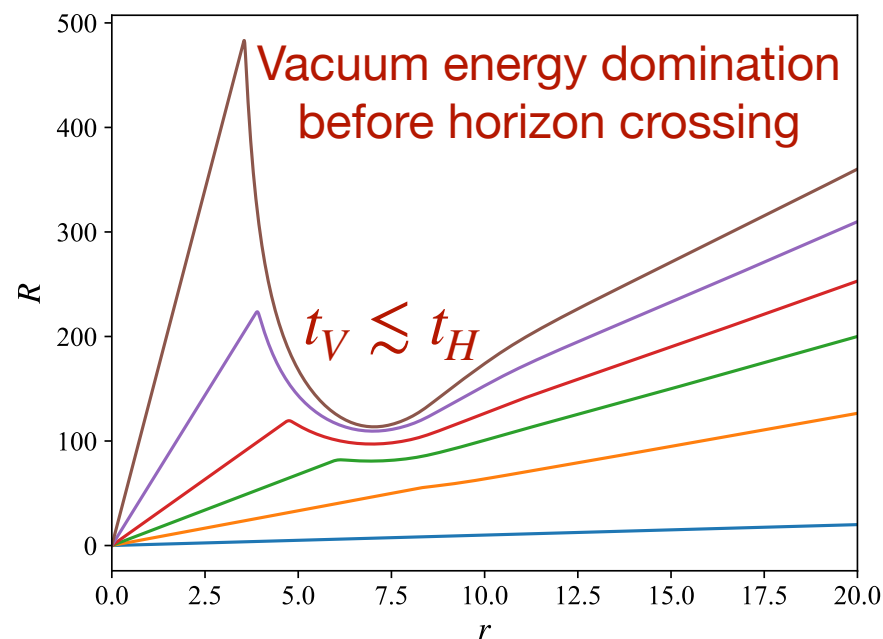
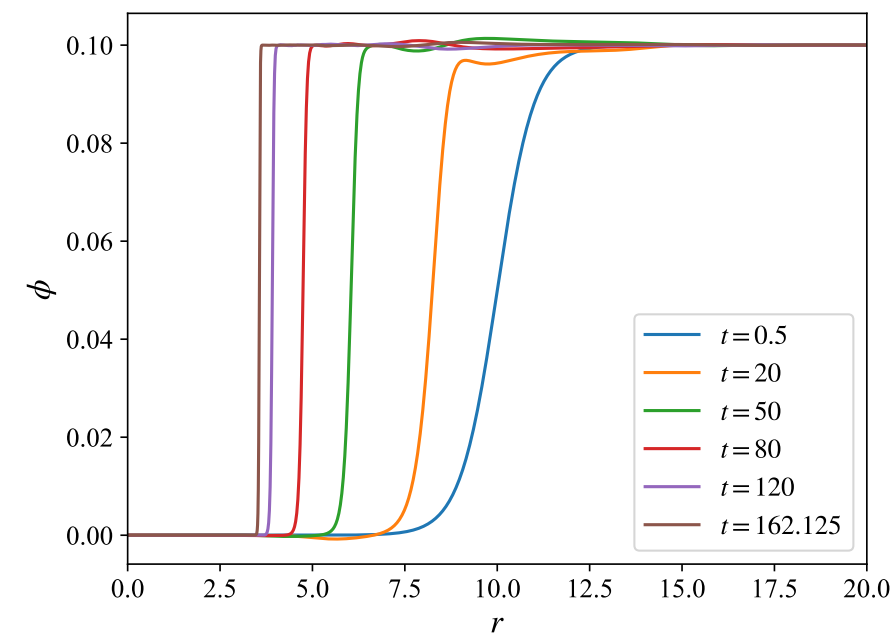
[139] Y. Gouttenoire, M. Lewicki, P. Toczek, and V. Vaskonen, Resurrecting PBH formation in first-order phase transitions, (2026), to appear.

2402.13341 Marcos M. Flores, Alexander Kusenko, and Misao Sasaki, **Revisiting formation of primordial black holes in a supercooled first-order phase transition**, **PRD 110 (2024) 1, 015005**

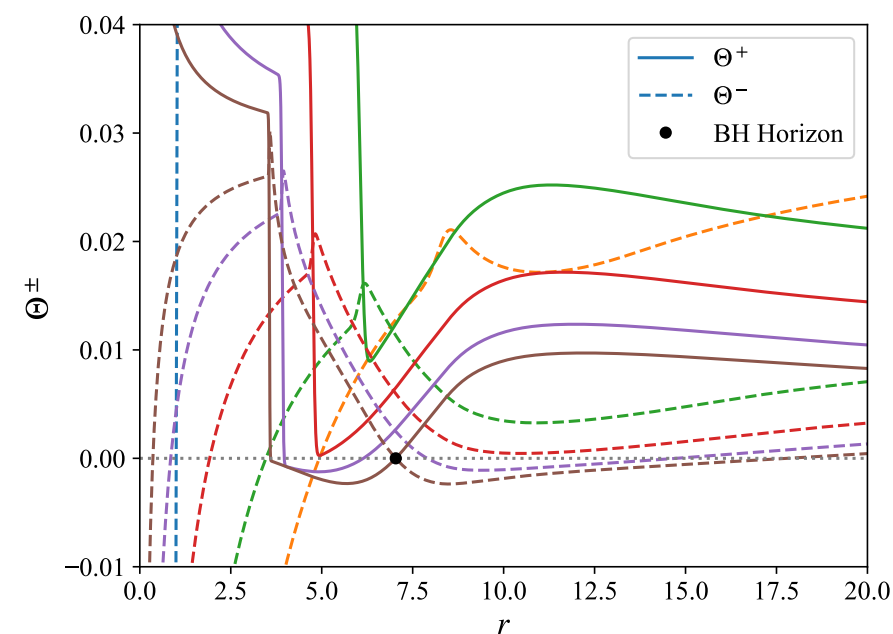


Type-B supercritical PBHs

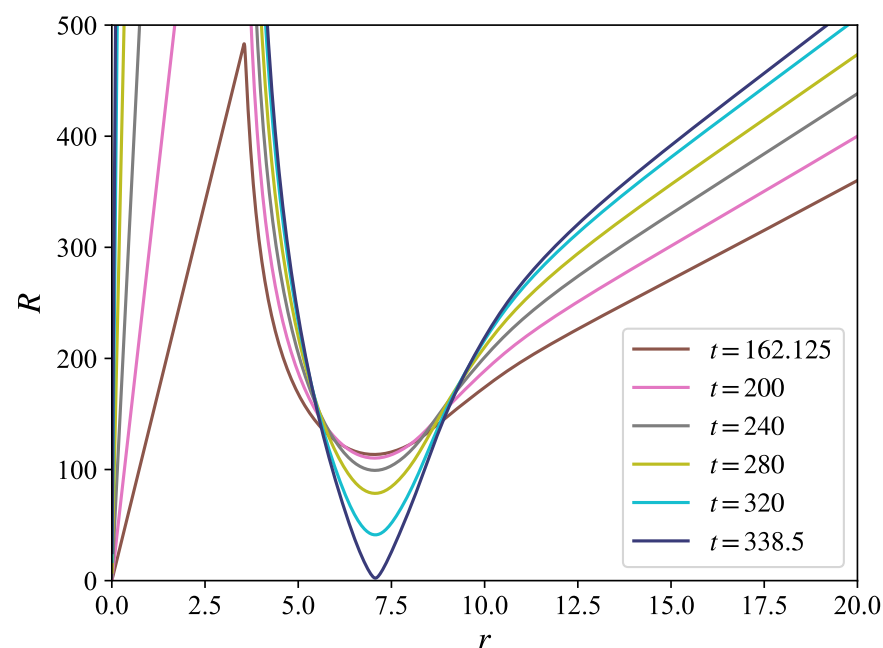
2601.21878 Zhuan Ning, Xiang-Xia Zeng, Rong-Gen Cai, Shao-Jiang Wang, **Numerical simulations of primordial black hole formation via delayed first-order phase transitions**



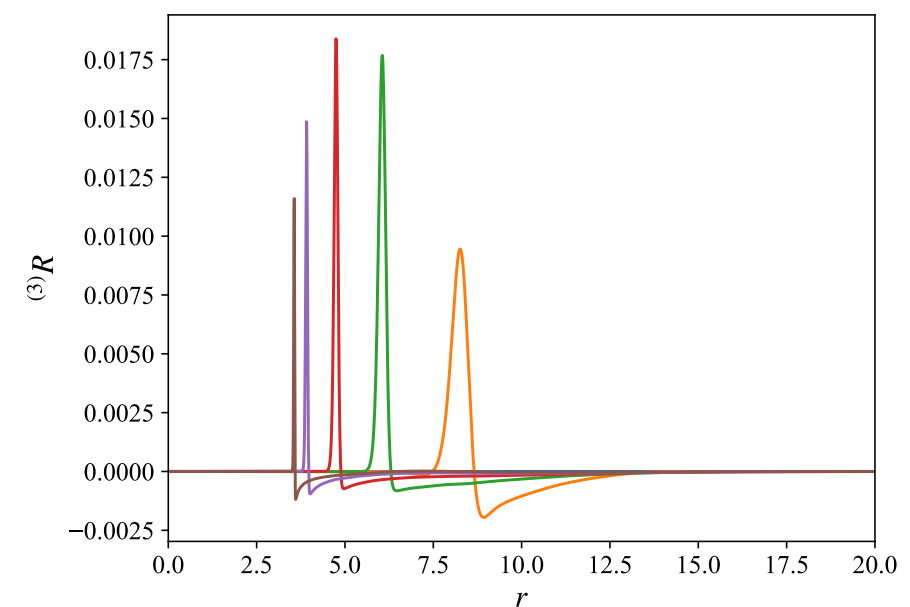
FVD shrink



Inflation inside



Overdensity



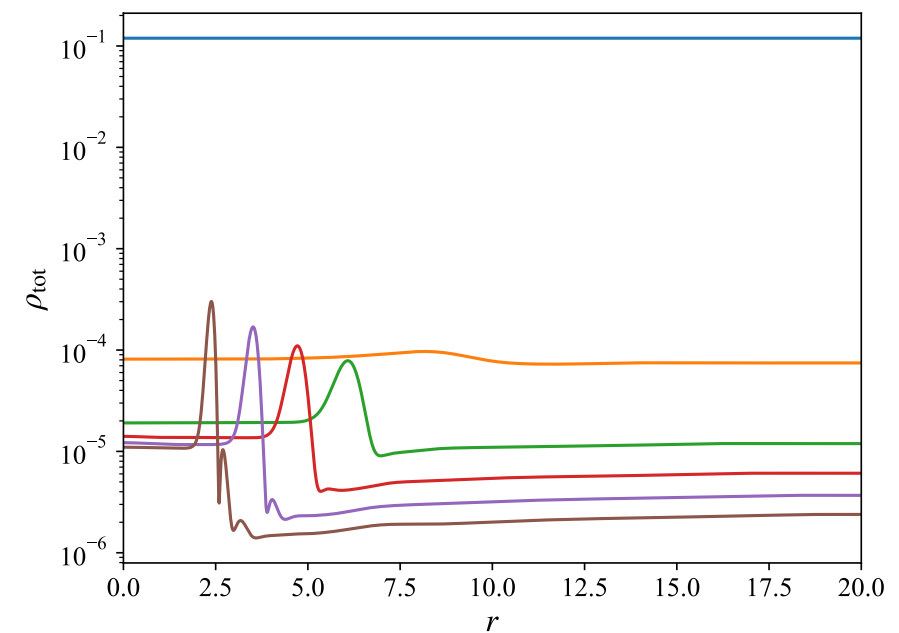
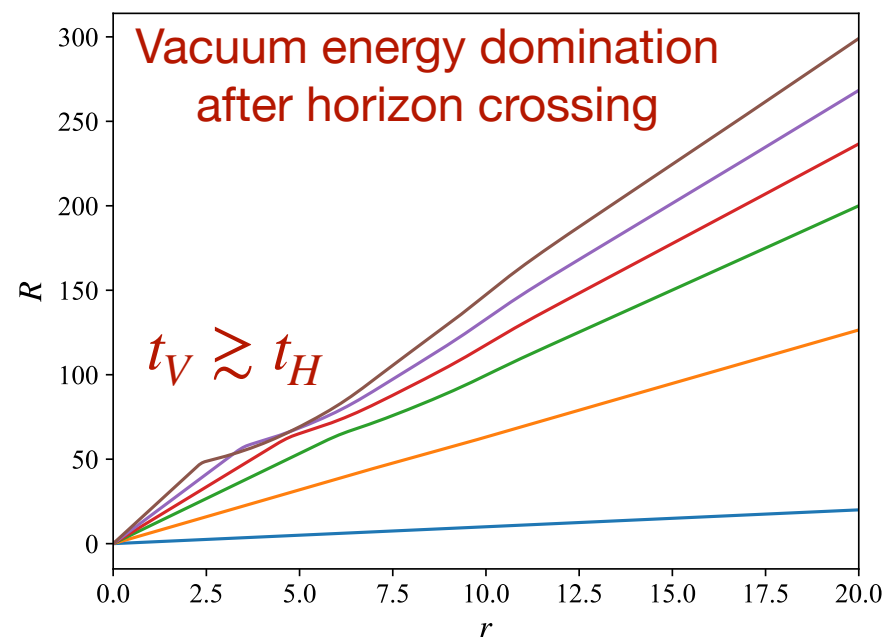
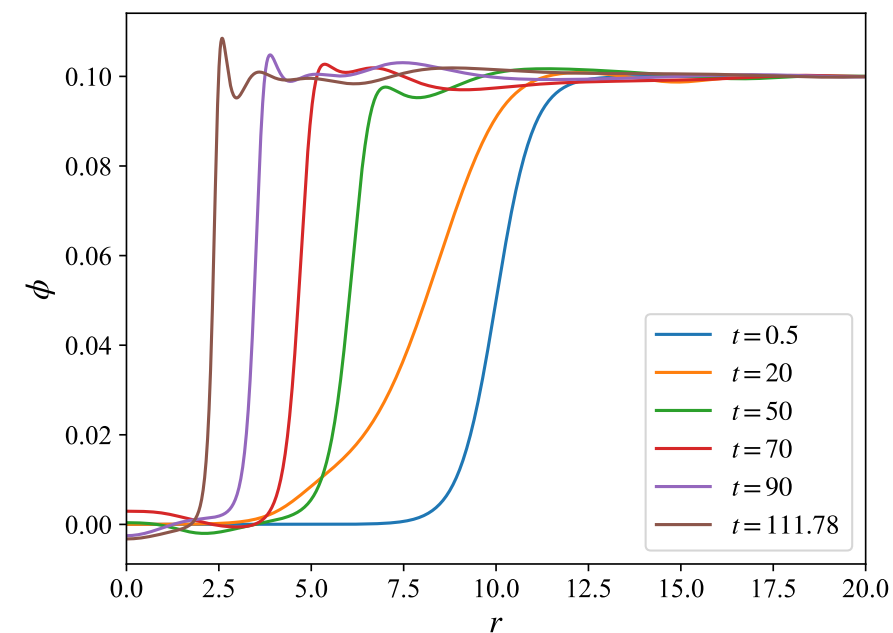
bifurcating trapping horizon

Wormhole pinched off

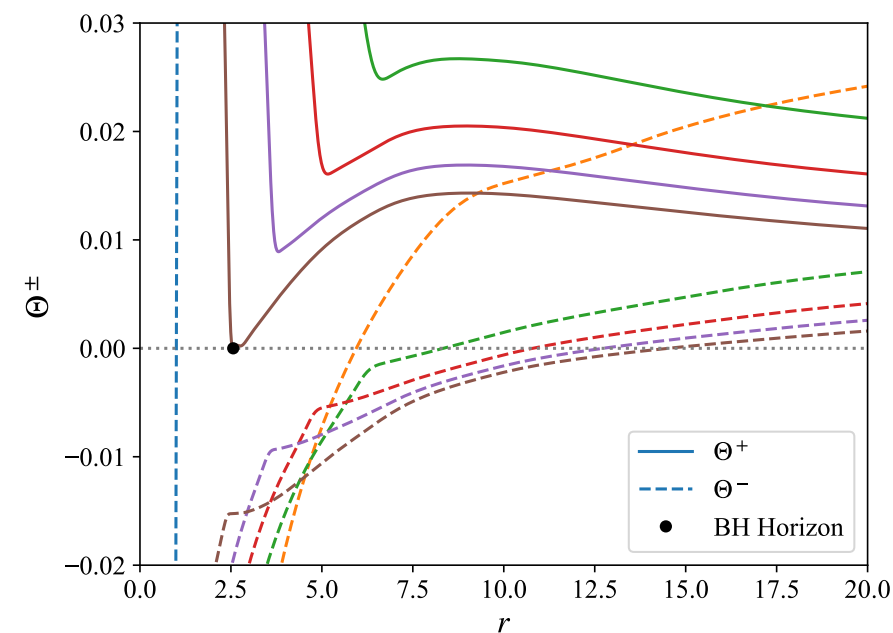
Local spatial curvature

Type A subcritical PBHs

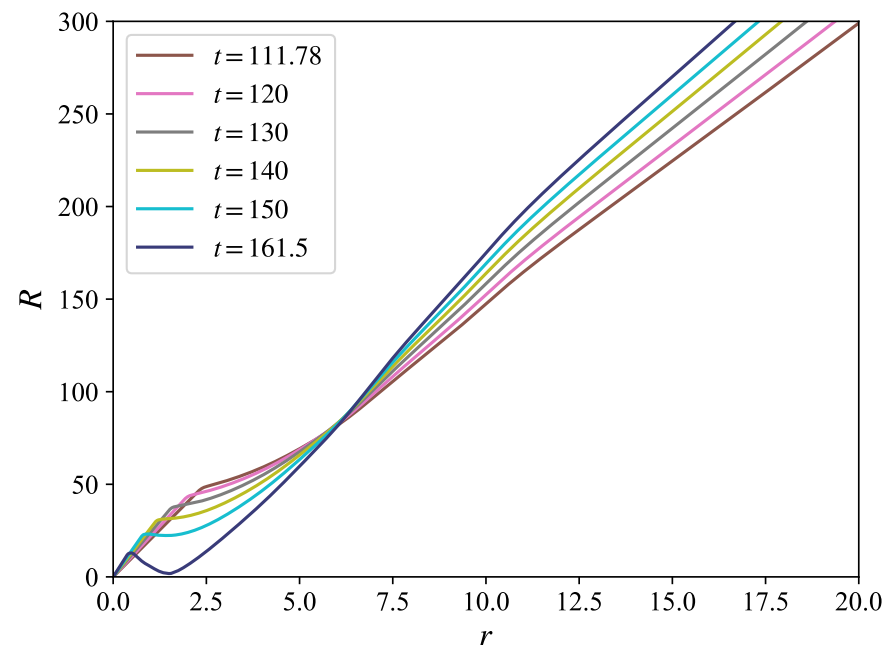
2601.21878 Zhuan Ning, Xiang-Xia Zeng, Rong-Gen Cai, Shao-Jiang Wang, **Numerical simulations of primordial black hole formation via delayed first-order phase transitions**



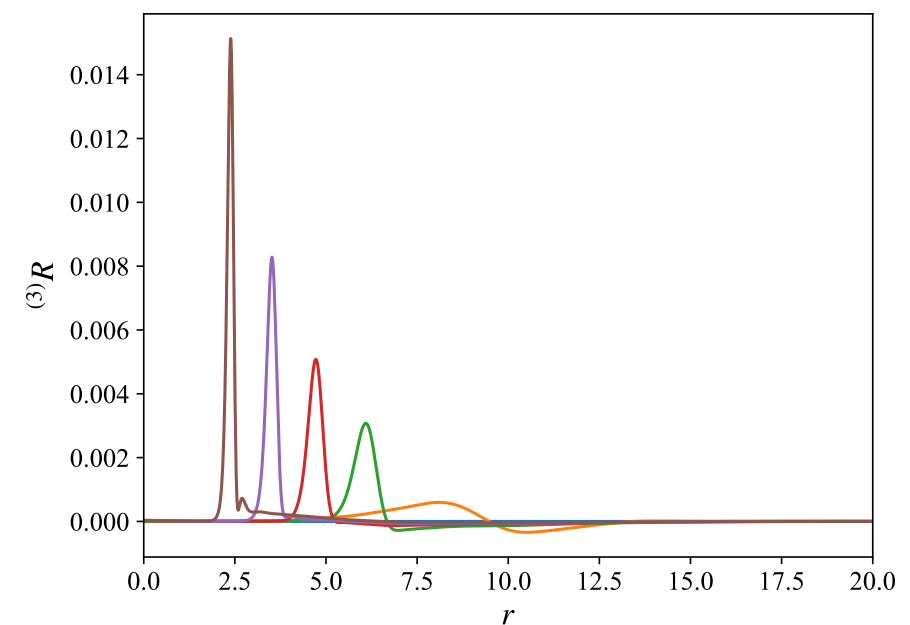
FVD shrink



no-inflation inside



Wall-energy concentration



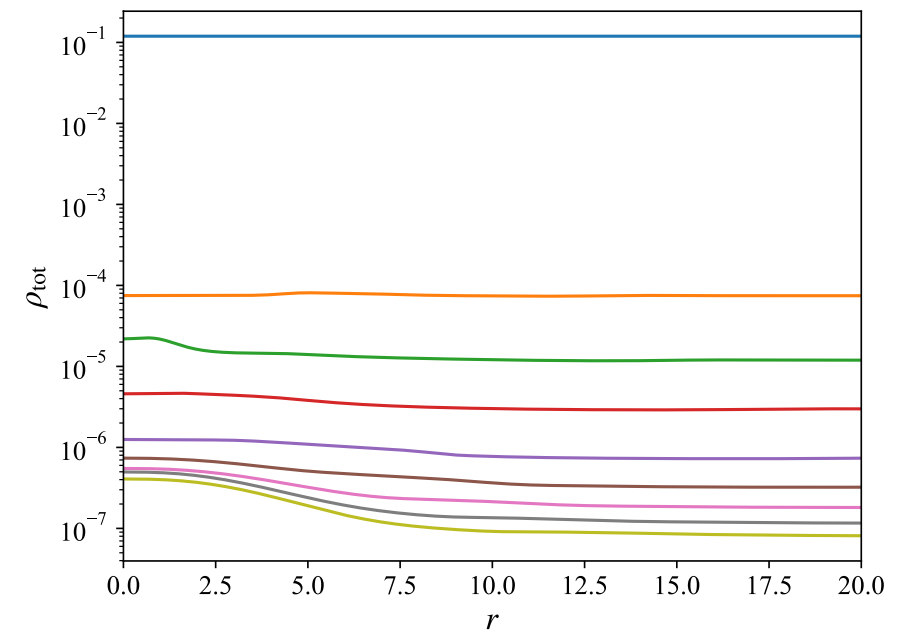
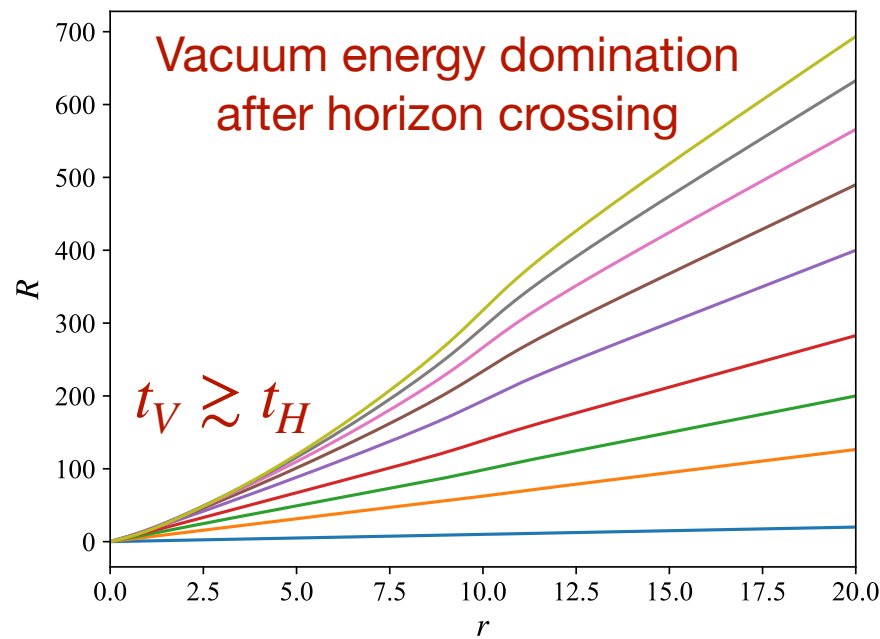
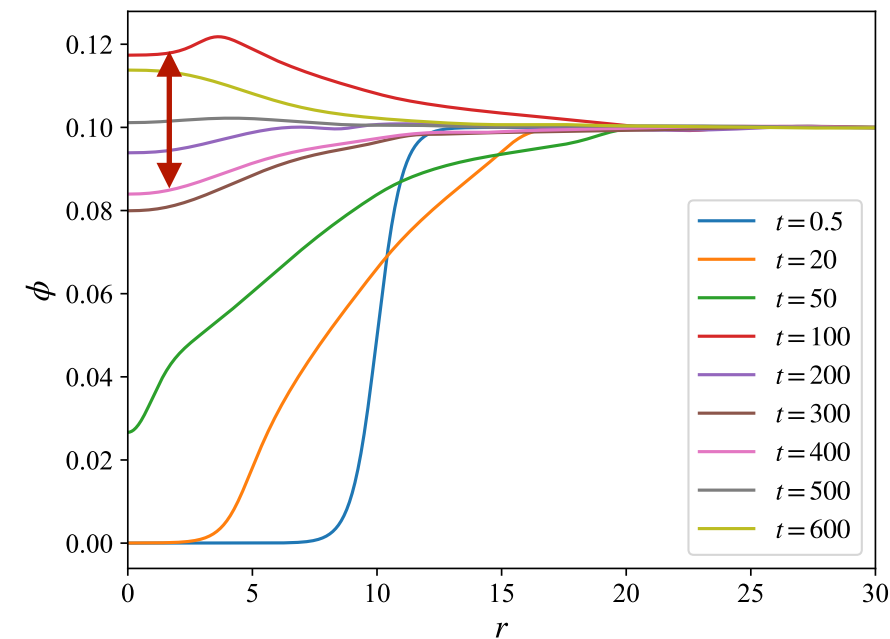
Apparent horizon

BH singularity

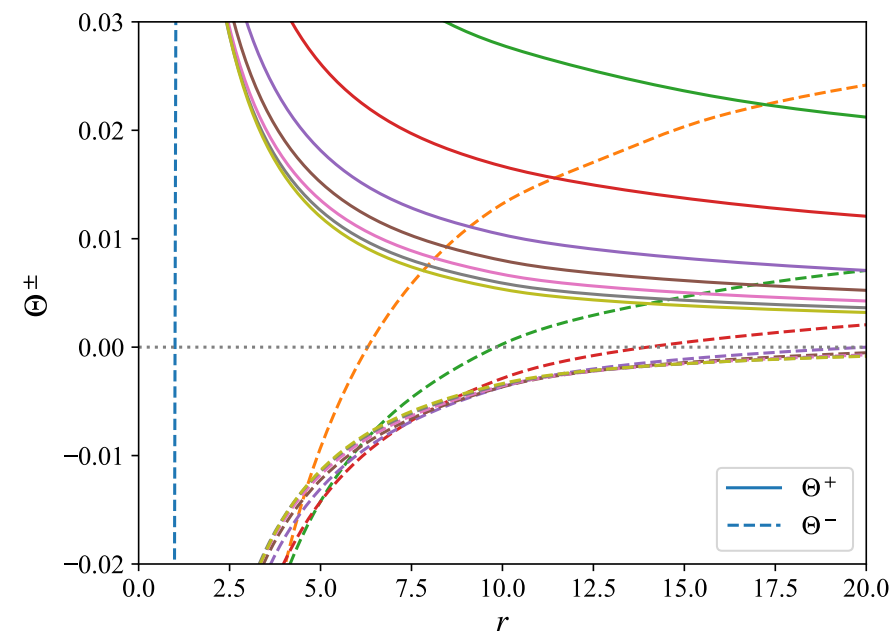
Local spatial curvature

No PBH (Dispersal)

2601.21878 Zhuan Ning, Xiang-Xia Zeng, Rong-Gen Cai, Shao-Jiang Wang, **Numerical simulations of primordial black hole formation via delayed first-order phase transitions**



Oscillate about true vacuum



no trapped surface

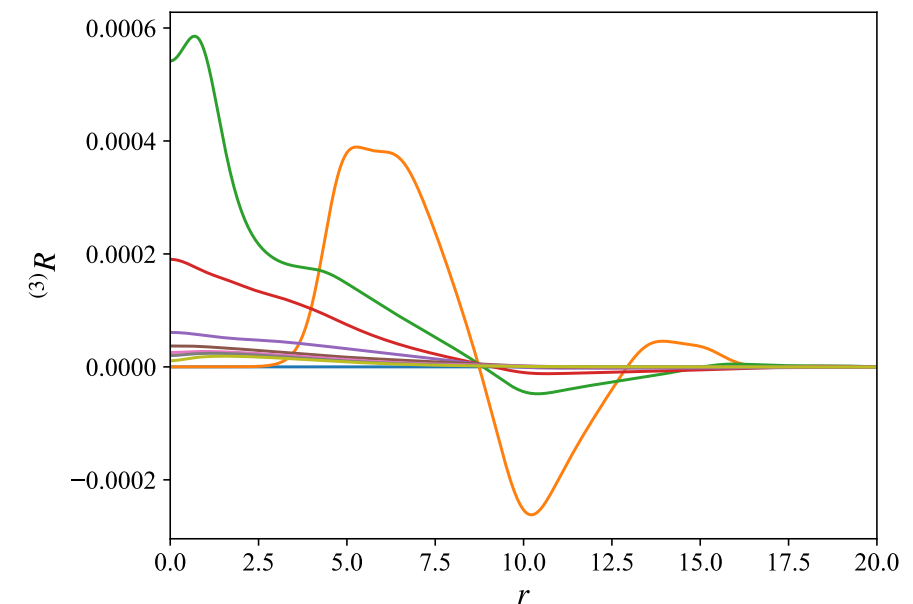
monotonical

Highlight

- 1. Initial condition**
- 2. Metric perturbation**
- 3. Geodesic gauge**

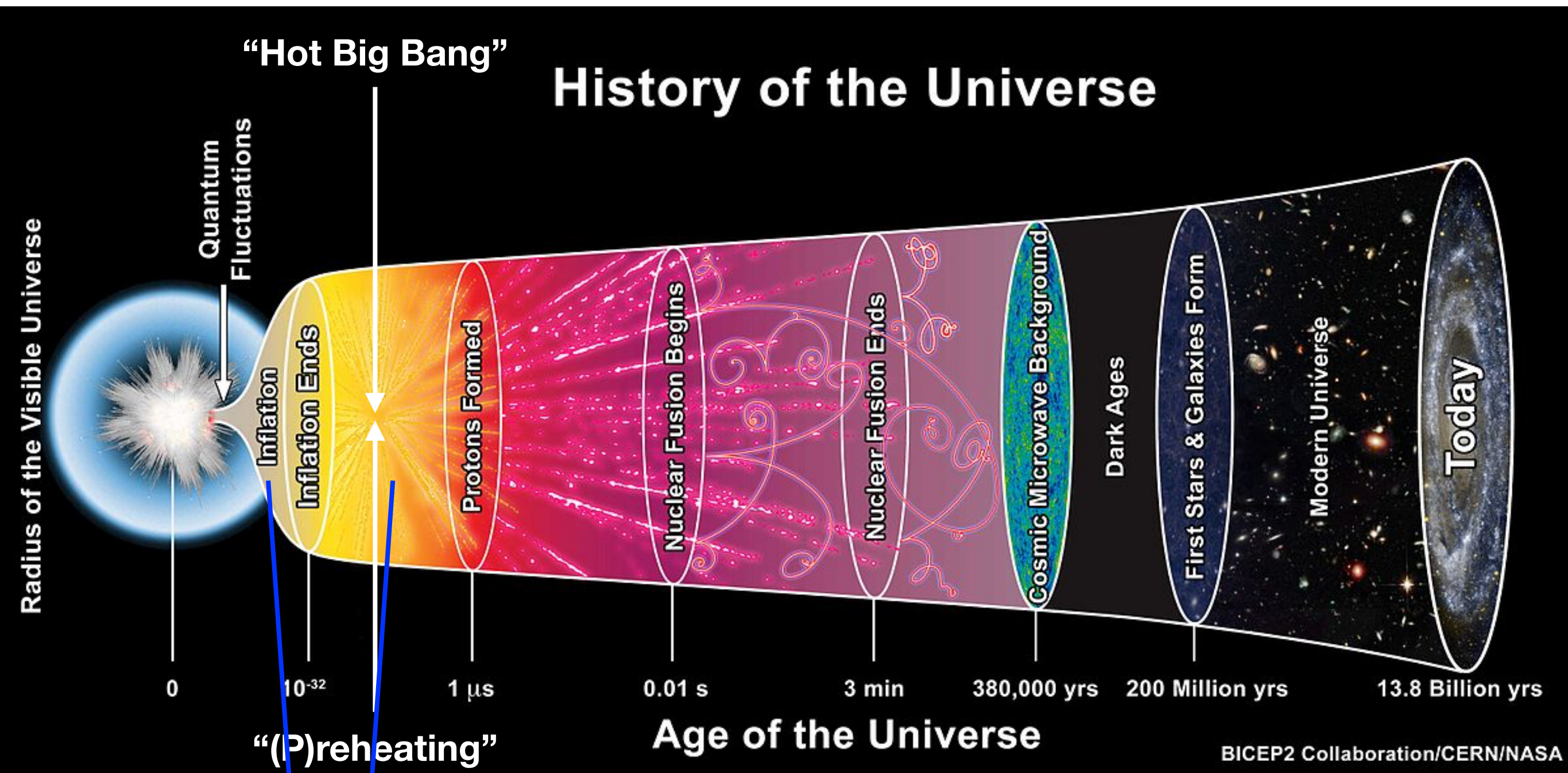
Need further investigation

small density contrast



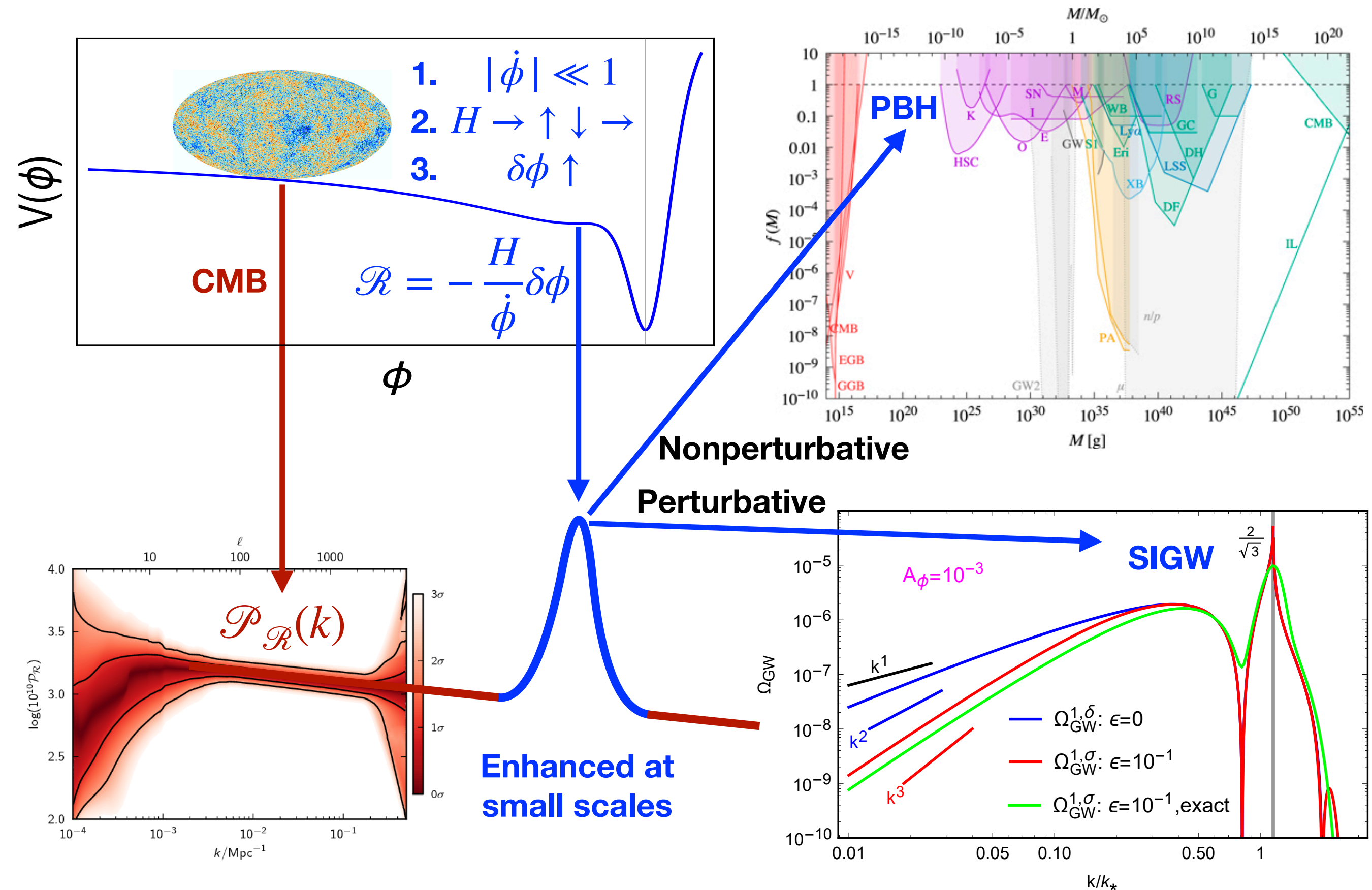
relax back toward zero

SIGWs from inflation



Induced GWs

Induced GWs



All-order non-Gaussianity

2508.10812 Xiang-Xi Zeng, Zhuan Ning, Rong-Gen Cai, Shao-Jiang Wang,
Scalar-induced gravitational waves with non-Gaussianity up to all orders

SimuSIGW: <https://github.com/ZengXiang-Xi/SimuSIGW/tree/main>

Standard approach: $\zeta = \zeta_g + F_{\text{NL}}\zeta_g^2 + G_{\text{NL}}\zeta_g^3 + H_{\text{NL}}\zeta_g^4 + \dots$ but high-dim integral is hard

SIGW: $ds^2 = a^2(\eta) \left\{ -(1 + 2\Phi)d\eta^2 + \left[(1 - 2\Phi)\delta_{ij} + \frac{1}{2}h_{ij} \right] dx^i dx^j \right\}$

$$\Omega_{\text{GW}} = \frac{\rho_{\text{GW}}}{\rho_c(\eta)} = \frac{1}{48\mathcal{H}^2} \langle h'_{ij} h'_{ij} \rangle \quad h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = -4S_{ij}^{\text{TT}}$$

$$S_{ij} = 4\Phi\partial_i\partial_j\Phi + 2\partial_i\Phi\partial_j\Phi - \frac{4}{3(1+\omega)\mathcal{H}^2}\partial_i(\Phi' + \mathcal{H}\Phi)\partial_j(\Phi' + \mathcal{H}\Phi)$$

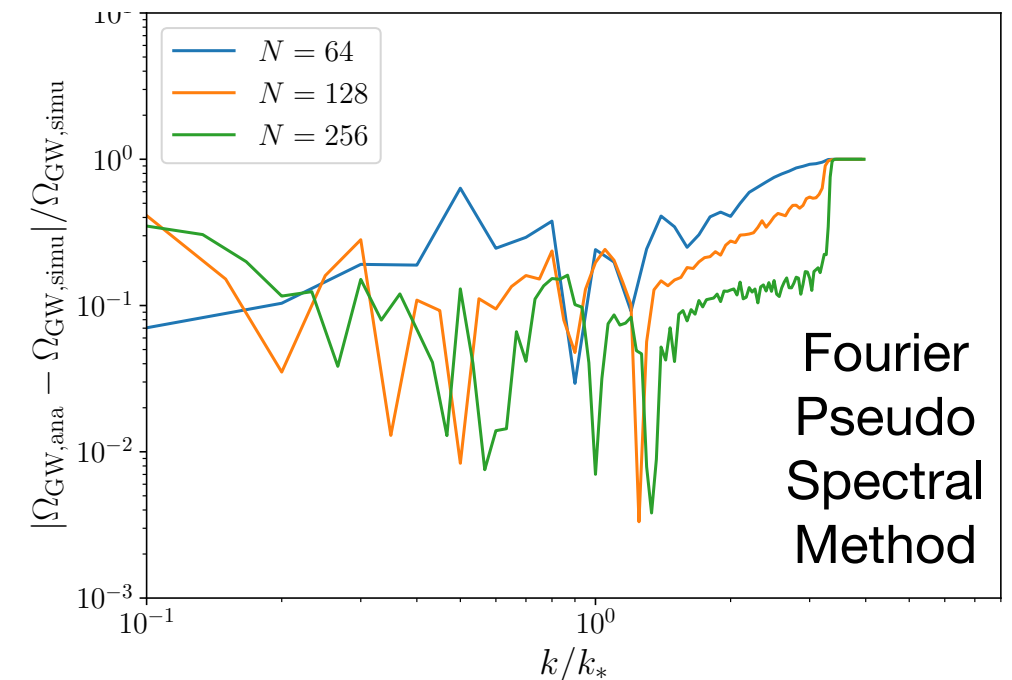
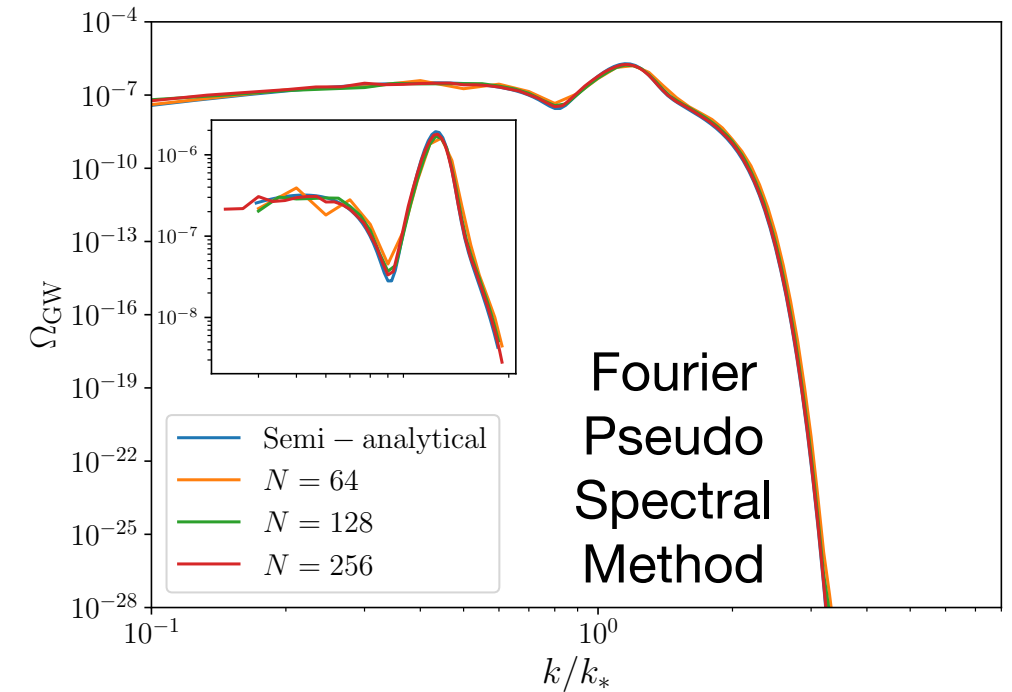
$$\Phi'' + 3\mathcal{H}(1 + c_s^2)\Phi' + (2\mathcal{H}' + (1 + 3c_s^2)\mathcal{H}^2 - c_s^2\nabla^2)\Phi = 0$$

Real-space simulation: $\Phi_i = \frac{3 + 3\omega}{5 + 3\omega}\zeta_i$ and $\zeta = F[\zeta_g]$

$$\zeta_g(\vec{x}) = \int \frac{d^3x}{(2\pi)^3} \zeta_g(\vec{k}) e^{i\vec{k}\cdot\vec{x}} \quad \mathcal{P}[\zeta_g(\vec{k})] = \frac{1}{\pi\sigma_k^2} \exp \left[-\frac{|\zeta_g(\vec{k})|^2}{\sigma_k^2} \right]$$

$$\langle \zeta_g(\vec{k}) \zeta_g(\vec{q}) \rangle = \sigma_k^2 \delta^3(\vec{k} + \vec{q}) \quad \sigma_k^2 = (2\pi)^3 P_{\zeta_g}(k)$$

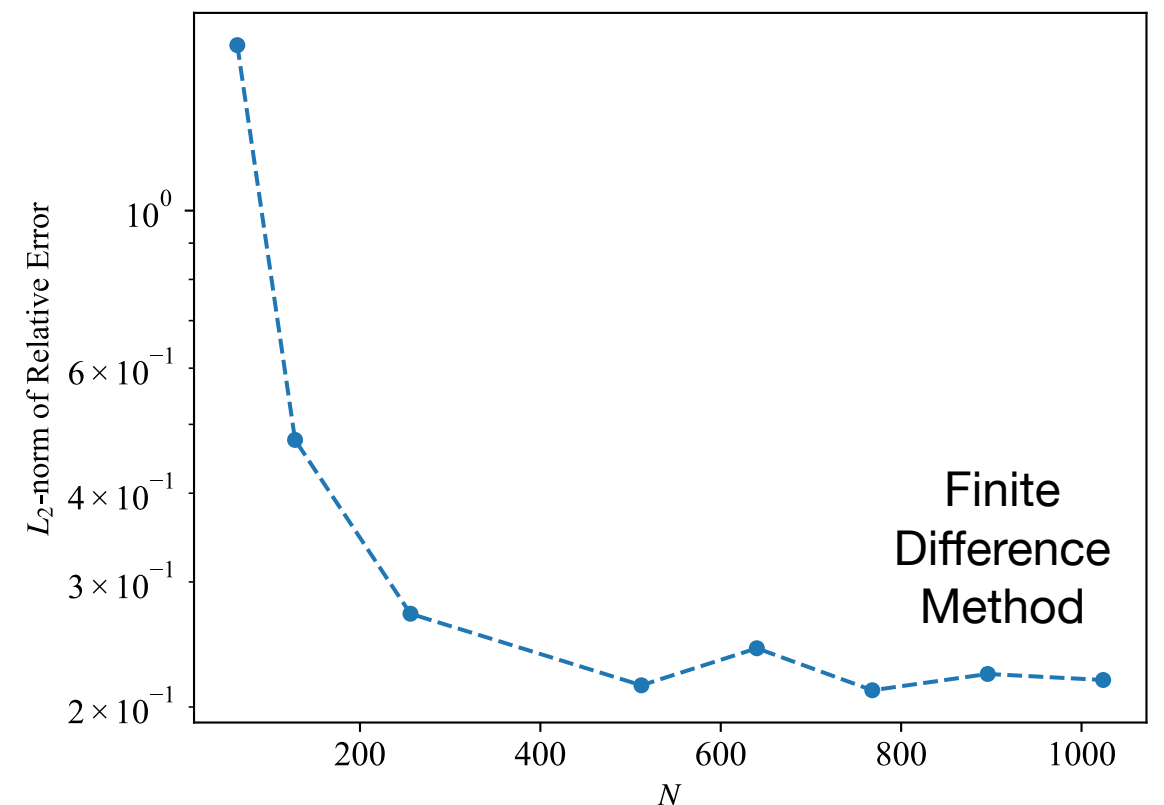
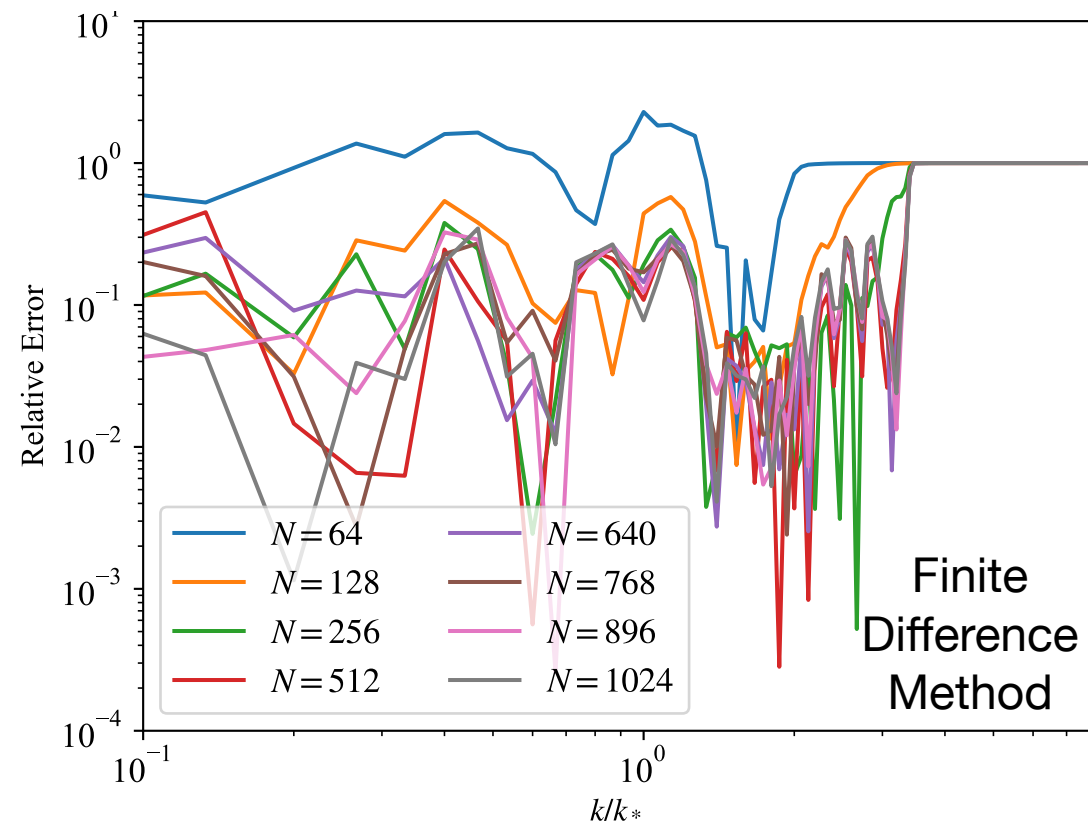
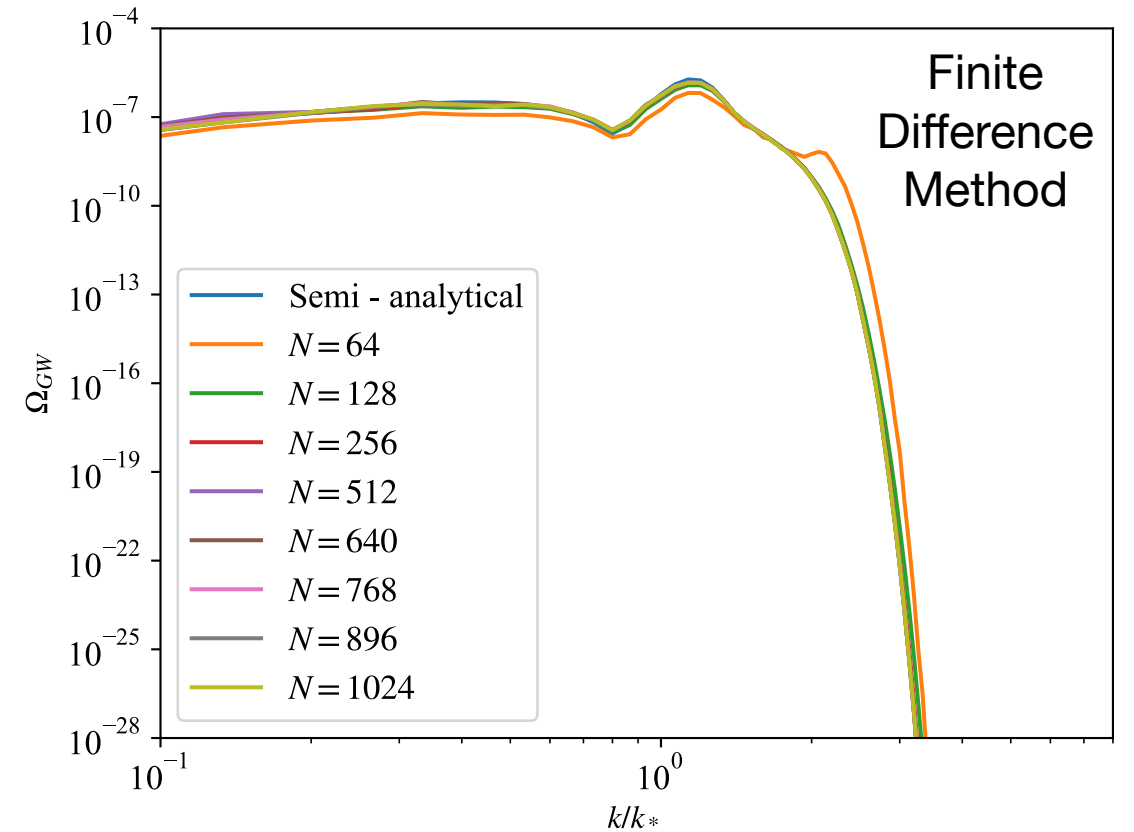
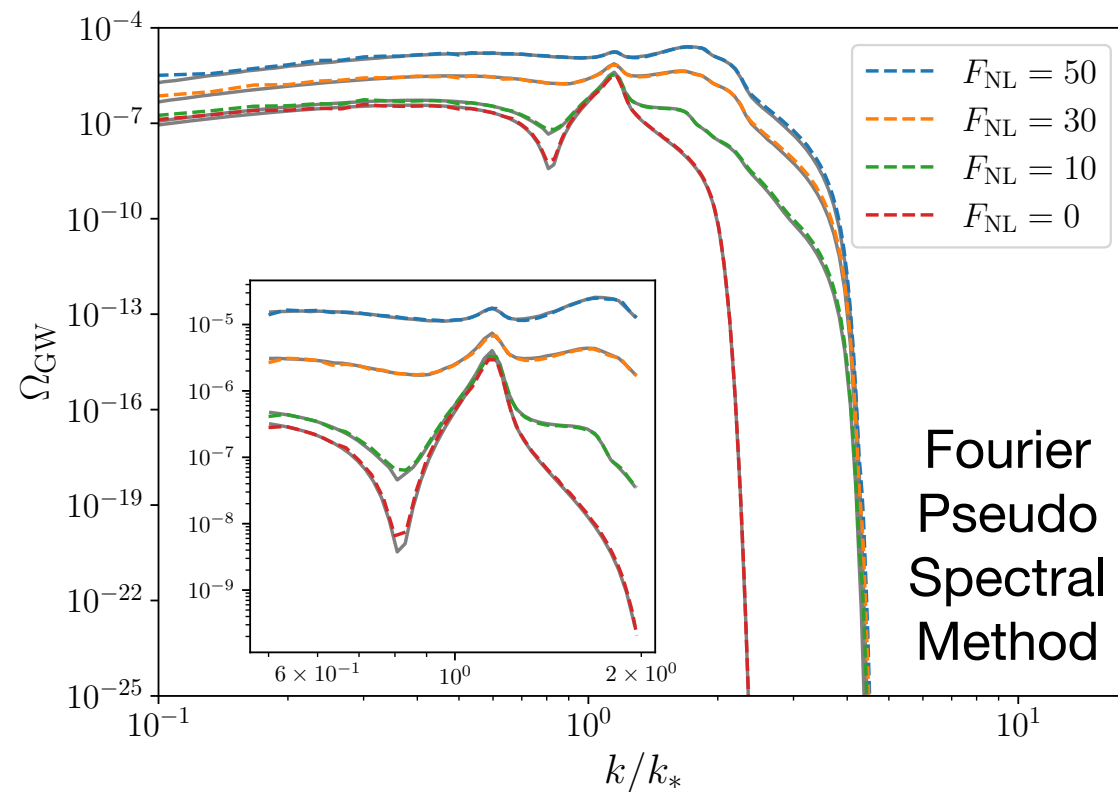
Test: $\mathcal{P}_{\zeta_g}(k) = A \frac{(k/k_*)^3}{\sqrt{2\pi}e} \exp \left[-\frac{(k/k_* - 1)^2}{2e^2} \right]$ $\zeta = \zeta_g + F_{\text{NL}}(\zeta_g^2 - \langle \zeta_g^2 \rangle)$



All-order non-Gaussianity

2508.10812 Xiang-Xi Zeng, Zhuan Ning, Rong-Gen Cai, Shao-Jiang Wang,
Scalar-induced gravitational waves with non-Gaussianity up to all orders

SimuSIGW: <https://github.com/ZengXiang-Xi/SimuSIGW/tree/main>

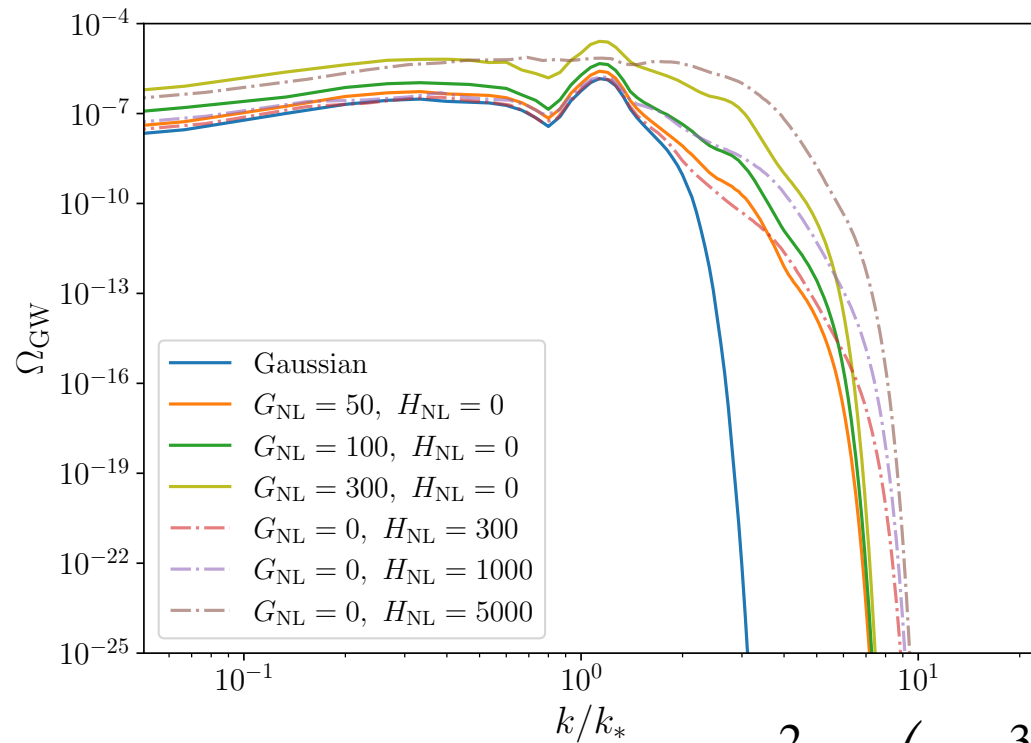


All-order non-Gaussianity

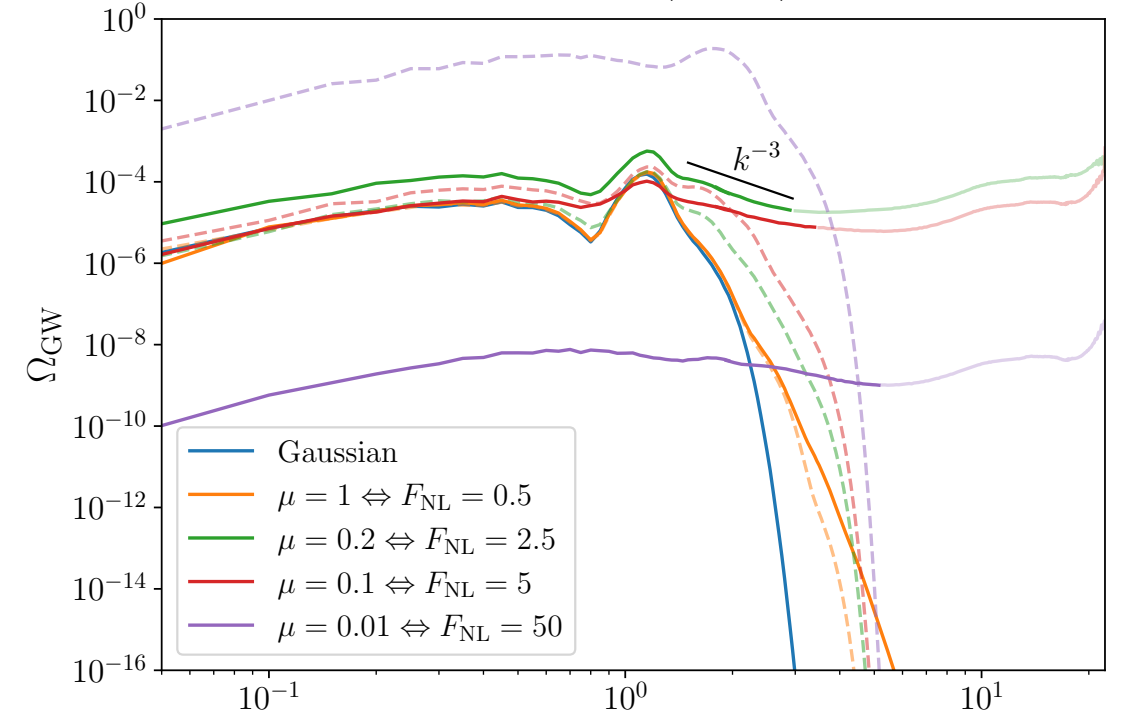
2508.10812 Xiang-Xi Zeng, Zhuan Ning, Rong-Gen Cai, Shao-Jiang Wang,
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SimuSIGW: <https://github.com/ZengXiang-Xi/SimuSIGW/tree/main>

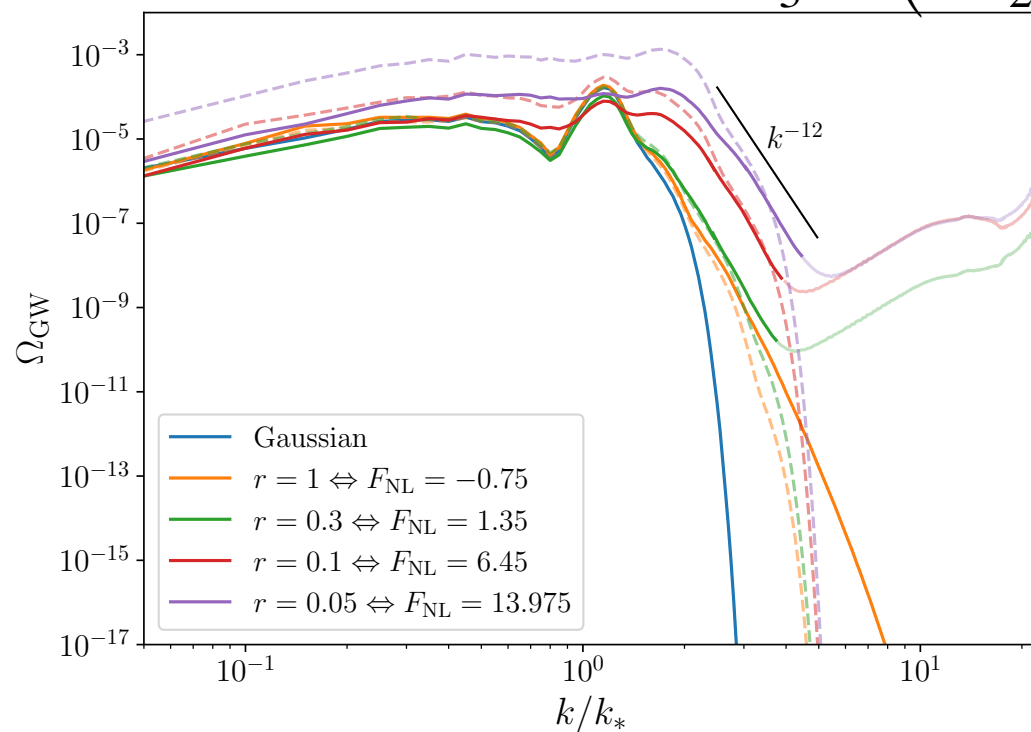
$$\zeta = \zeta_g + G_{\text{NL}}\zeta_g^3 + H_{\text{NL}}(\zeta_g^4 - 3\langle\zeta_g^2\rangle^2)$$



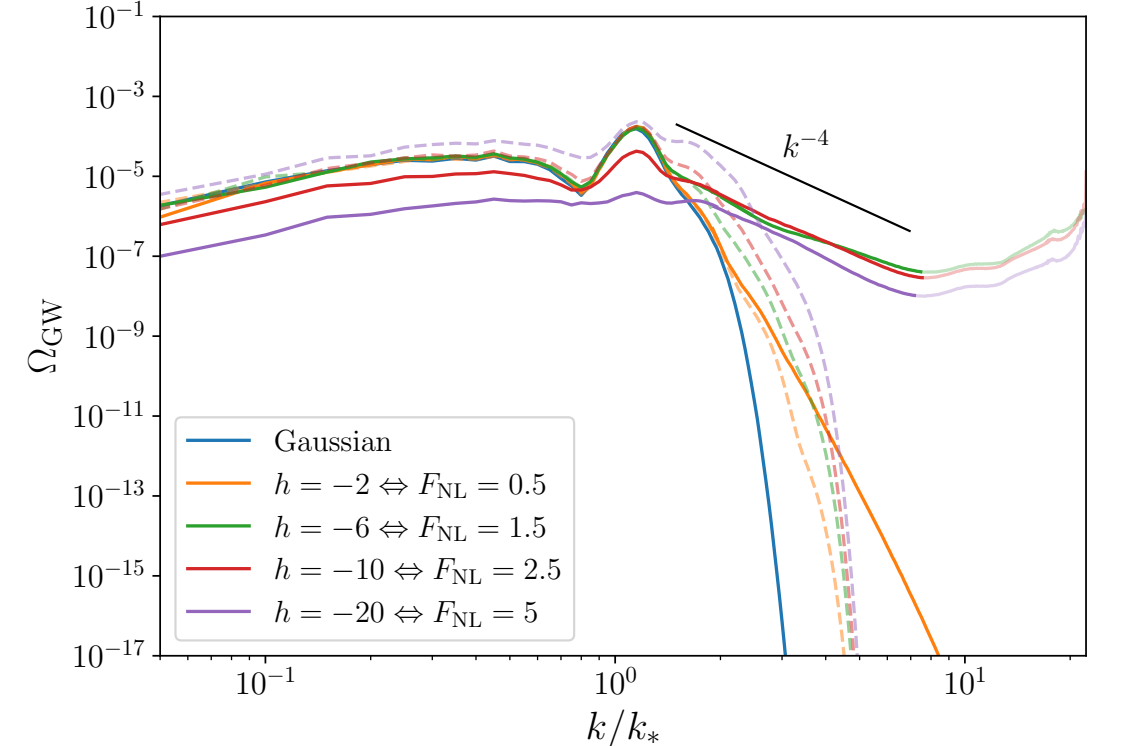
$$\zeta = -\mu \log \left| 1 - \frac{\zeta_g}{\mu} \right|$$



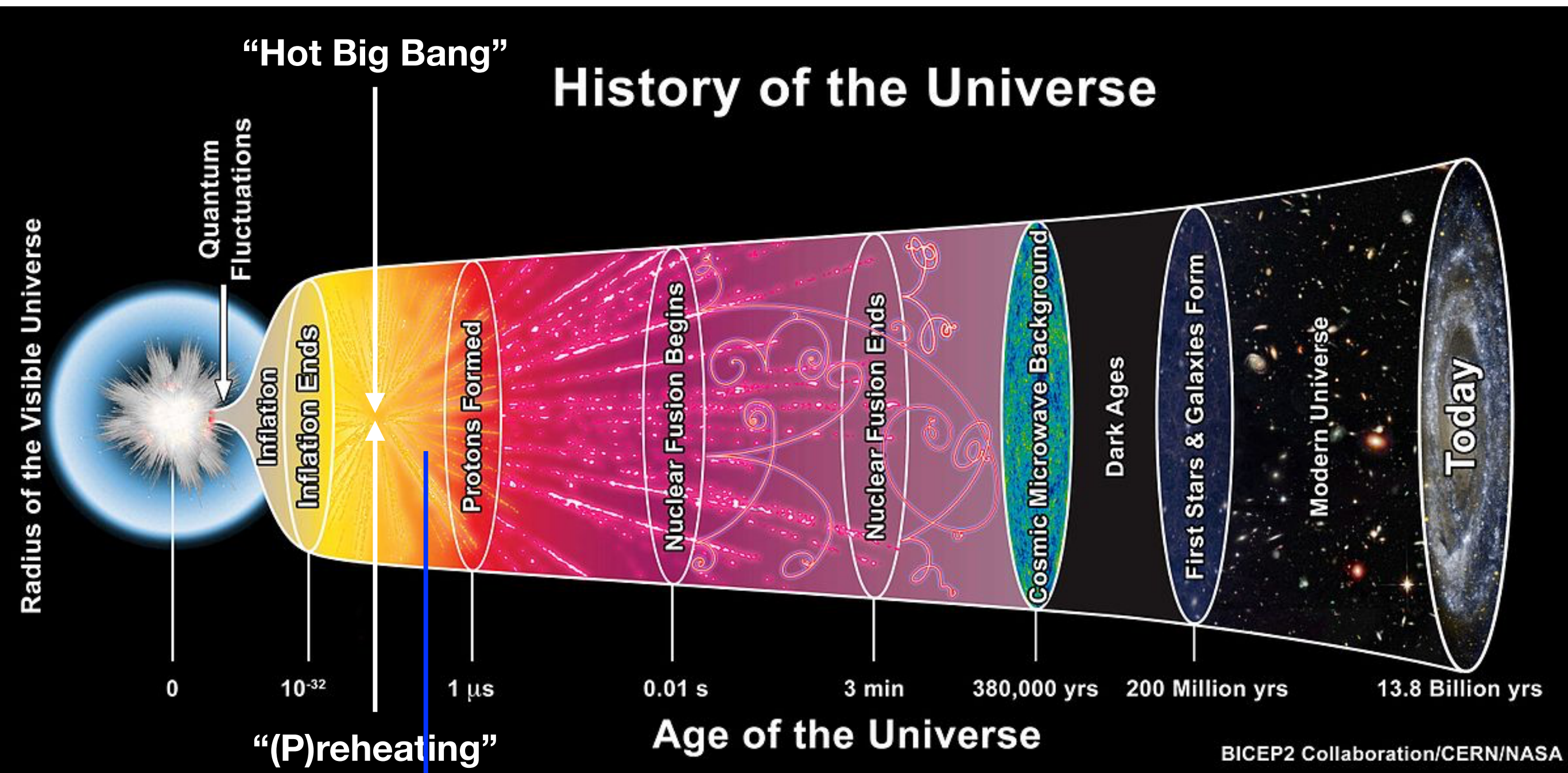
$$\zeta = \log[X(r, \zeta_g)] = \frac{2}{3} \log \left(1 + \frac{3}{2} \zeta_g \right)$$



$$\zeta = -\frac{2}{|h|} \left(\sqrt{|1 - |h|\zeta_g|} - 1 \right)$$



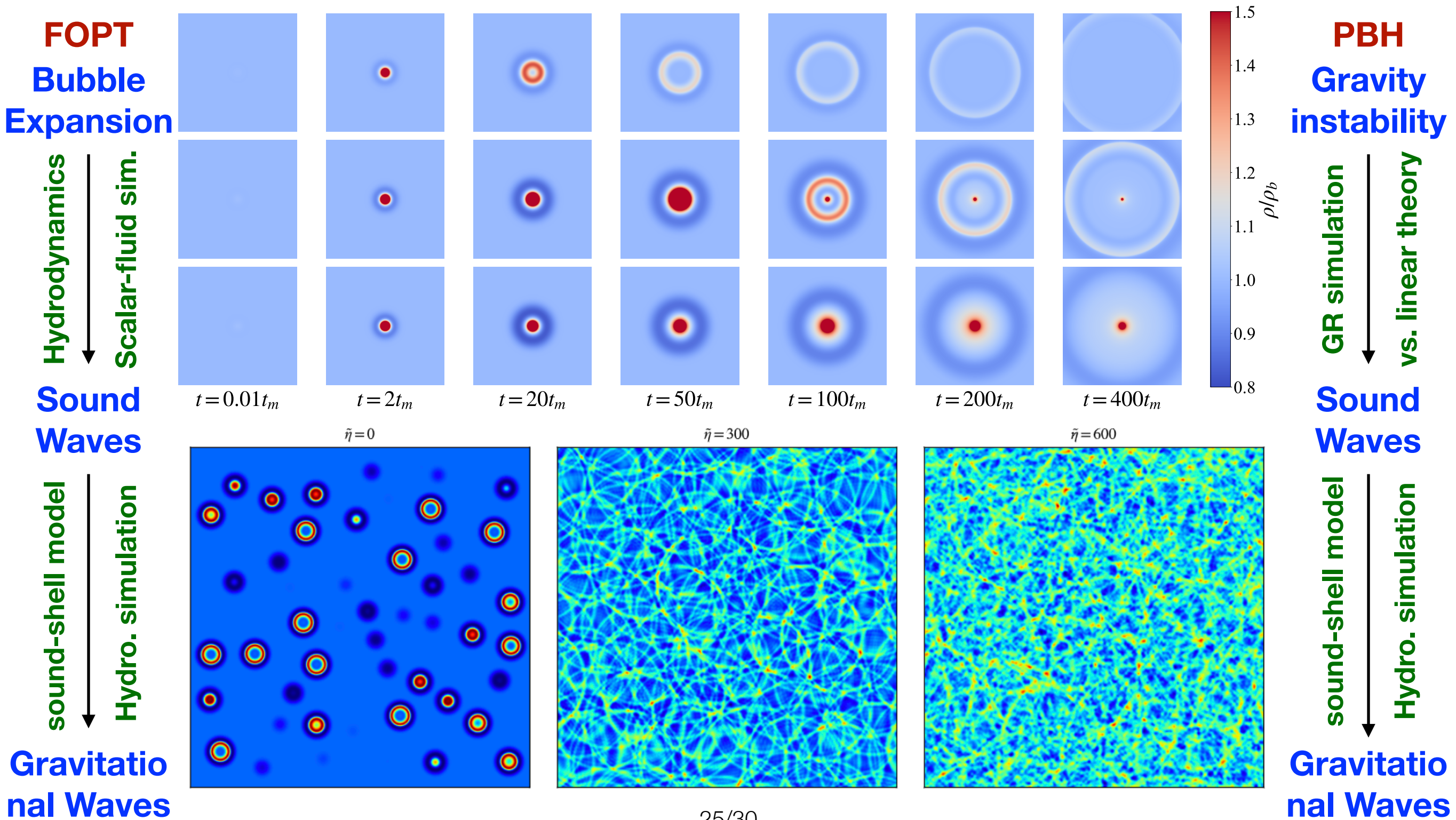
SWGWs from FOPTs



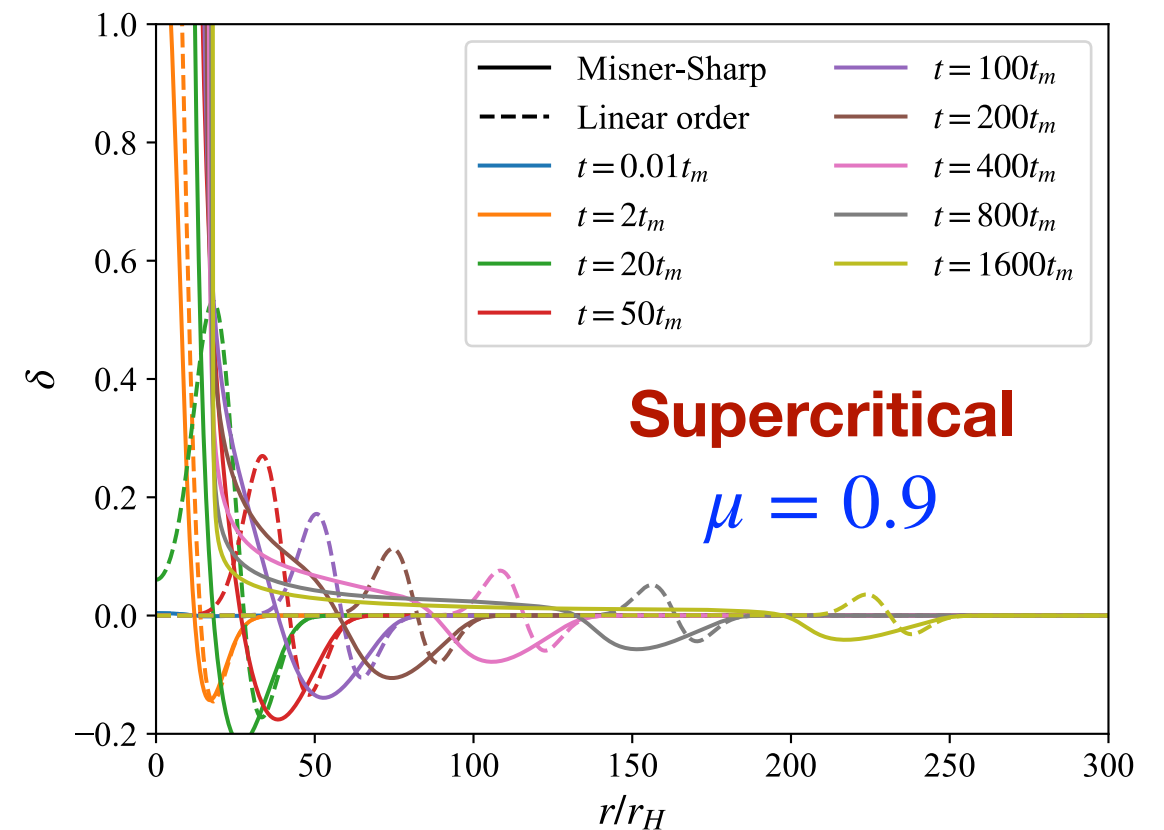
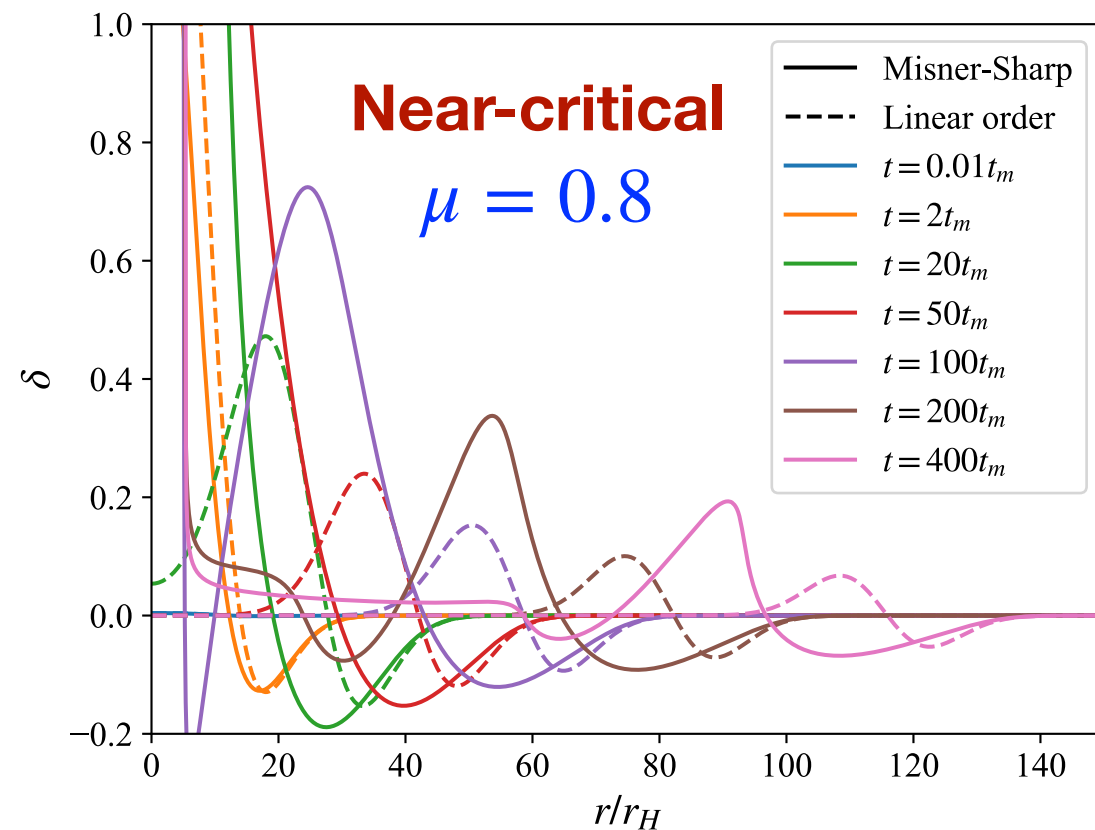
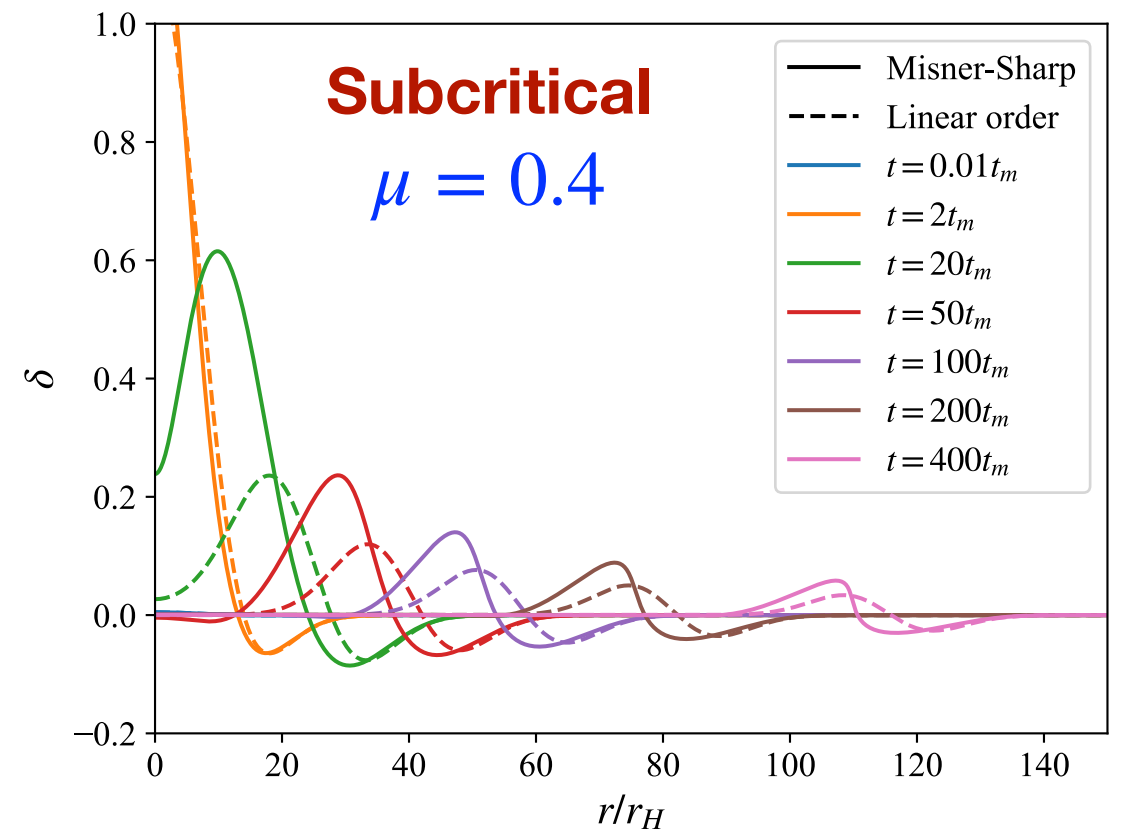
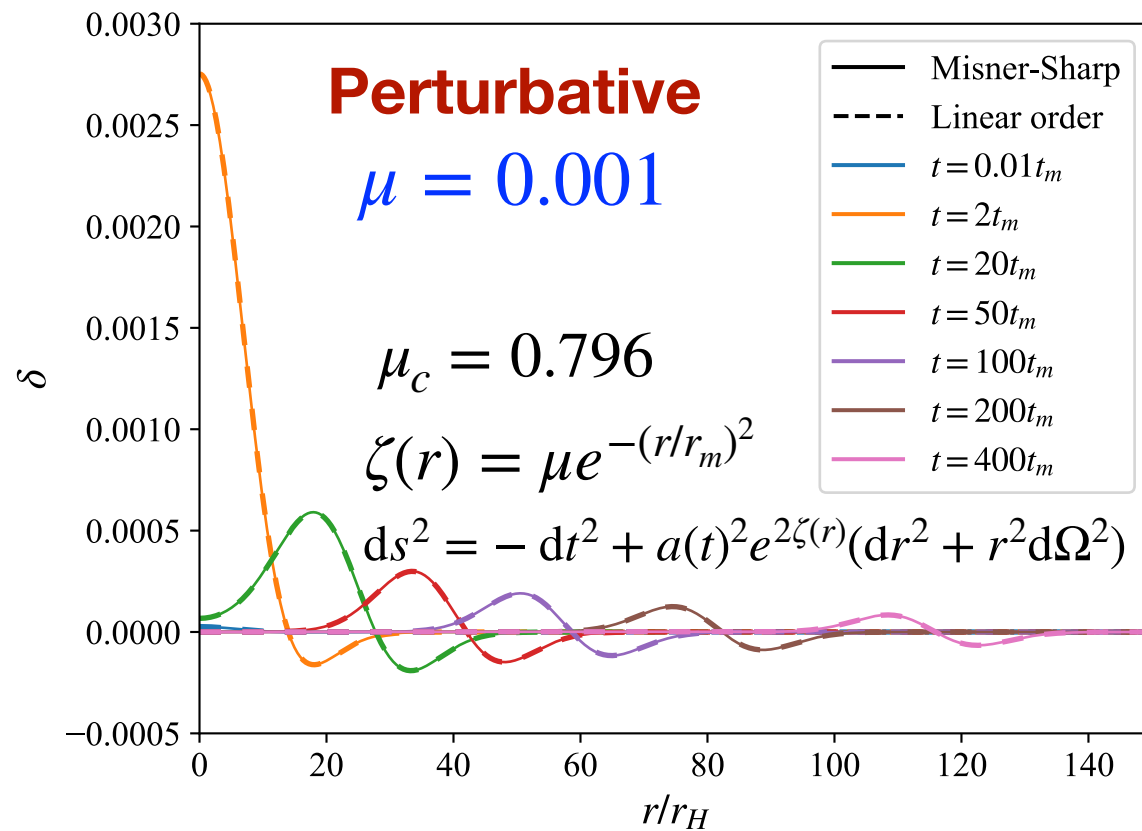
SWGWs from FOPTs

GWs from gravitational collapses

2504.11275 Zeng, Ning, Yuwen, SJW, Cai **Relic gravitational waves from primordial gravitational collapses**
2504.12243 Ning, Zeng, Yuwen, SJW, Cai **Sound waves from primordial black hole formations** **PRD 2026**
2512.21151 Ning, Yuwen, Zeng, Cai, SJW, **Acoustic gravitational waves from primordial curvature perturbations**

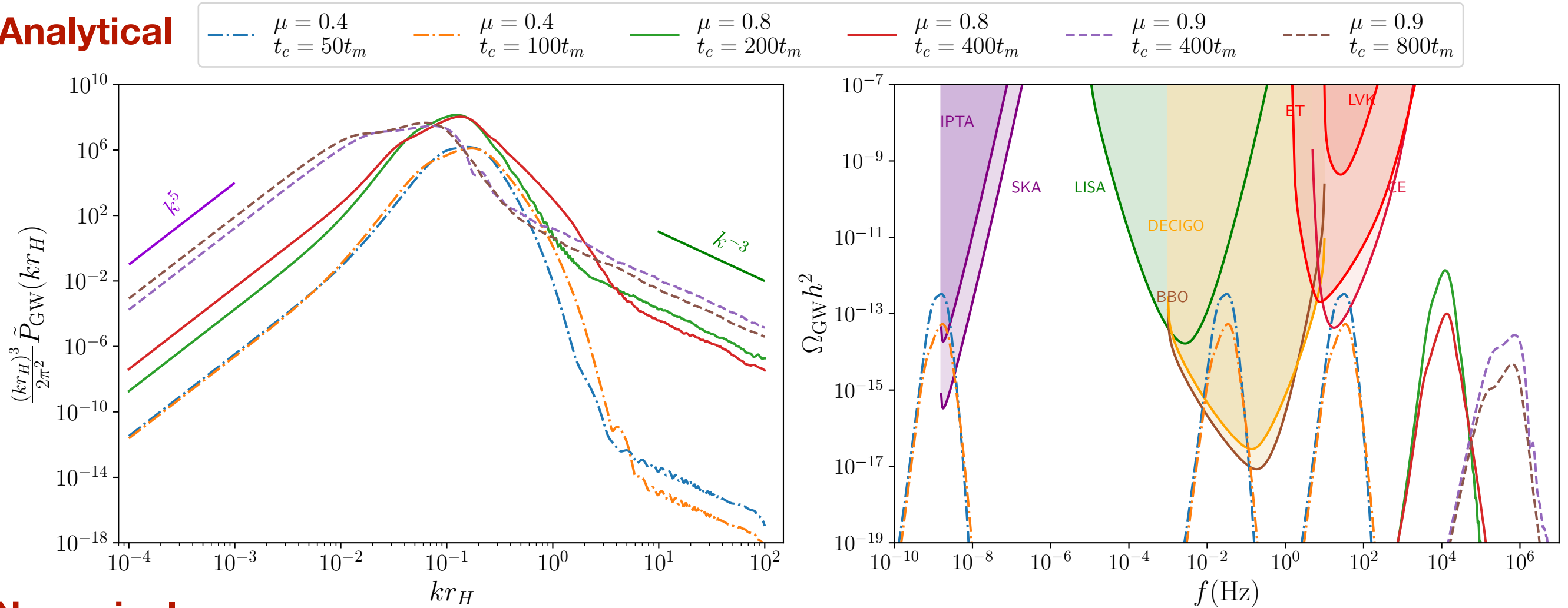


SWs from gravitational collapses

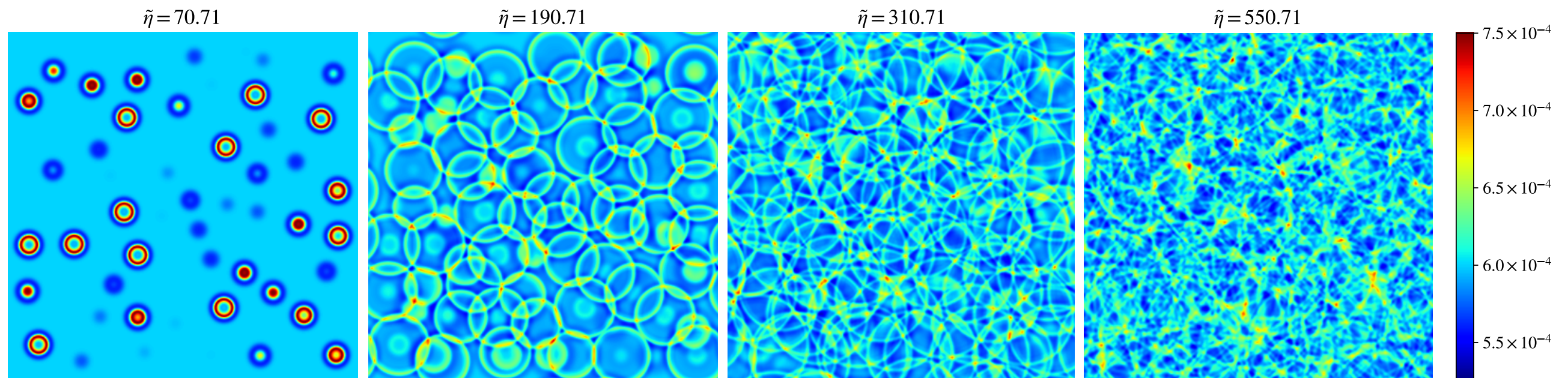


GWs from gravitational collapses

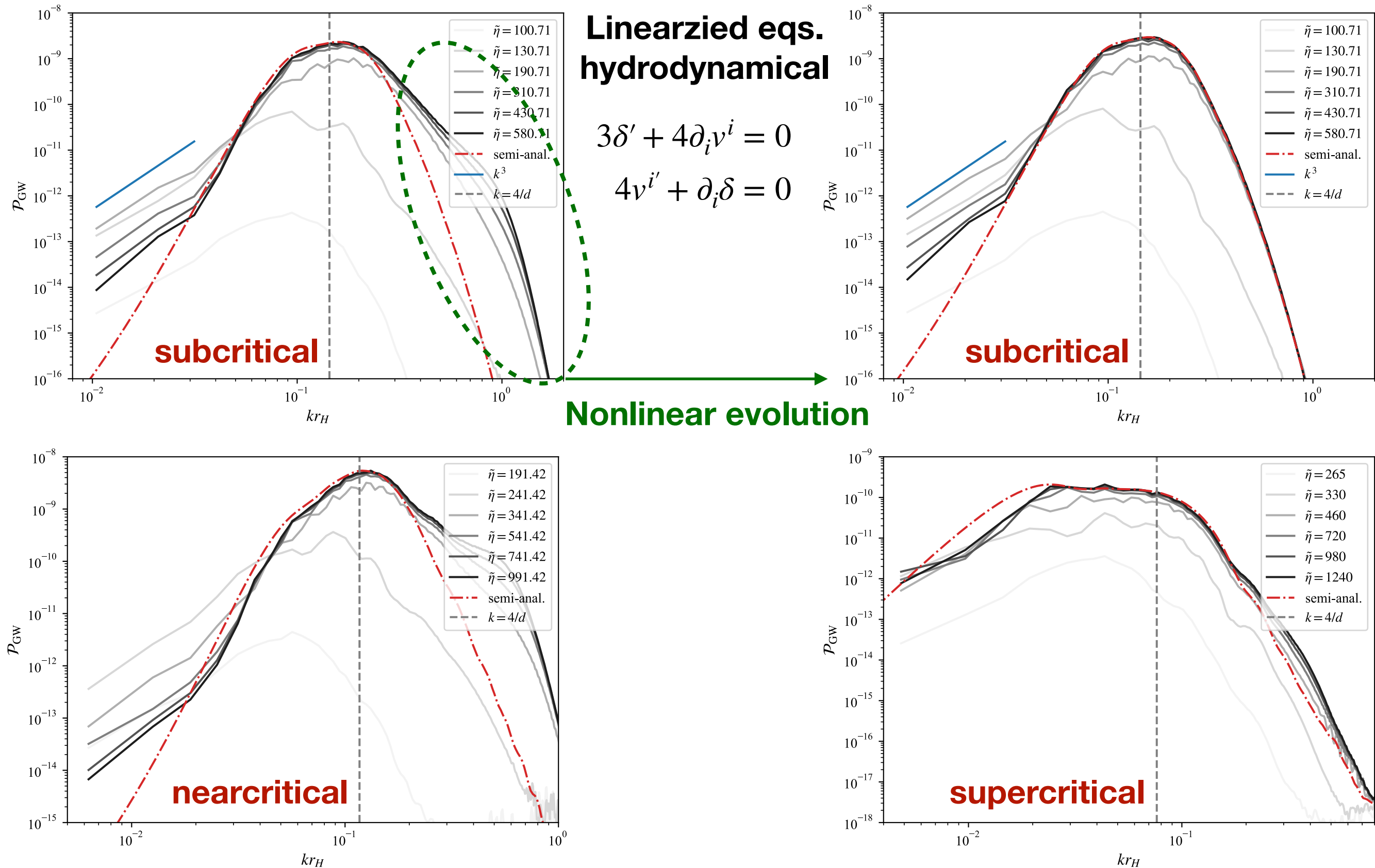
Analytical



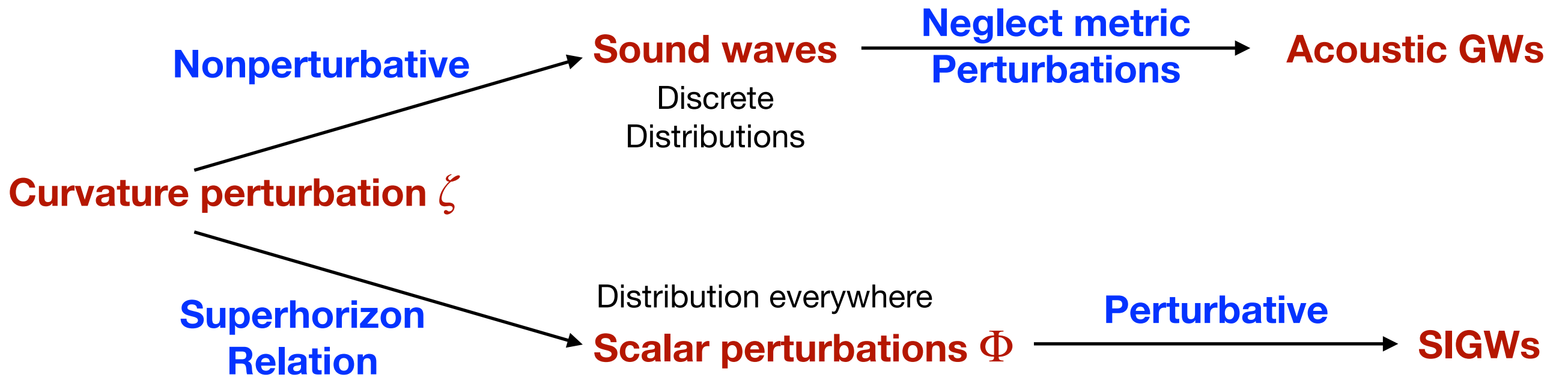
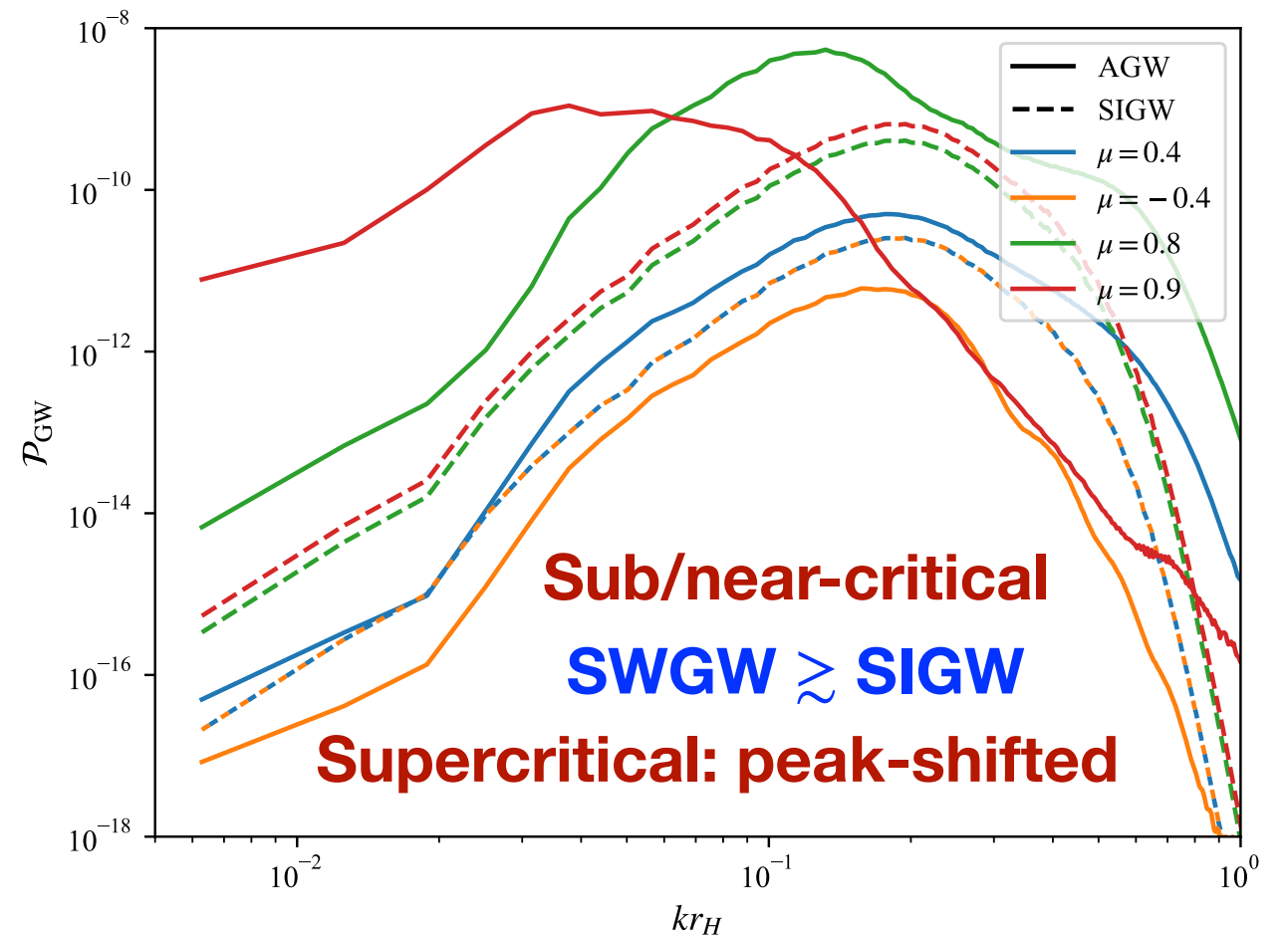
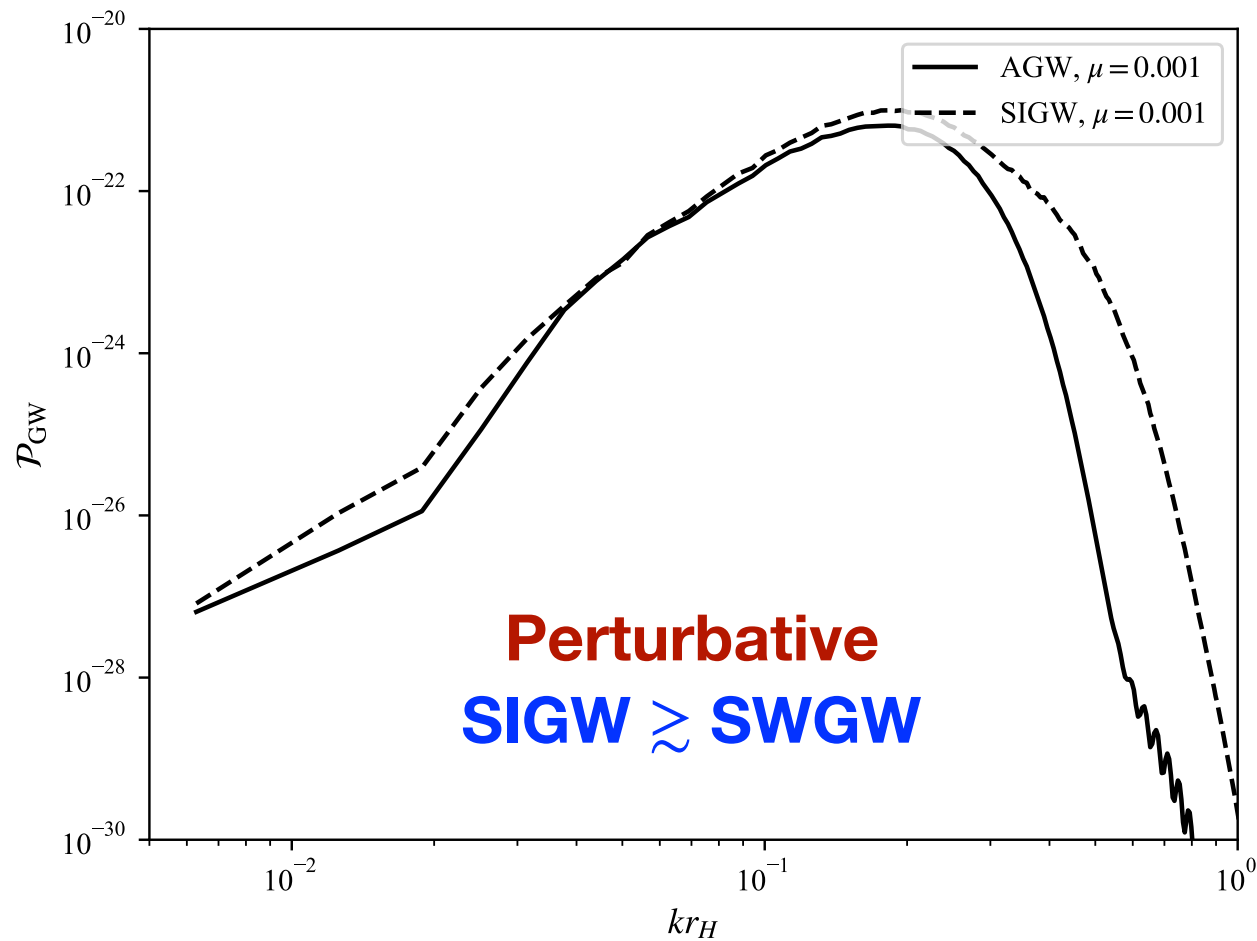
Numerical



GWs from gravitational collapses



SWGWs vs. SIGWs



Conclusions and discussions

SW from FOPT : IR casual scaling law $\leftrightarrow$ bubble-wall vs. sound-shell lifetimes

PBH from FOPT : likely no gauge issue to get density perturbations from FOPT

SIGW from initial : all-order non-Gaussianity can differ from finite truncations

AGW from FOPT : AGWs include nonperturbative effect/neglect metric vs. SIGW

We haven't reach full understandings of SWs, PBHs, SIGWs, AGWs from FOPTs

Still active of debates with interactive topics

PBH from bubble collisions/Fermi ball

Particle production from bubble collisions

GWs from PBH formation and evaporation

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Thank you

