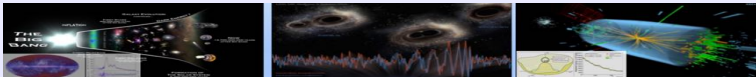


Superradiant instability of area-quantized Kerr black hole with discrete reflectivity

Speaker: Yun-Long Zhang (NAOC)

National Astronomical Observatories,
Chinese Academy of Sciences

Zhong-Hao Luo and Yun-Long Zhang Eur. Phys. J. C (2025) 85:507
2026 Aug 24 @UCAS [email: zhangyunlong@nao.cas.cn]



Axion-like Cloud and Black Hole Superradiance

Chiral GWs

Background

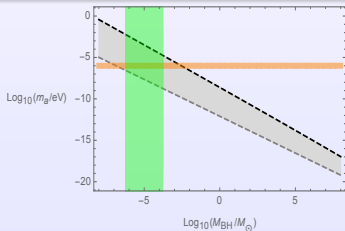
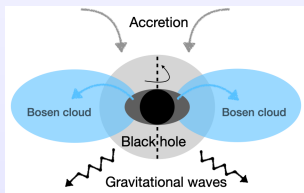
Superradiance

Ultralight DM

Area-quantized BHs

Schwarzschild $R_{BH} = G_N M_{BH} / c^2$, de Broglie wave length $\lambda_\vartheta = \hbar / (m_\vartheta v_\vartheta)$

- Characteristic $\alpha \equiv \frac{R_{BH}}{\lambda_\vartheta} \simeq \left(\frac{M_{BH}}{M_\odot} \right) \left(\frac{m_\vartheta}{10^{-10} \text{eV}} \right) \left(\frac{v_\vartheta}{c} \right)$.
- Formation time $\tau_\vartheta < \text{Universe's age } \tau_U \simeq 10^{23} \left(\frac{M_\odot}{M_{BH}} \right) R_{BH}$
- $\tau_{\vartheta\uparrow} \simeq 10^7 e^{1.84\alpha} R_{BH}$, $\alpha \gg 1$, $\tau_{\vartheta\downarrow} \simeq 24 \alpha^{-9} R_{BH}$, $\alpha \ll 1$.



Axion-like Dark Matter

Chiral GWs

Background

Superradiance

Ultralight DM

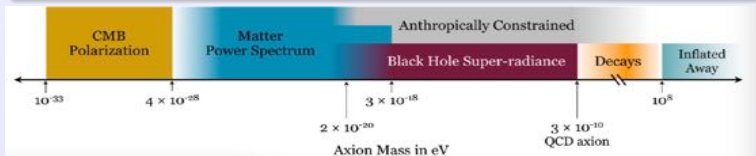
Area-quantized BHs

QCD axion

- $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^a F_a^{\mu\nu} - \alpha_\theta \theta F_{\mu\nu}^a \tilde{F}_a^{\mu\nu} + \bar{\psi} \left(i\gamma^\mu D_\mu - me^{i\theta'\gamma_5} \right) \psi$
- Strong CP problem: no CP violation in measurement
- Peccei–Quinn (1977): introduce new pseudoscalar
- Wilczek–Weinberg: relax the CP-violation parameter

Axion-like: ultra light dark matter

- e.g. Fuzzy Dark Matter, Dark photon dark matter



New physics in ultra-low energy

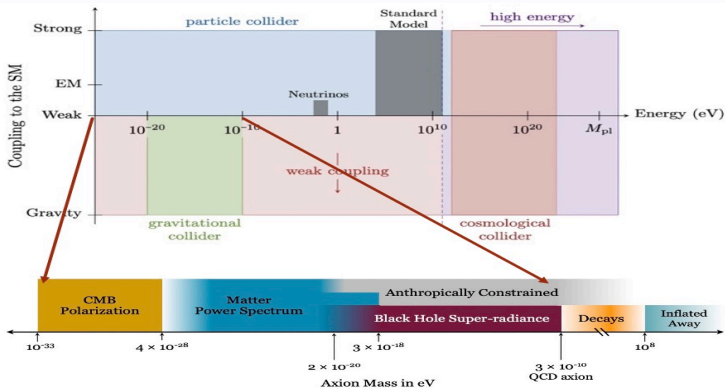
Chiral GWs

Background

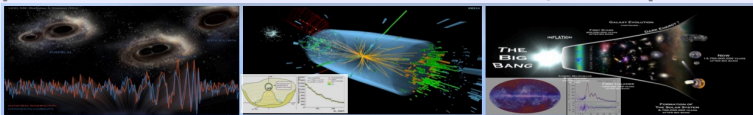
Superradiance

Ultralight DM

Area-quantized BHs



[cf. Baumann-Chia-Porto-Stout, Gravitational Collider Physics, 2019]



Multi-band gravitational wave detection

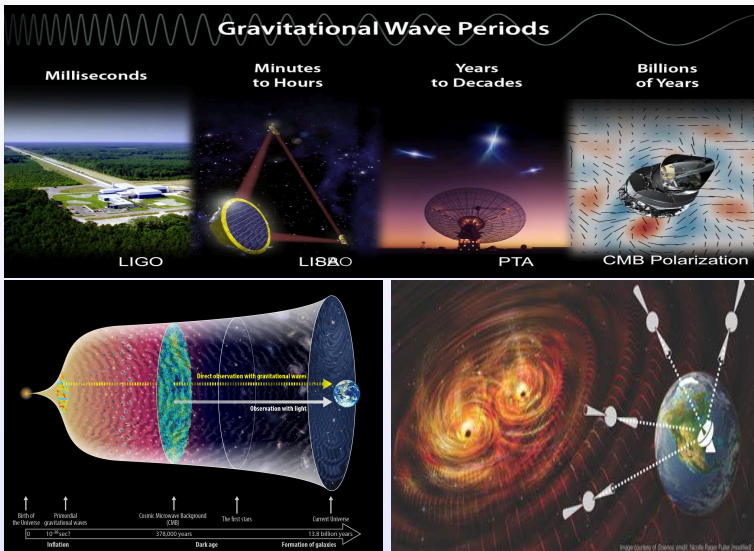
Chiral GWs

Background

Superradiance

Ultralight DM

Area-quantized BHs



Energy level transition and Monochromatic GW

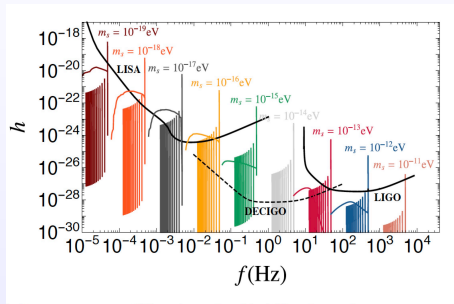
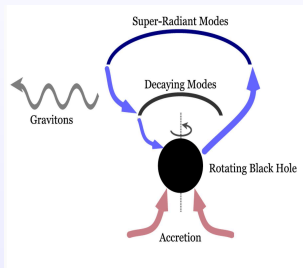
Chiral GWs

Background

Superradiance

Ultralight DM

Area-quantized BHs



- Energy transition $\vartheta^+ \rightarrow \vartheta^- + h$, Strain $h \sim 10^{-19} - 10^{-27}$
- Monochromatic GW [cf. Brito-Cardoso-Pani, Superradiance 2020]

Axion annihilation and Stochastic GWs

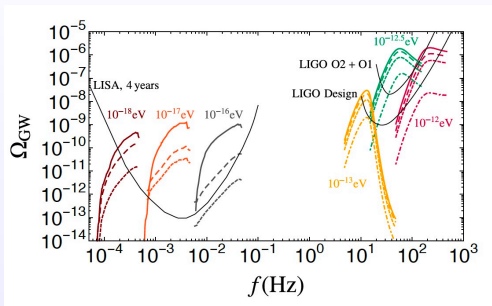
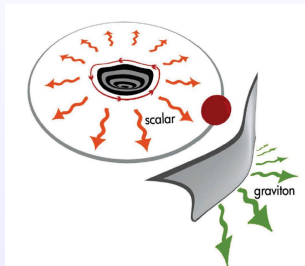
Chiral GWs

Background

Superradiance

Ultralight DM

Area-quantized BHs



- Axion annihilation $\vartheta + \vartheta \rightarrow hh$, Strain $h \sim 10^{-21} - 10^{-32}$.
- Stochastic GW [cf. Brito-Cardoso-Pani, Superradiance 2020]

Area-quantized black holes and spectrum

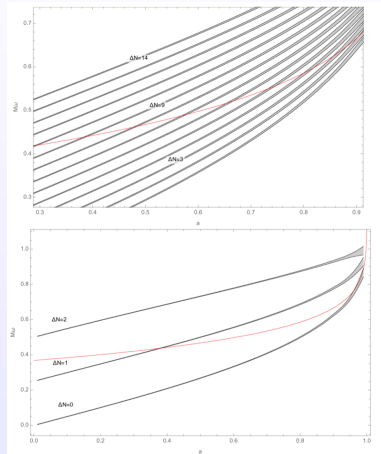
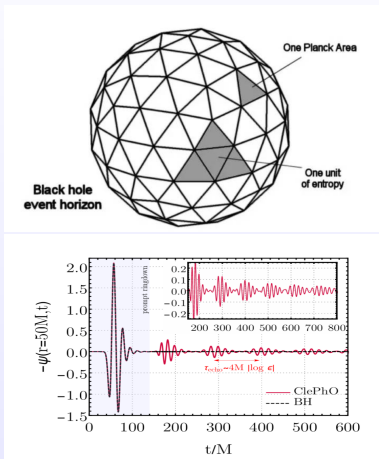
Chiral GWs

Background

Superradiance

Ultralight DM

Area-quantized BHs



■ cf. Ivan Agullo, Vitor Cardoso et al. (2021)

Area-quantized black holes

Chiral GWs

Background

Superradiance

Ultralight DM

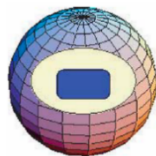
Area-quantized BHs

$$H(t) = H_{\text{BH}} + H_{\text{int}}(t), \quad M_f = M_i \pm \hbar\omega,$$

$$\begin{aligned} \hbar\omega_{\hat{n}} &\simeq \frac{\partial M_{N,j}}{\partial N} \Delta N + \frac{\partial M_{N,j}}{\partial j} \Delta j + O\left(\frac{\Delta N}{N}, \frac{\Delta j}{j}\right) \\ &\simeq \hbar\hat{n} \frac{\alpha\kappa}{8\pi} + \boxed{\hbar m \Omega_H} + O(N^{-1}). \end{aligned}$$

$$dM = \frac{\kappa}{8\pi} dA + \Omega_H dJ = \hbar\hat{n} \frac{\kappa\alpha}{8\pi} + \hbar m \Omega_H,$$

$$\Delta E \cdot \Delta t_O \geq \frac{\hbar}{2}, \quad \left| \frac{dE}{dt} \right| = |\dot{M}| = -\dot{M},$$



$$\Gamma_{\text{width}} = \frac{\hbar}{\Delta t_H} = \hbar \dot{N} = \frac{\hbar f(a)}{M(M\langle\omega\rangle)}.$$

$$\Gamma = \frac{1.005}{M} e^{-6.42 + 1.8a^2 + 1.9a^{12} - 0.1a^{14}}.$$

Superradiance of Area-quantized black holes

Chiral GWs

Background

Superradiance

Ultralight DM

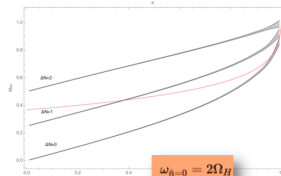
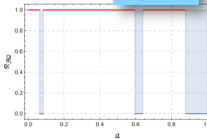
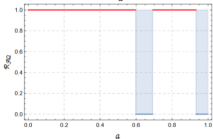
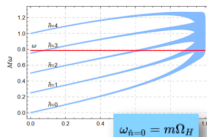
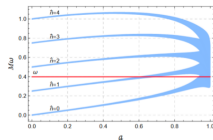
Area-quantized BHs

$$\mathcal{R}_{AQ}(\omega, M, a) = \begin{cases} 1 - |\Theta(\sin \zeta_1) - \Theta(\sin \zeta_2)|, & \Delta\omega > 2\pi\Gamma \\ 0, & \Delta\omega \leq 2\pi\Gamma \end{cases}$$

$$\Delta\omega = \alpha\kappa/8\pi$$

$$\zeta_1 = \frac{\pi(\omega - m\Omega_H + 2\pi\Gamma/2)}{\Delta\omega},$$

$$\zeta_2 = \frac{\pi(\omega - m\Omega_H - 2\pi\Gamma/2)}{\Delta\omega},$$



$$M_{N,j} \rightarrow M_{N+\Delta N, j+2}$$

[Ivan Agullo, Vitor Cardoso et al. (2021)]

cf. Z.H.Luo & Y.L. Zhang (2025)

Superradiance of Area-quantized black holes

Chiral GWs

Background

Superradiance

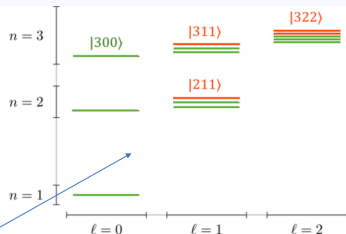
Ultralight DM

Area-quantized BHs

$$\text{Im}[\delta\omega] = \frac{\delta\nu_*}{M} \left(\frac{\varepsilon}{\nu_0}\right)^3 \left[1 - \left(\frac{\varepsilon}{\nu_0}\right)^2\right]^{-1/2} \times \frac{1 - \mathcal{R}^2}{1 + \mathcal{R}^2 + 2\mathcal{R} \cos(i \sum_{j=1}^l \ln \frac{j+2ip}{j-2ip})}$$

$$\delta\nu_* = \frac{2p}{n!} \left[\frac{l!}{(2l)!(2l+1)!} \right]^2 \prod_{j=1}^l (j^2 + 4p^2) \times (2l + n + 1)! [2k(r_+ - r_-)]^{2l+1}.$$

[Detweiler(1980)]



[Baumann et al, 2019, 2020]

➤ Klein-Gordon方程

Yoshino & Kodama, PTP 128 (2012), 153–190

$$(\nabla^\nu \nabla_\nu + \mu^2) \Phi = 0 \quad \Phi = \sum_{nlm} e^{-i\omega_{nlm}t} e^{im\phi} S_{lm}(\theta) R_{nlm}(r) + \text{c.c.}$$

Press & Teukolsky, (1973)

$$\frac{1}{\sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{dS_{lm}}{d\theta} \right) + \left[a^2 (\omega^2 - \mu^2) \cos^2 \theta - \frac{m^2}{\sin^2 \theta} + \Lambda_{lm} \right] S_{lm}(\theta) = 0$$

$$\Delta \frac{d}{dr} \left(\Delta \frac{dR}{dr} \right) + \left[\omega^2 (r^2 + a_*^2)^2 - 4\omega M r m a_* + m^2 a_*^2 - \Delta (\mu^2 r^2 + a_*^2 \omega^2 + \lambda) \right] R(r) = 0.$$

Asymptotic matching method for the solutions

Chiral GWs

Background

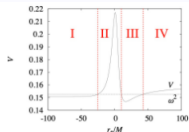
Superradiance

Ultralight DM

Area-quantized BHs

$$\Delta \frac{d}{dr} \left(\Delta \frac{dR}{dr} \right) + \left[\omega^2 (r^2 + a_*^2)^2 - 4\omega M r m a_* + m^2 a_*^2 - \Delta (\mu^2 r^2 + a_*^2 \omega^2 + \lambda) \right] R(r) = 0.$$

$$\lambda \simeq l(l+1)$$



$$z = \frac{r - r_+}{r_+ - r_-} \quad p = \frac{m a_* - 2\omega M r_+}{r_+ - r_-}$$

$$\rho = 2kr = 2\sqrt{\mu^2 - \omega^2} r \quad v = M\mu^2/k$$

$$z(z+1) \frac{d}{dz} \left[z(z+1) \frac{dR}{dz} \right] + [p^2 - l(l+1)z(z+1)] R = 0,$$

$$\frac{d^2(\rho R)}{d\rho^2} + \left[-\frac{1}{4} + \frac{v}{\rho} - \frac{l(l+1)}{\rho^2} \right] (\rho R) = 0,$$



$$R(z) = (Ac_1 + Bc_3)U_1(0)z^{ip} + (Ac_2 + Bc_4)U_2(0)z^{-ip}.$$

$$U_1(0) = F(-l, l+1, 1-2ip; 1)$$

$$U_2(0) = F(-l-2ip, l+1-2ip, 1-2ip; 1)$$

$$R(r) = (2kr)^l e^{-kr} F(-n - \delta\nu, 2l+2; 2kr) \\ \simeq (-1)^n \frac{\Gamma(2l+n+2)}{\Gamma(2l+2)} (2kr)^l + \\ (-1)^{n+1} \Gamma(n+1) \Gamma(2l+1) \delta\nu (2kr)^{-l-1},$$

Reduced mass of the axion cloud

Chiral GWs

Background

Superradiance

Ultralight DM

Area-quantized BHs

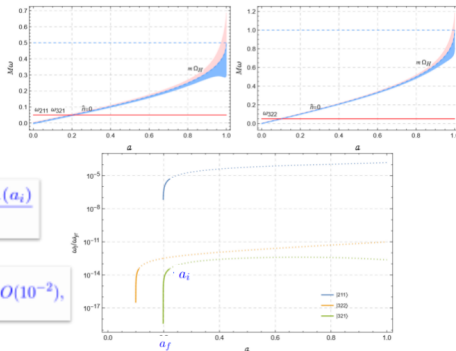
$$\frac{M_{\text{cloud}}}{M_i} \simeq \frac{\mu M_i}{m} \left(a_i - \frac{4\mu M_i}{m} \right) + O(\varepsilon^3).$$

$$M_{\text{cloud}}/M \simeq (a_i - a_f)\mu M/m, \quad a_f \simeq 4\mu M/m$$

$$\Delta E_{322}(a_f(211)) > \Delta E_{322}(a_i(322))$$

$$\frac{2\mu r_+(a_f)}{m} \leq a \leq \frac{2[\mu + \pi\Gamma(a_i)]r_+(a_i)}{m}$$

$$\frac{M_{\text{cloud}}}{M_i} \simeq \frac{2\pi\Gamma(a_i)r_+(a_i)\mu M_i}{m^2} \sim \frac{\mu M_i}{m} \times O(10^{-2}),$$



Summary of area-quantized black holes

Chiral GWs

Background

Superradiance

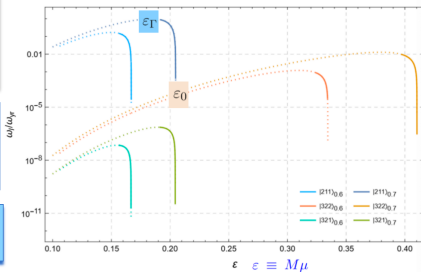
Ultralight DM

Area-quantized BHs

$$\omega_I = \text{Im}[\delta\omega] = \frac{\delta\nu_*}{M} \left(\frac{\varepsilon}{\nu_0} \right)^3 \left[1 - \left(\frac{\varepsilon}{\nu_0} \right)^2 \right]^{-1/2} \times \frac{1 - |\mathcal{R}|^2}{1 + |\mathcal{R}|^2 + 2|\mathcal{R}| \cos \phi_\omega},$$

$$\delta\nu_* = \frac{2p}{n!} \left[\frac{l!}{(2l)!(2l+1)!} \right]^2 \prod_{j=1}^l (j^2 + 4p^2) \times (2l + n + 1)! [2k(r_+ - r_-)]^{2l+1}.$$

$$\mathcal{R}_{AQ}(\omega, M, a) = \begin{cases} 1 - |\Theta(\sin \zeta_1) - \Theta(\sin \zeta_2)|, & \Delta\omega > 2\pi\Gamma \\ 0, & \Delta\omega \leq 2\pi\Gamma \end{cases}$$



Scalar fields	Instability	BHs ($ \mathcal{R} = 0$)	ECOs ($ \mathcal{R} = 1$)
Massive	Ergoregion	Stable [10]	Stable
Massive	Superradiant	Unstable [10]	Stable [10]
Massless	Ergoregion	Stable [10,62]	Unstable [10,62]
Massless	Superradiant	Stable [10]	Stable [10]

Thanks a lot for your attention!