

Can the Universe actually Inflate?

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w/ Josu Aurrekoetxea (MIT), Katy Clough (QMUL), Raphael Flauger (UCSD), Matt Elley (Prague), Nicole Righi (Bologna), Panos Giannadakis (KCL), Sonia Paban (UTAustin), Willy Fischler (UTAustin), Brandon DiNunno (UTAustin)

2nd New Horizons to PSI Workshop, Beijing 2026

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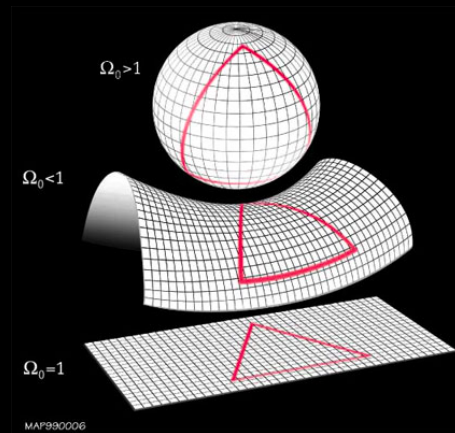


Inflation

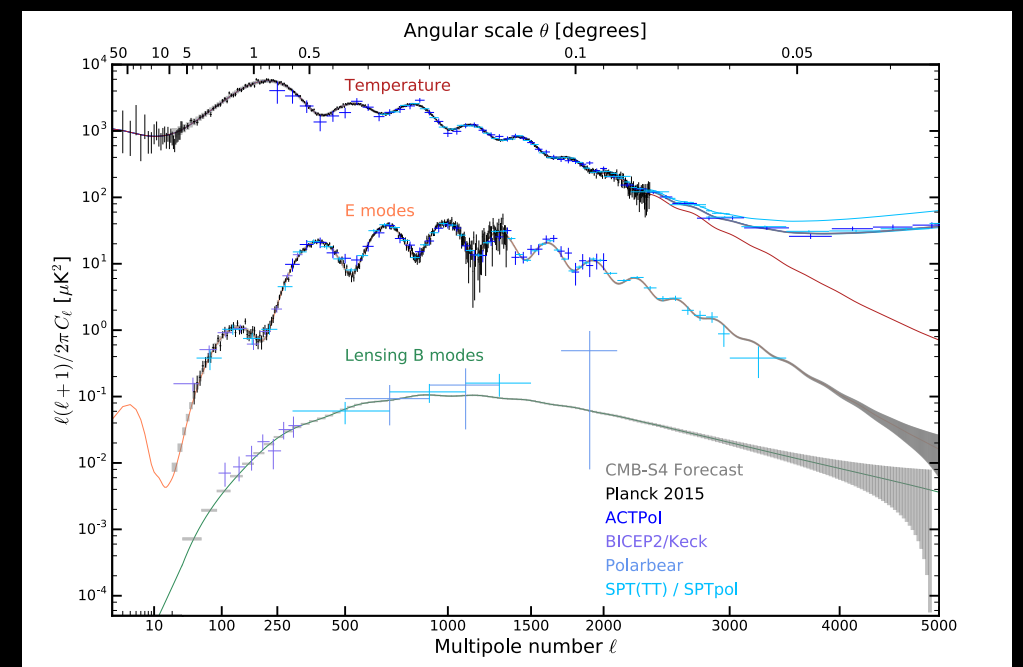
Inflation is a *dynamical* solution to an *initial conditions* problem.

Old “Background” Problems

Horizon
Flatness
Monopoles



“Structure” Problems



CMB S4 Science Book

They are logically *different* initial conditions problems.

Inflation

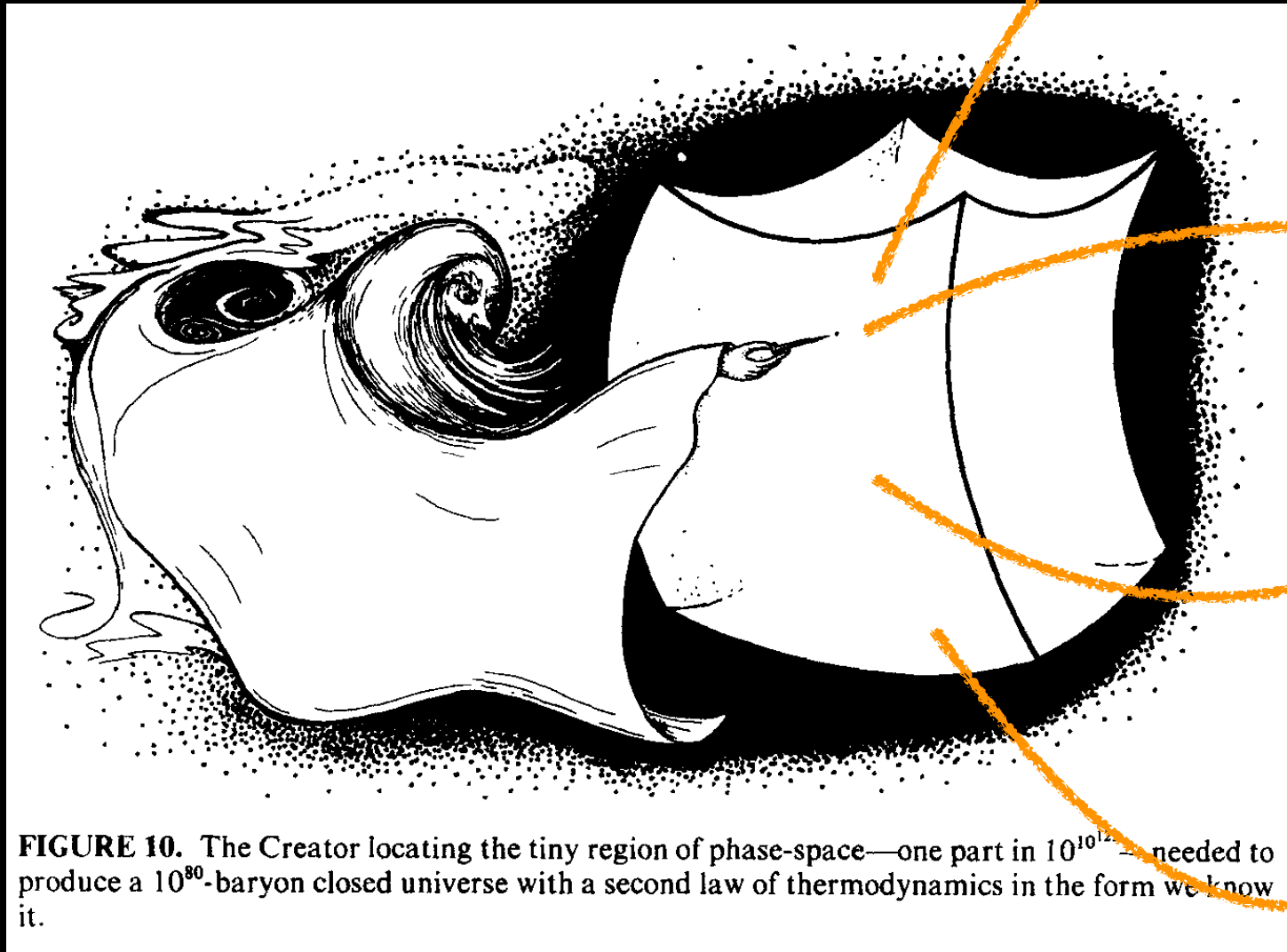
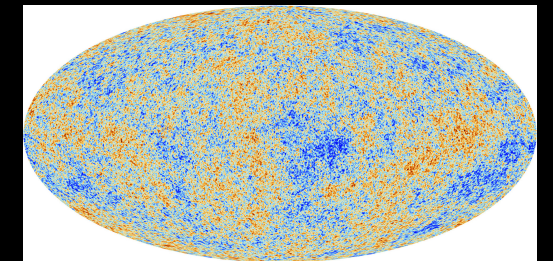


FIGURE 10. The Creator locating the tiny region of phase-space—one part in $10^{10^{12}}$ —needed to produce a 10^{80} -baryon closed universe with a second law of thermodynamics in the form we know it.



Planck

Penrose 1989

“Regardless of initial conditions, inflation dynamically smooths out all inhomogeneities.”

Part 1

**Does Inflation always inflate
regardless of Initial conditions?**

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regardless of Initial conditions?

No (Sorry.)

It depends on inflationary models *and* initial conditions.

Inflation itself has an initial conditions problem

Betteridge's Law

Part 1



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regardless of Initial conditions?

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Inflation itself has an initial conditions problem

How to test this?

Step 1 : Pick an inflationary model (dynamics)

Step 2 : Choose **inhomogeneous** initial conditions for this model

Step 3 : Do calculations (numerics/analytics), do we get >60 folds of inflation for some patch?

This is the hard part!

GRCHOMBO + GRTeclyn!

Numerical Relativity with Adaptive Mesh Refinement

Aurrekoetxea et al.

GRTL Code Collaboration

(formerly known as the “GRCHOMBO people”)

www.grtlcollaboration.org



Open Source BSD-3 License

arXiv:1503.03436
arXiv:2112.10567
arXiv:2201.03458

These are expensive simulations...

Solve full Einstein equations : no approximations made up to numerical error.

Adaptive mesh refinement allows for precision.

Each simulation run takes 10000-100000 core/hrs

....(14.2 - 142 Tree-months of CO₂).



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Cosmology using numerical relativity

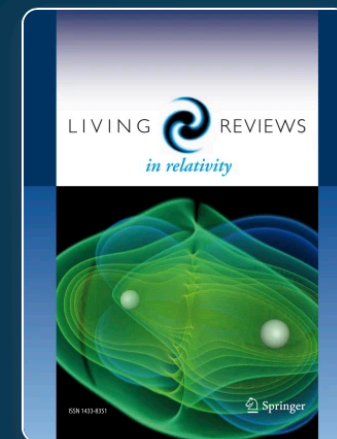
Review Article | [Open access](#) | Published: 23 June 2025

Volume 28, article number 5 (2025) [Cite this article](#)

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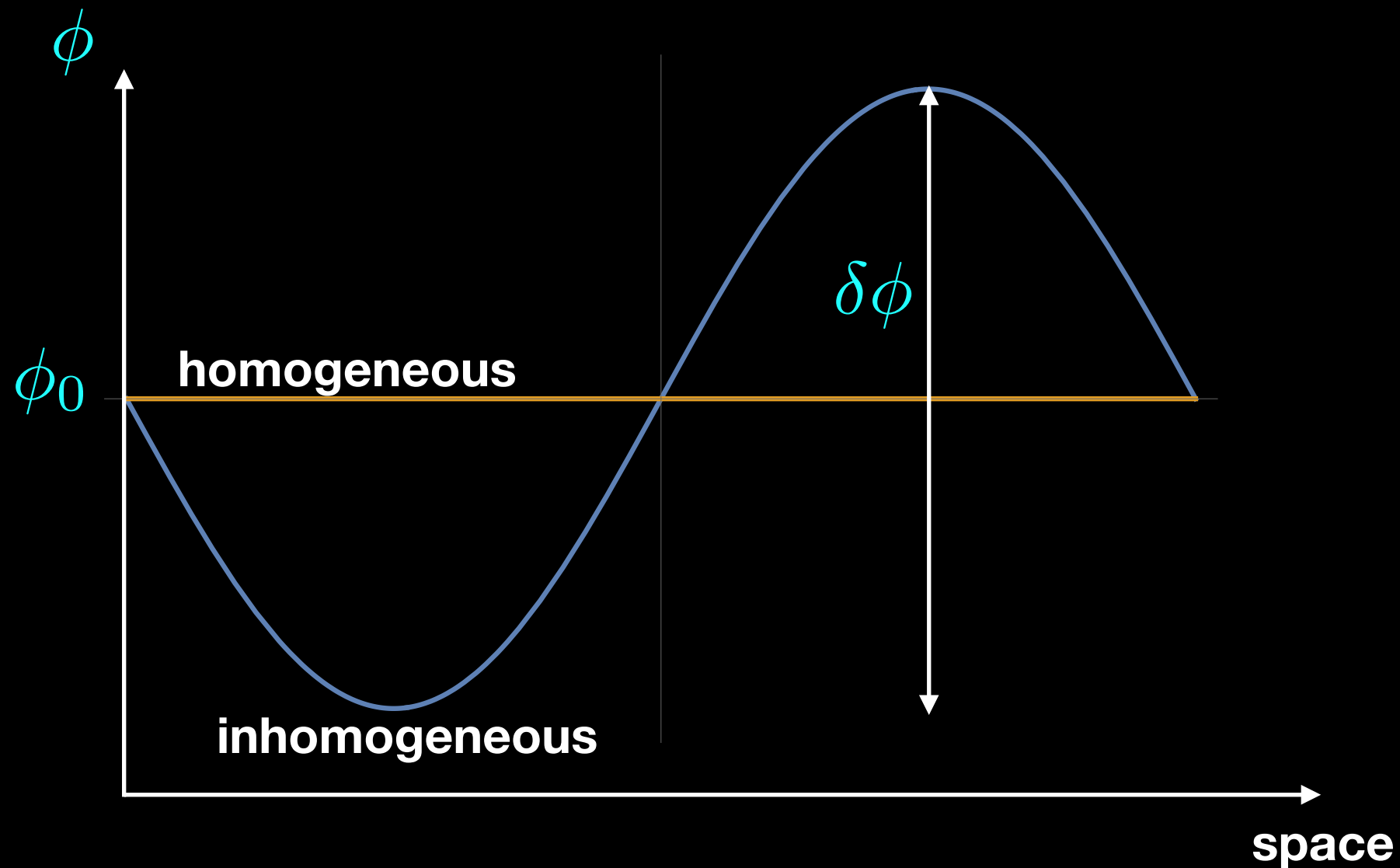
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[Aims and scope](#) →

[Josu C. Aurrekoetxea](#) , [Katy Clough](#) & [Eugene A. Lim](#)

Part of a collection:
[Numerical Relativity](#)

Inhomogeneous Initial Fields



An Inhomogeneous Mode
Can be very big $\rho_{\text{mode}} \gg \rho_{\text{vacuum}}$

Huge parameter space...

ADM decomposition $ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)$

Gravity Sector

3-metric

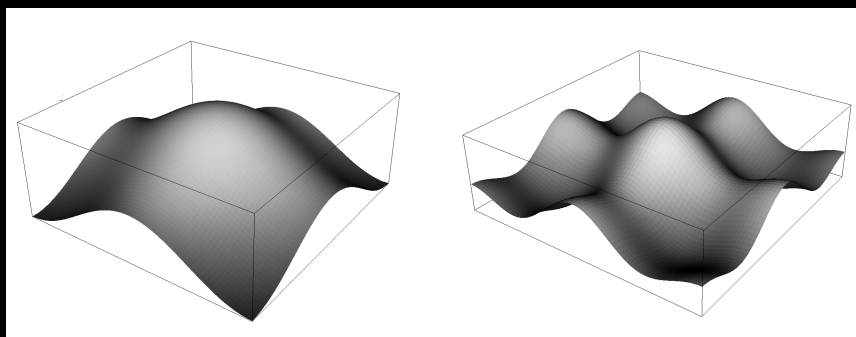
$$\gamma_{ij}(\mathbf{x})$$

Extrinsic Curvature

$$K_{ij}(\mathbf{x}) \sim \dot{\gamma}_{ij}$$

Scalar/Matter Sector

$$\phi(\mathbf{x}) \quad \dot{\phi}(\mathbf{x})$$



$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Traceless

$$A_{ij} + \frac{1}{3}\gamma_{ij}K$$

Trace

Tensor "GW"
Modes

Expansion/
Contraction

Freedom to specify $(\phi, \dot{\phi}, \gamma_{ij}, K_{ij})$
subject to Hamiltonian and 3 Momentum constraints

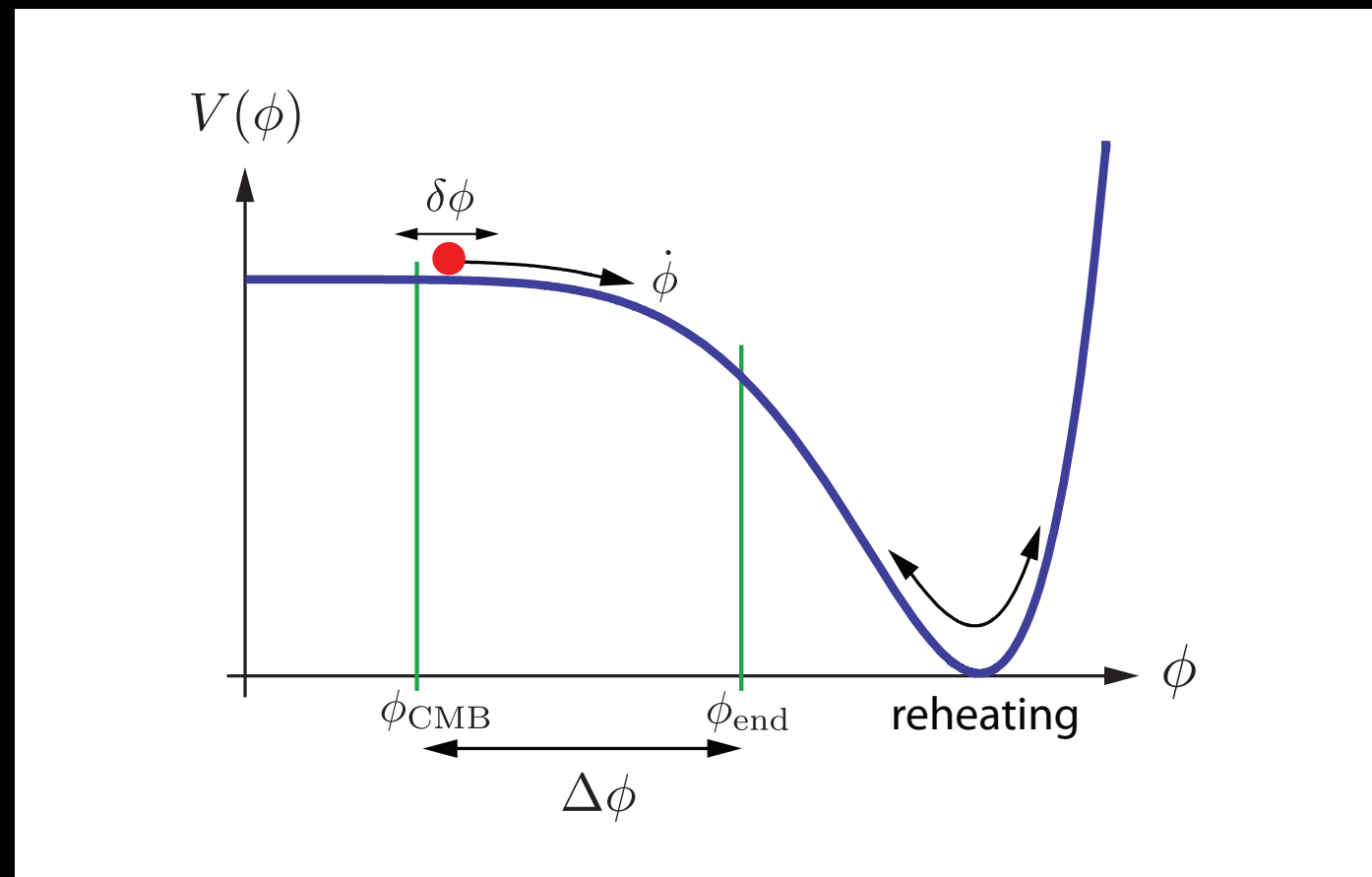
Part 2

Seven Lessons of Inhomogenous Inflation

Lesson 1 : Large Field Inflation

very robust

Lots of room at the inflating plateau! $\delta\phi \gg M_p$



arXiv: 1608.04408

arXiv: 1511.05143

arXiv: 2212.04479

arXiv: 2011.12190

Clough, Lim, DiNunno, Fischer, Paban, Flauger
East, Kleban, Linde, Senatore

Corman + East

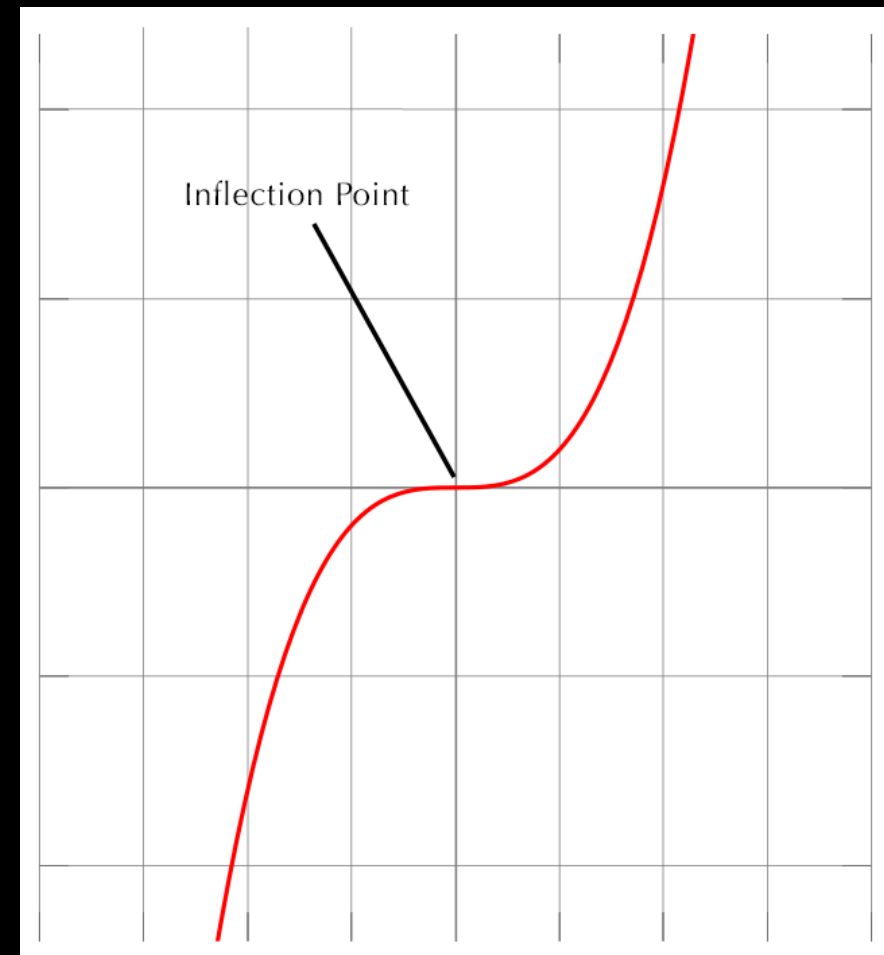
Joana + Clesse

arXiv: 2304.12150 Garfinkle, Ijjas, Steindhardt **Disagrees**

Lesson 2 : Small Field inflation very fragile

Slow-roll domain too small, inflation gets pushed off the slow-roll region.

$$\rho_{grad} \sim 10^{-4} \rho_V$$

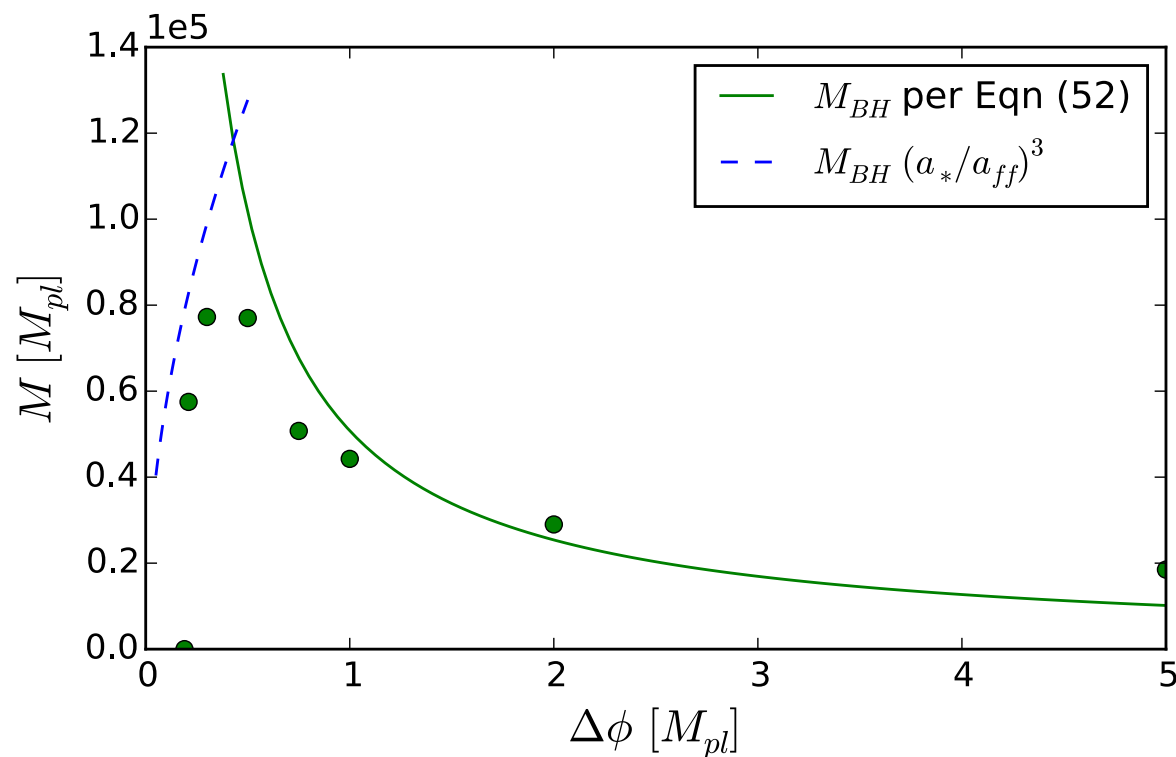


arXiv: 1511.05143
arXiv: 1608.04408

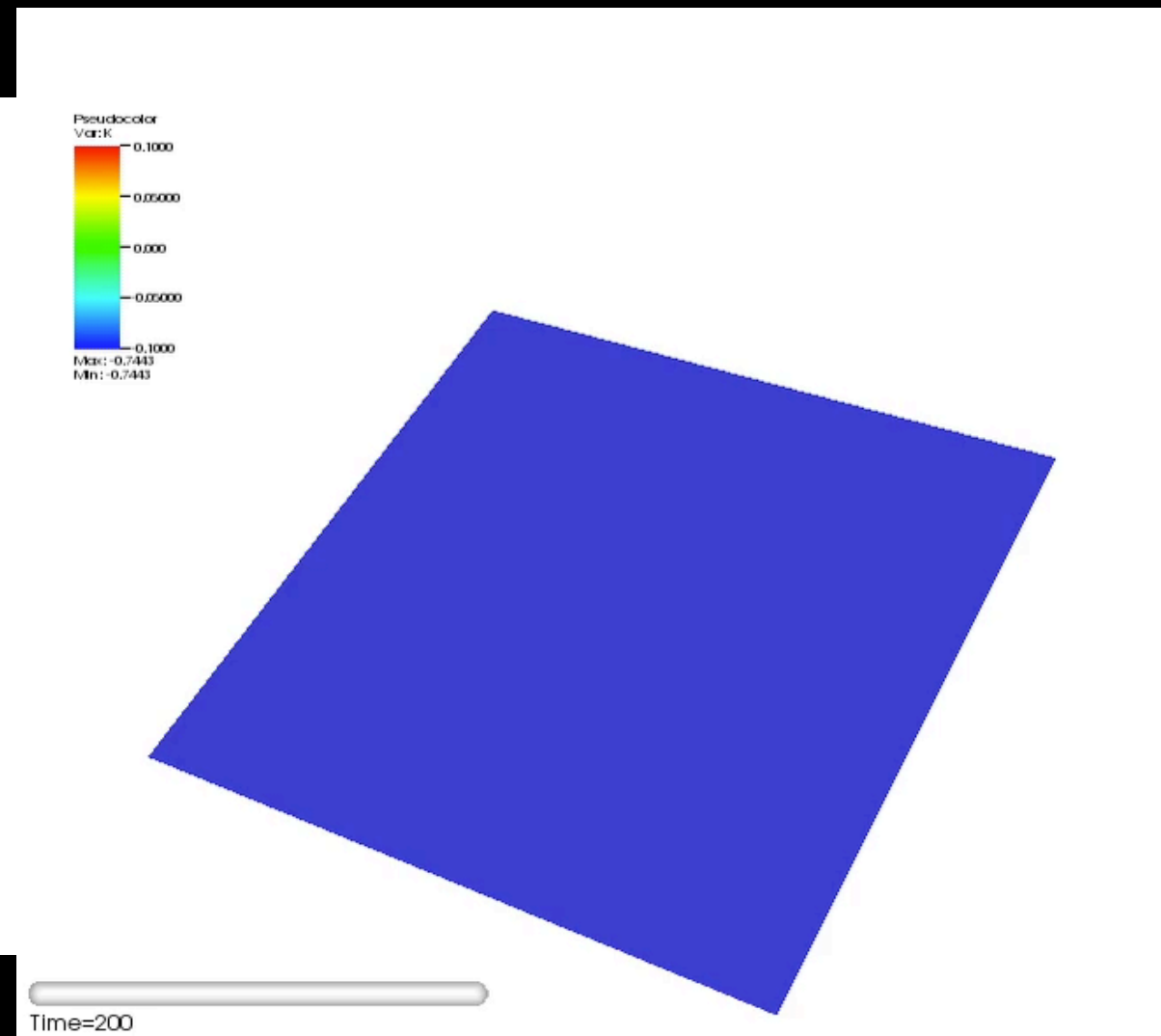
East, Kleban, Linde, Senatore,
Clough, Lim, DiNunno, Fischer, Paban, Flauger

Lesson 3 : no Giant Death Black Holes

You can't make a BH so big it swallows the universe.

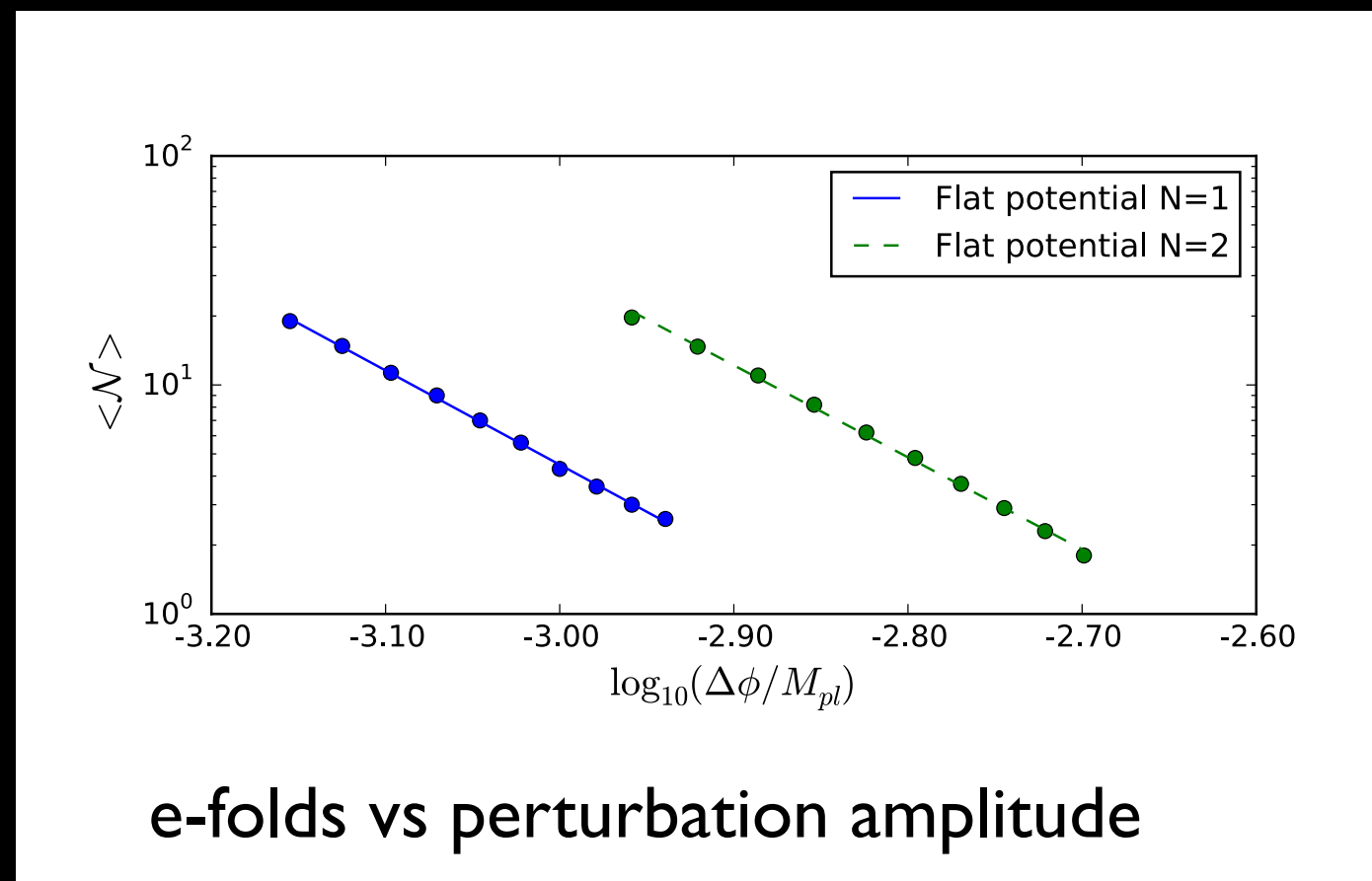


BH Mass vs Perturbation Amp



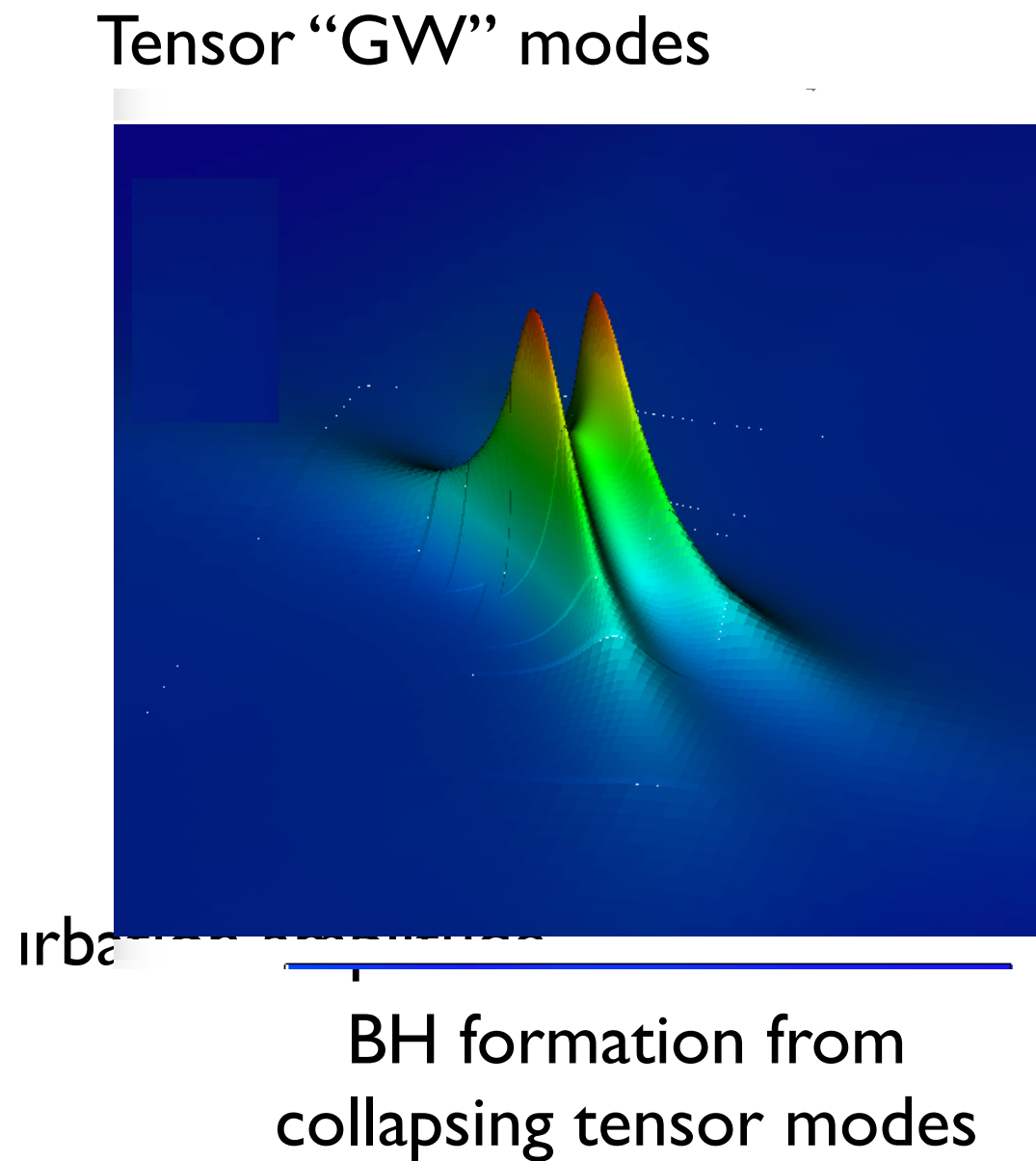
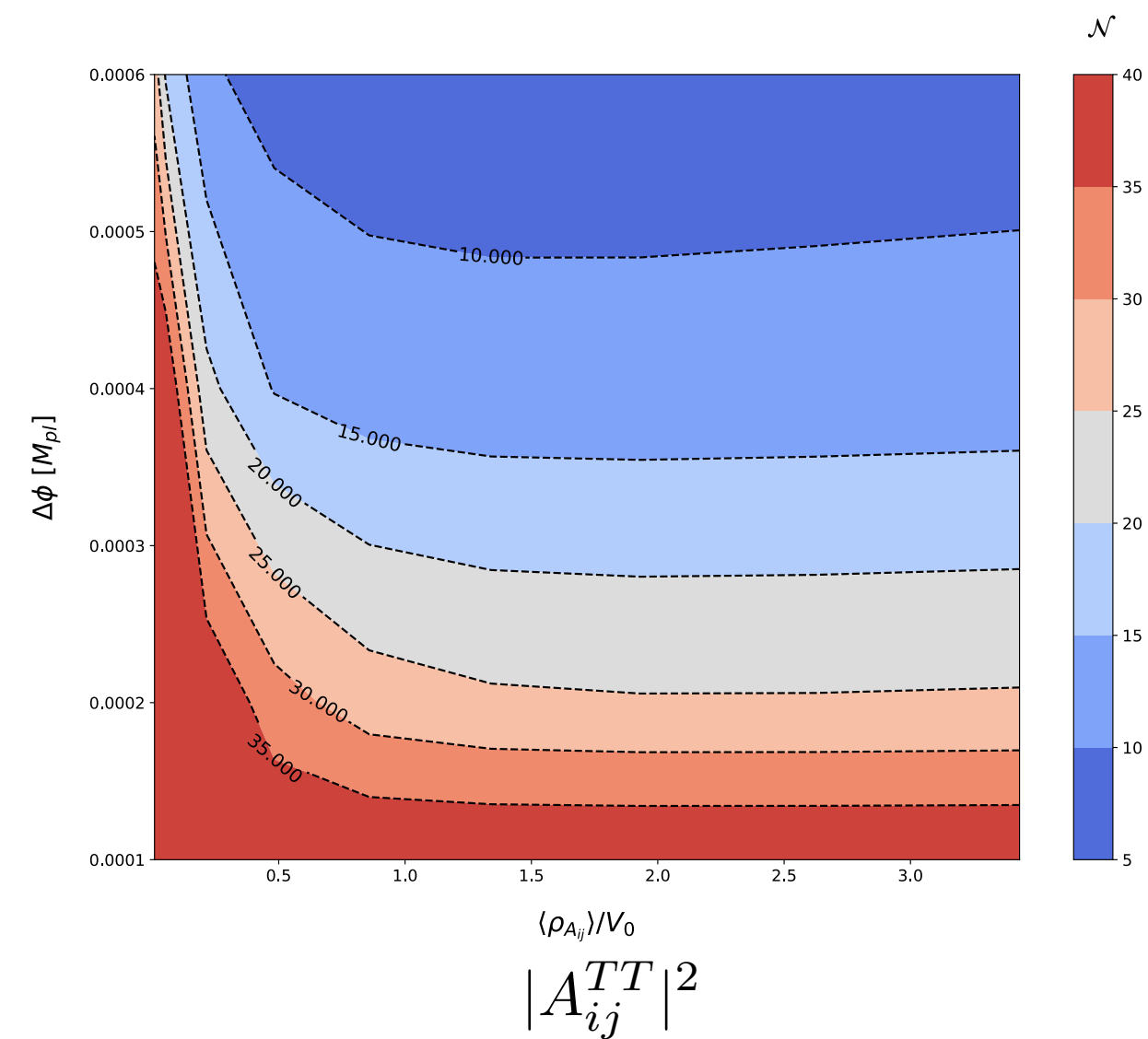
Lesson 4 : IR modes deadlier than UV modes

Hubble-scale wavelengths are deadlier than short wavelength perturbations (given same energy)



arXiv: 1608.04408 Clough, Lim, DiNunno, Fischer, Paban, Flauger
arXiv: 2405.03490 Elley, Aurrekoetxea, Clough, Giannidakis, Lim, Flauger

Lesson 5 : Tensor metric modes are not dangerous

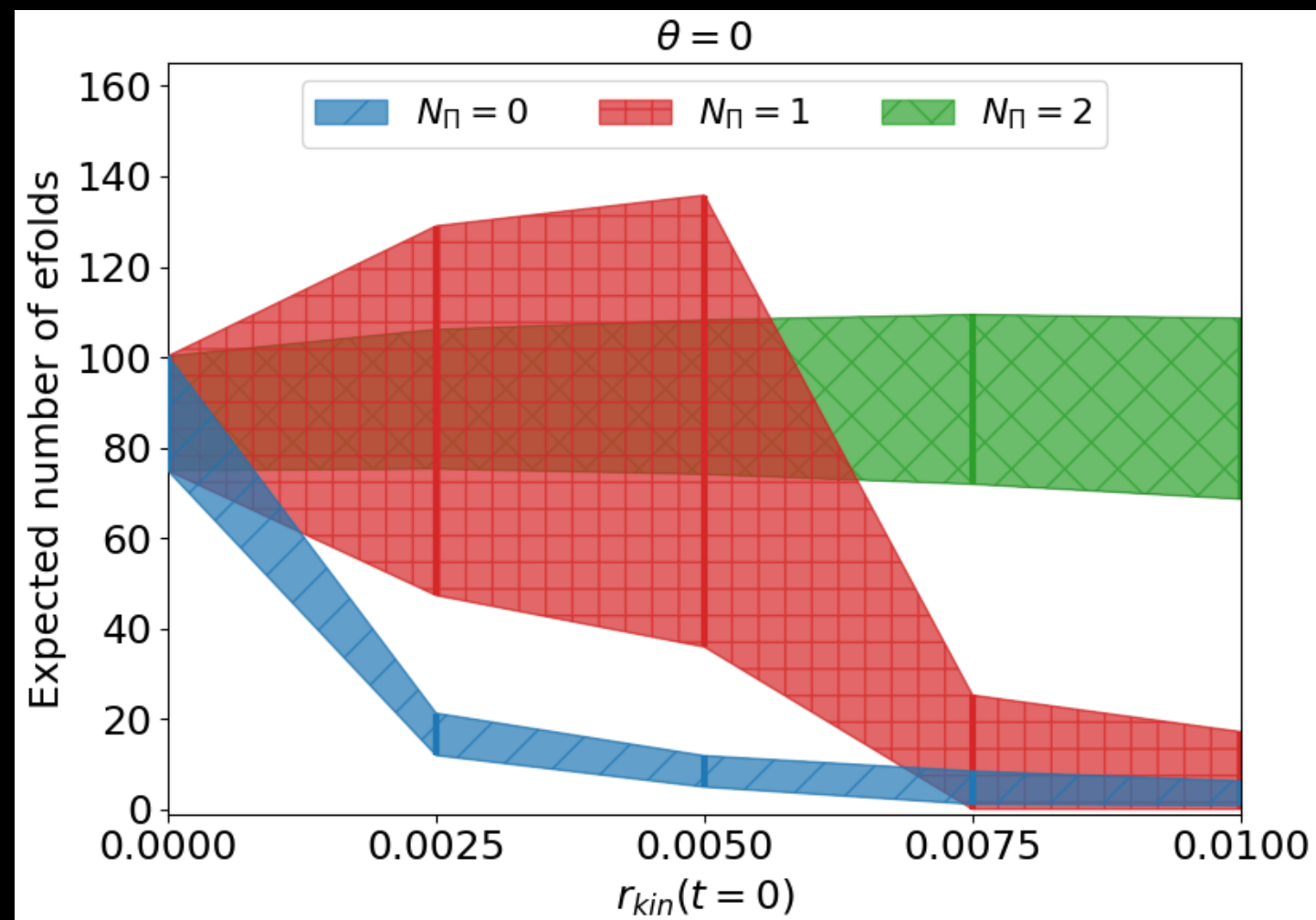


arXiv: 1712.07352

Clough, Flauger, Lim

Lesson 6 : Fast roll makes inflation (not much) less robust

Adding kinetic perturbations $\dot{\phi} \neq 0$

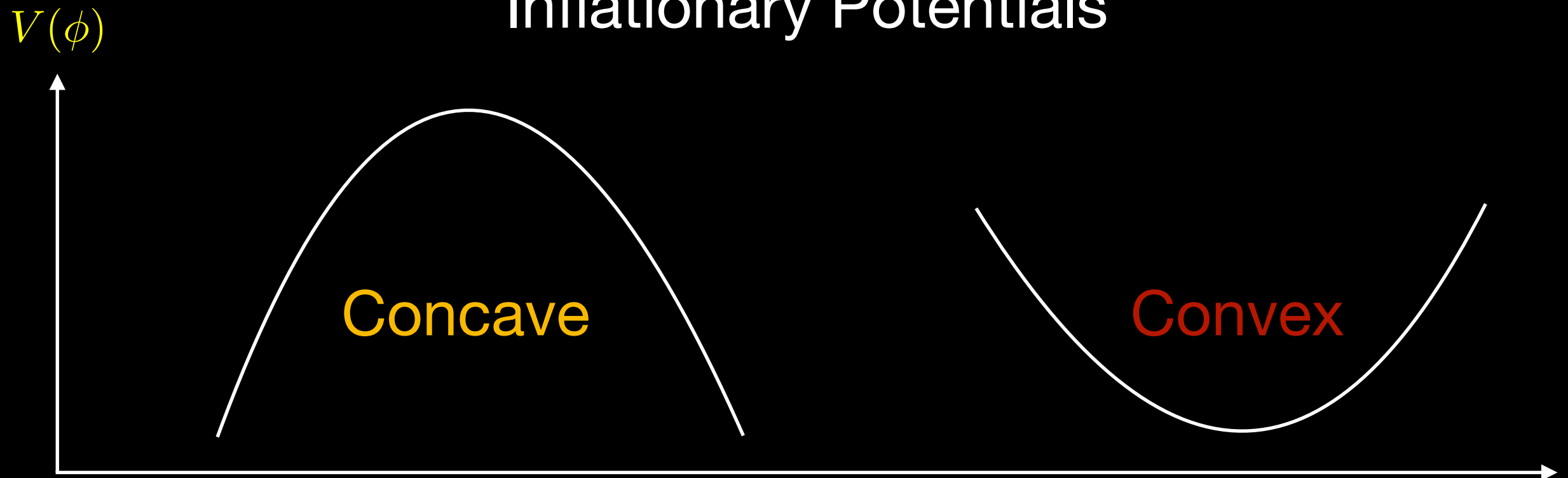


arXiv: 2212.04479 Corman + East

arXiv: 2405.03490 Elley, Aurrekoetxea, Clough, Giannidakis, Lim, Flauger

Lesson 7 : Concave potentials are less robust

Inflationary Potentials



$$\frac{d^2V}{d\phi^2} < 0$$

$$\frac{d^2V}{d\phi^2} > 0$$

~~Does Inflation always inflate
regardless of initial conditions?~~

Better question : *Suppose inflation happened, what
can it tell us about the universe?*

Part 3

A lower bound on scalar-tensor ratio r

Giannadakis, Elley, Flauger and Lim (arXiv : 2512.13673)

Observables or Predictions?

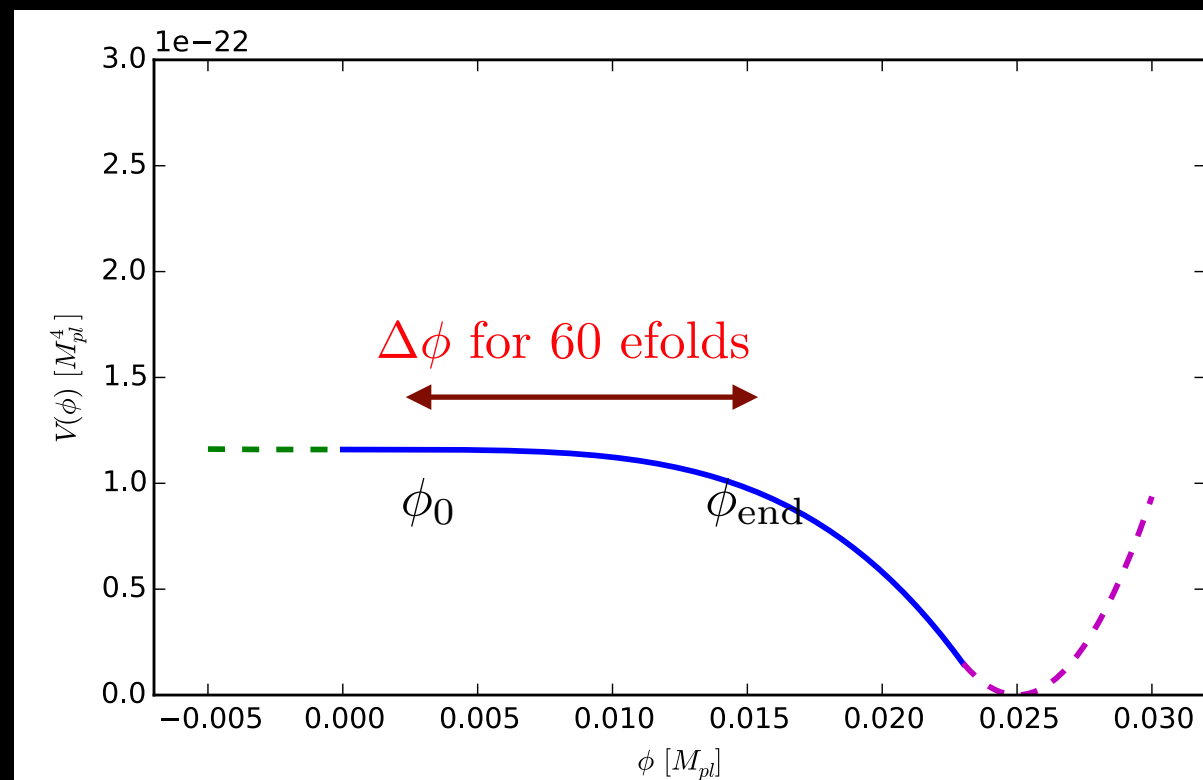
Are there generic observable predictions for inflation if the inflation starts inhomogeneously?

1. For **fixed inhomogeneous initial conditions**, some inflation models can fail.
2. Suppose, for a given class of models, the failure depends on **the model parameter**.
3. If an **observable** depends on this model parameter, then this implies a bound on it.

“Characteristic scale” of inflation

Roest (2014), Creminelli et al (2015), CMB-S4 White paper (2016)

For a given N efolds till end of inflation, how much field space did the inflation traverse?

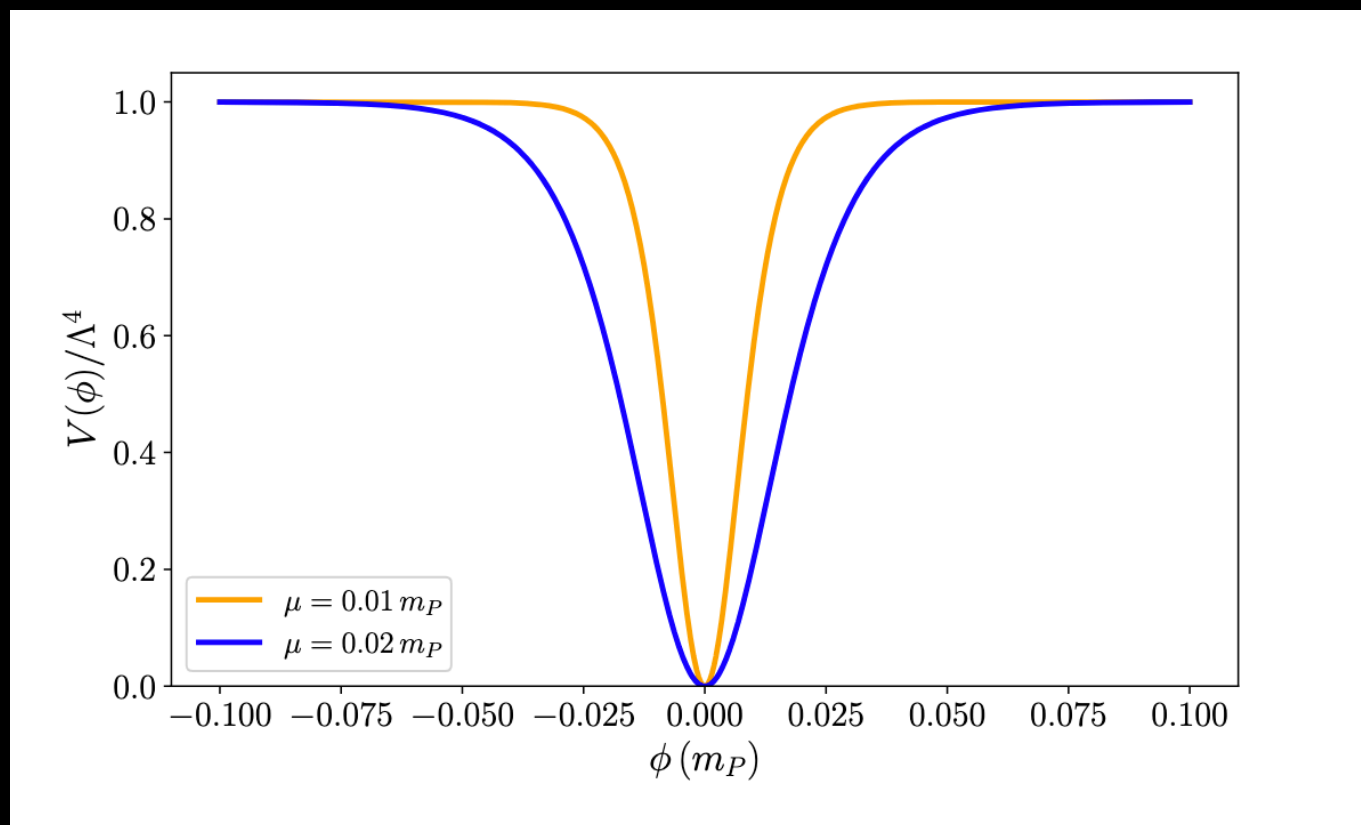


The bigger the traverse, the bigger its characteristic scale, the more robust inflation is.

“Characteristic scale” of inflation

α – attractor T – model

Kallosh+Linde (2013)



$$V(\phi) = \Lambda^4 \tanh\left(\frac{\phi}{\mu}\right)$$

characteristic scale μ

It has definite prediction for scalar-tensor index

$$r = \frac{16\pi\mu^2}{M_p^2 N_*^2}$$

Our NR Lessons predict : higher μ , more robust inflation.

What initial conditions?

Without a fundamental theory, we make guesses.

Consider *volume averaged* inflaton field energies

Gradient Energy

$$\rho_{\nabla}$$

Kinetic Energy

$$\rho_{\text{kin}}$$

Potential Energy

$$V(\phi)$$

What initial conditions?

Without a fundamental theory, we make guesses.

Consider *volume averaged* inflaton field energies

Gradient Energy

$$\rho_{\nabla}$$

Kinetic Energy

$$\rho_{\text{kin}}$$

Potential Energy

$$V(\phi)$$

Start with $\rho_{\nabla} \gg V$, $\rho_{\text{kin}} \gg V$ and evolve...

If V dominates $\Rightarrow$ inflation!

What initial conditions?

Without a fundamental theory, we make guesses.

Consider *volume averaged* inflaton field energies

Gradient Energy

$$\rho_{\nabla}$$

Kinetic Energy

$$\rho_{\text{kin}}$$

Potential Energy

$$V(\phi)$$

Start with $\rho_{\nabla} \gg V$, $\rho_{\text{kin}} \gg V$ and evolve...

If V dominates $\Rightarrow$ inflation!

If no inflation, then “worse” case is some kind of equipartition $\rho_{\nabla} \sim \rho_{\text{kin}} \sim V$

Initial Metric choices

(γ_{ij}, K_{ij}) must obey Hamiltonian and Momentum constraints

Still need to make 8 choices

Conformally flat initial spatial metric $\gamma_{ij} = \delta_{ij}$

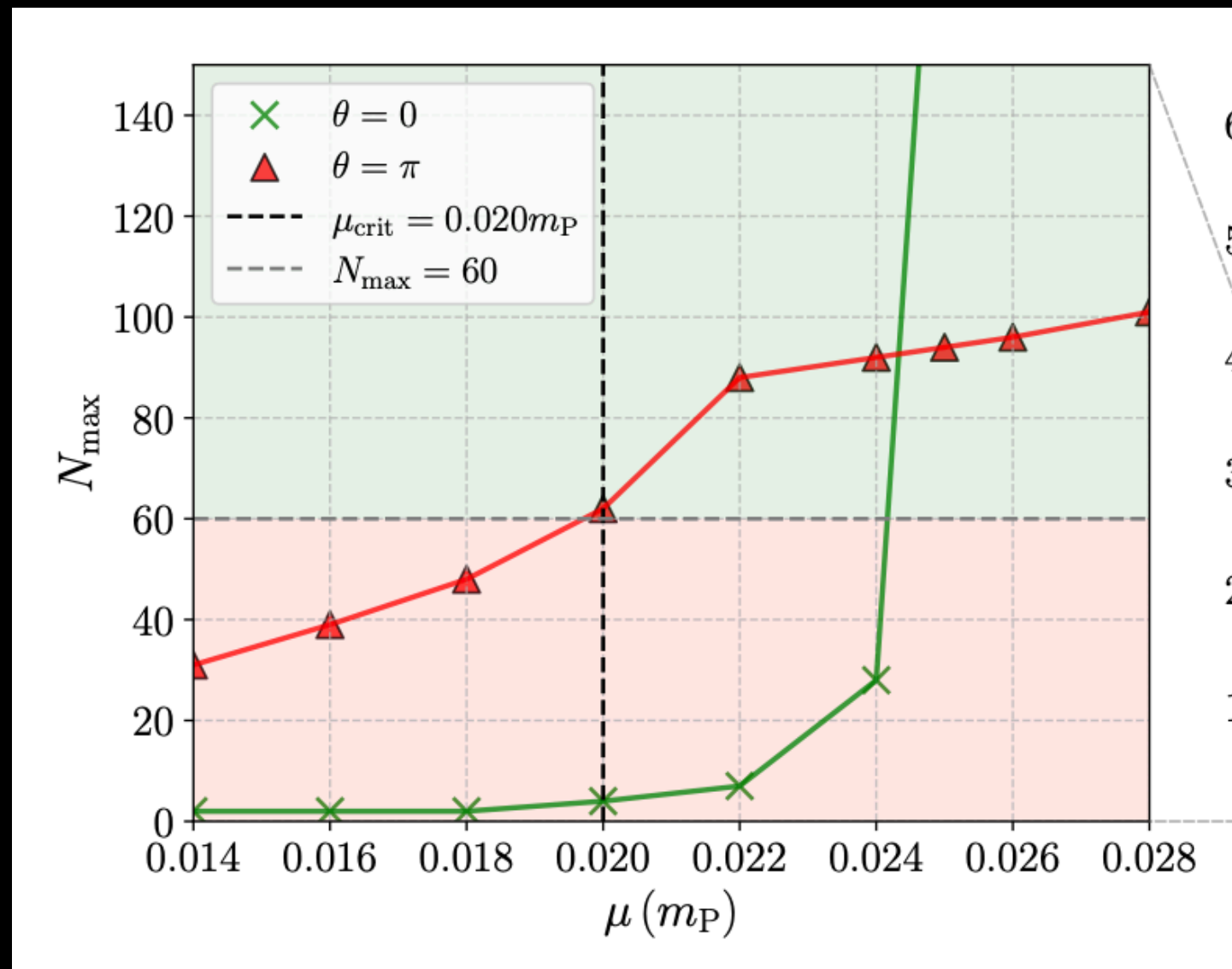
Traceless-traverse components of K_{ij} set to zero
(Lesson 5)

4 remaining components of K_{ij} solved consistently
via Hamiltonian and Momentum constraints.

Aurrekoetxea, Clough, Lim (2023)

Aurrekoetxea et al (+ Lim) (2025)

Results



For in-phase $\theta = 0$ $\mu_{\text{crit}} = 0.024 M_p \Rightarrow r = 5.6 \times 10^{-6}$

There is a $\mu < \mu_{\text{crit}}$ where inflation always fails.

A lower bound on r

Increasing $\rho_{\nabla} \sim \rho_{\text{kin}} > V$ makes inflation *less* robust.

This means μ_{crit} has to be *bigger* for successful inflation
(Lessons 1+2) $\Rightarrow r_{\text{crit}}$ must increase.

Inhomogeneous inflation predicts a *lower* bound $r \gtrsim 10^{-6}$

(At least for this model of inflation.)

Summary

- NR can now precisely test models of inflation starting from inhomogeneous initial conditions.
- Successful inflation depends on IC and inflationary models, and there are general rules which are model independent.
- Requiring successful inhomogeneous inflation can put *lower* bound on the scalar-tensor ratio r .

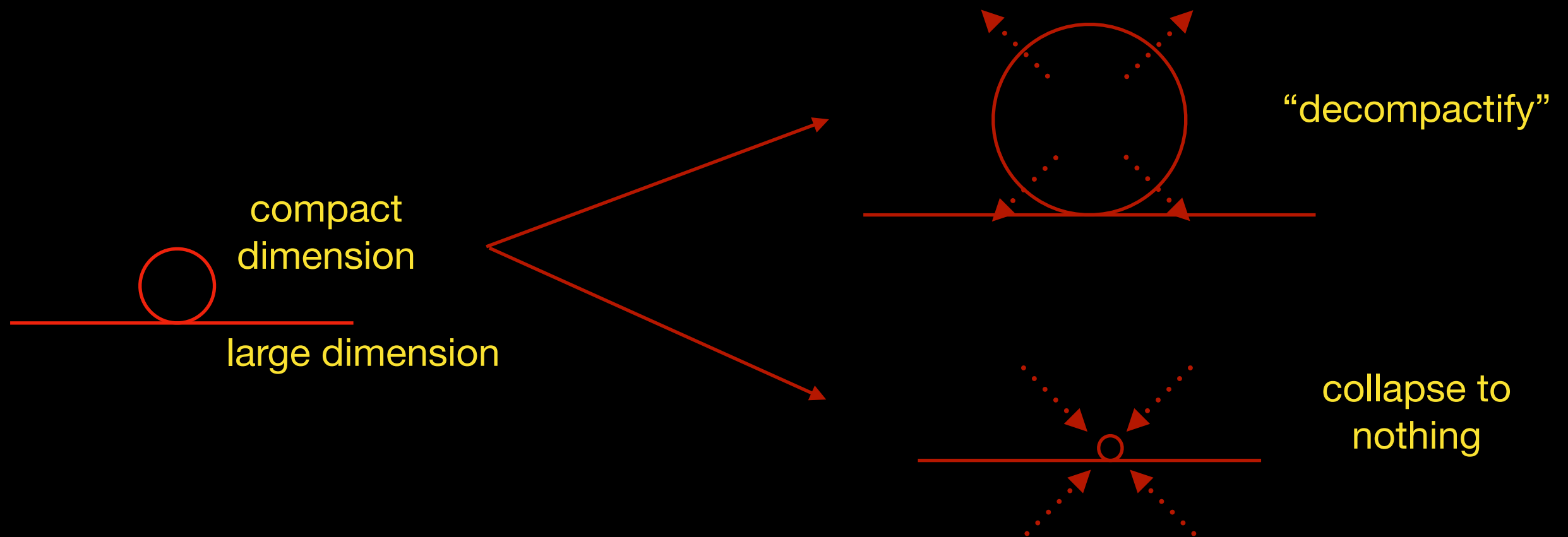
Thank you!

Post-credits Part 4

Beyond the Big Bang into Extra Dimensions?

Giannadakis, Righi, Flauger and Lim (soon to be submitted!)

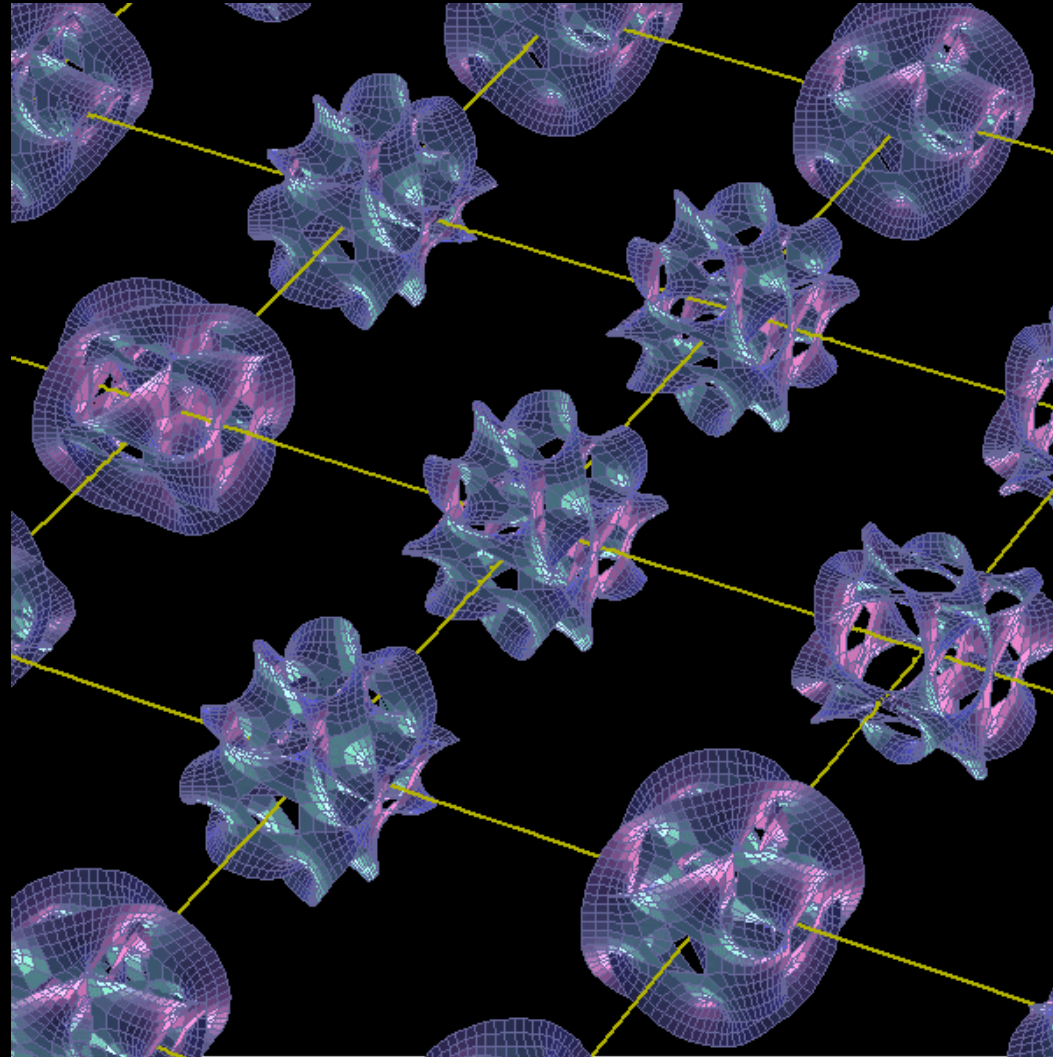
Flux stabilized compactified Extra Dimensions



Need fluxes (gauge fields) to stabilize them.

Key use : stabilization for Calabi-Yau in string theories!

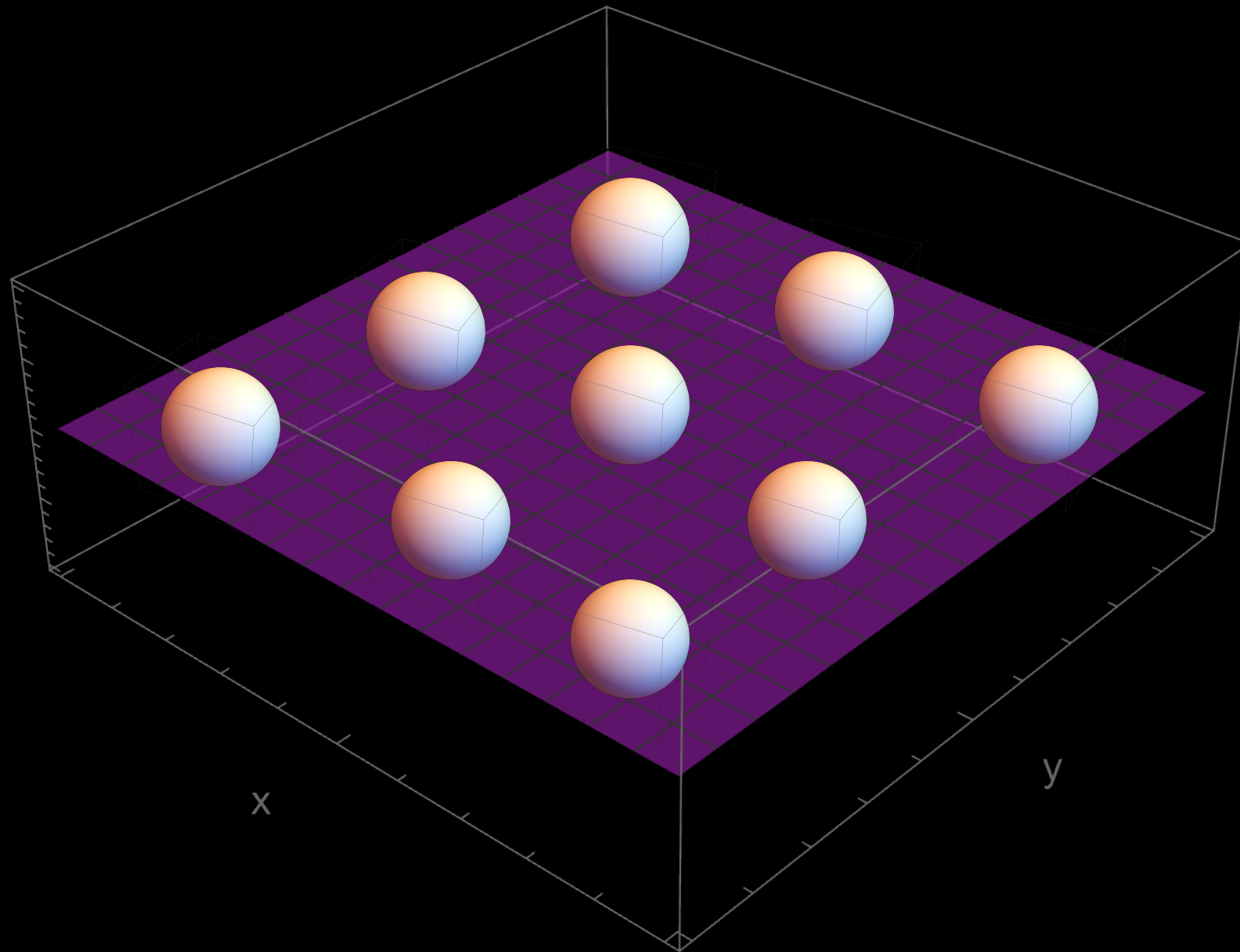
Flux stabilized compactified Extra Dimensions



Question : what is the **Non-Linear stability** of these models?

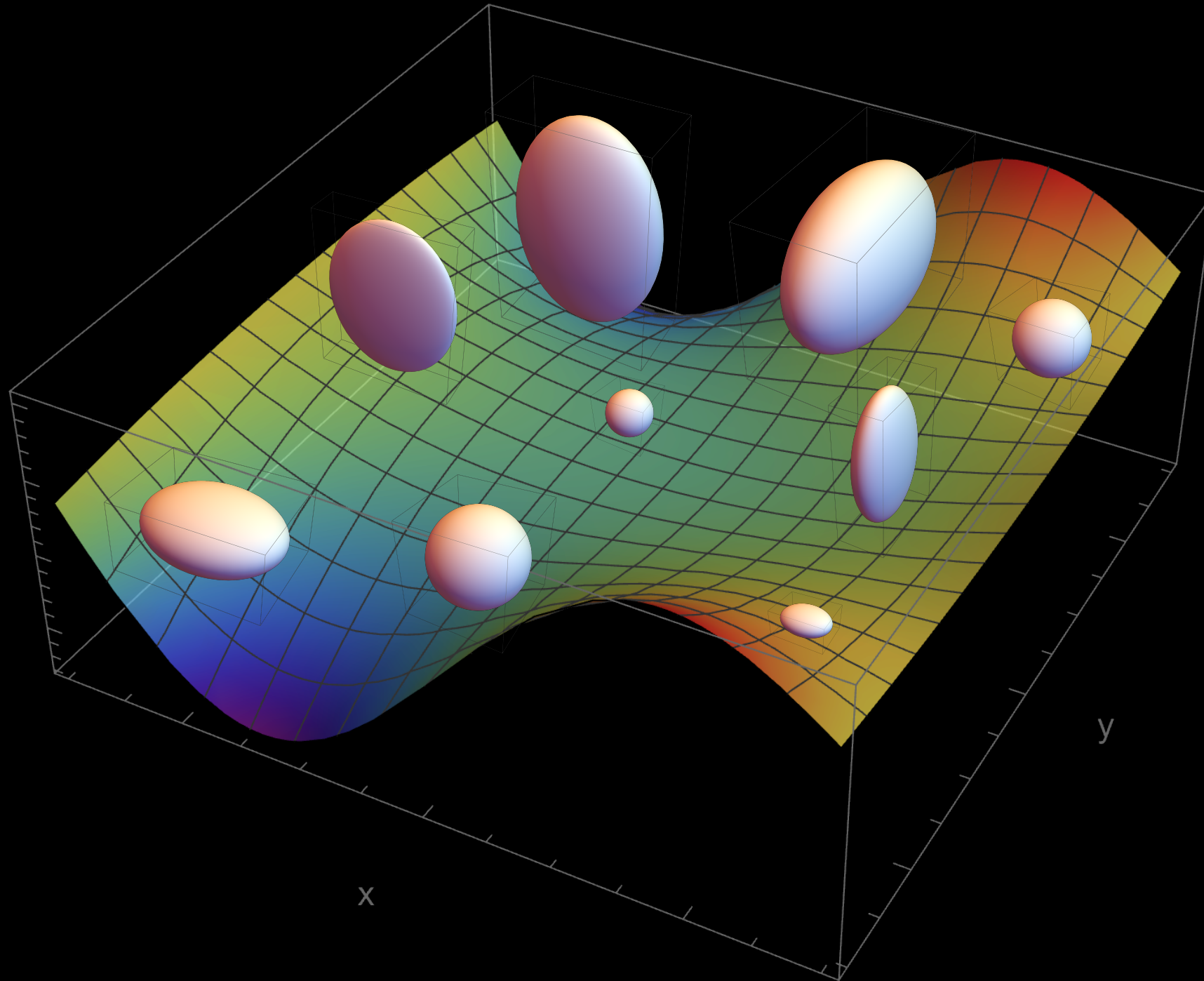
Investigate with Numerical Relativity!

Toy : Freund-Rubin



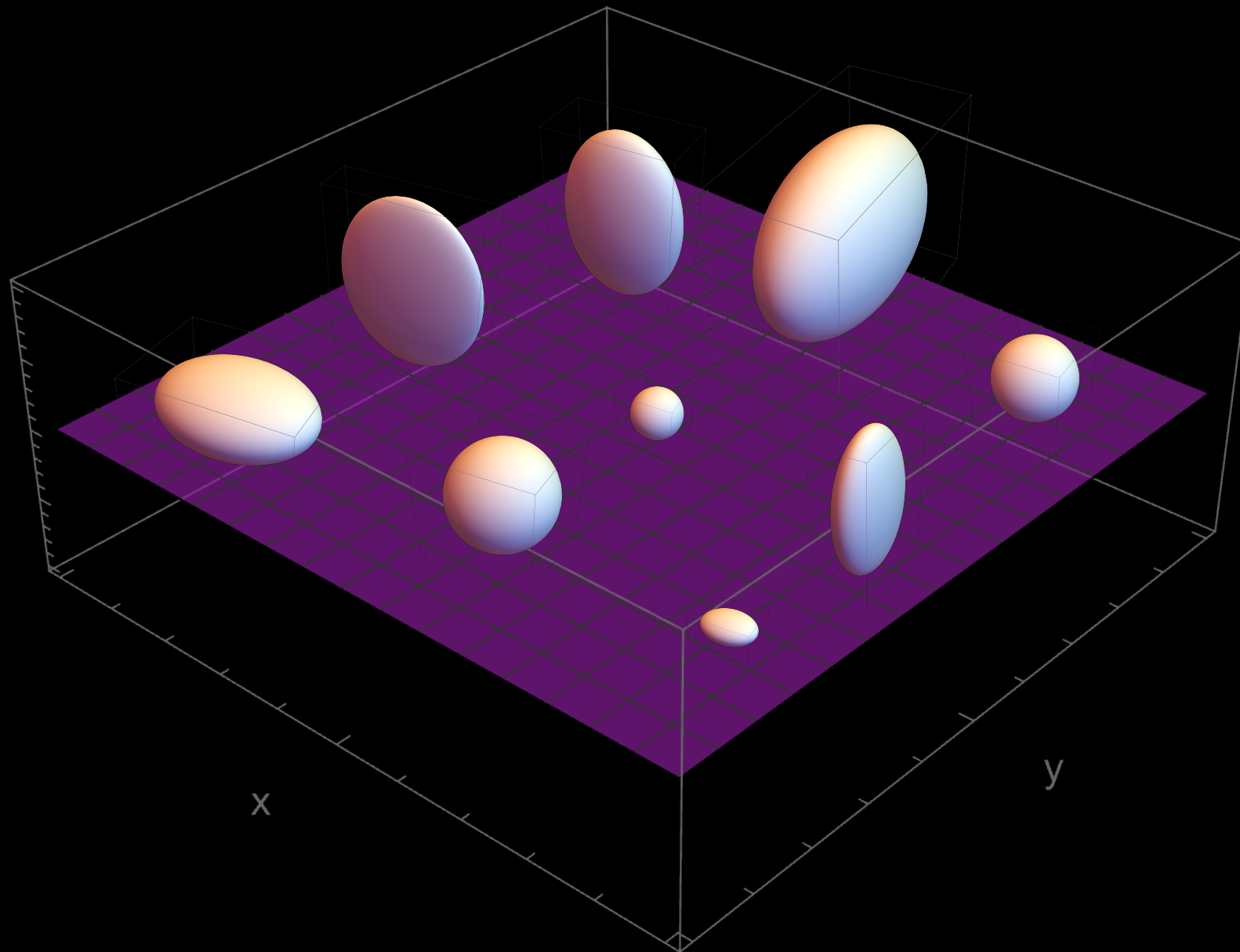
Linear stability assumes homogenous large dimensions (FRW) and small perturbations of compact dimensions.

Toy : Freund-Rubin



Non-linear stability = no FRW and large compact dim perturbations.

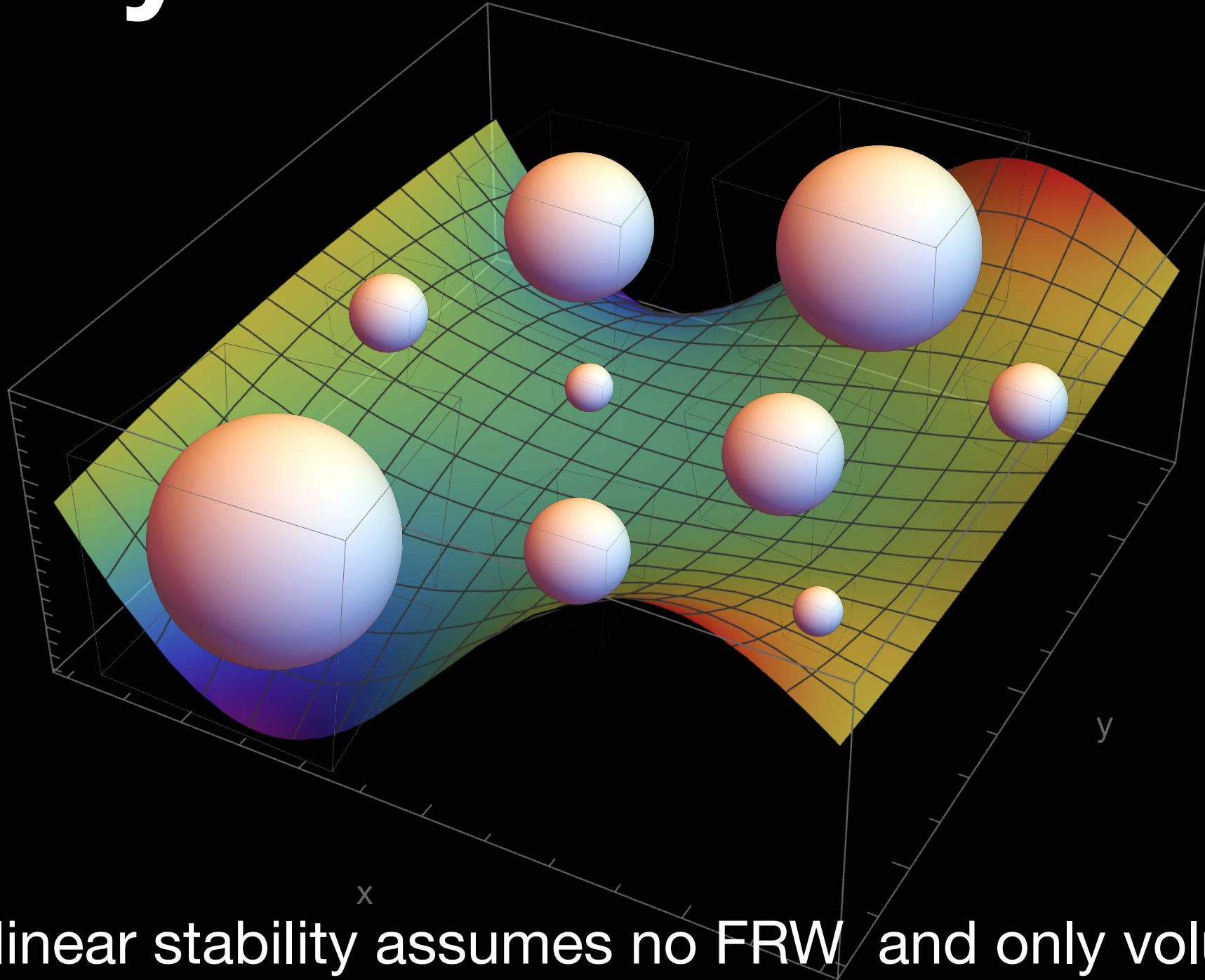
Toy : Freund-Rubin



Non-Linear stability assuming FRW and large compact perturbations with full GR.

Corman, East, Johnson (2022)

Toy : Freund-Rubin



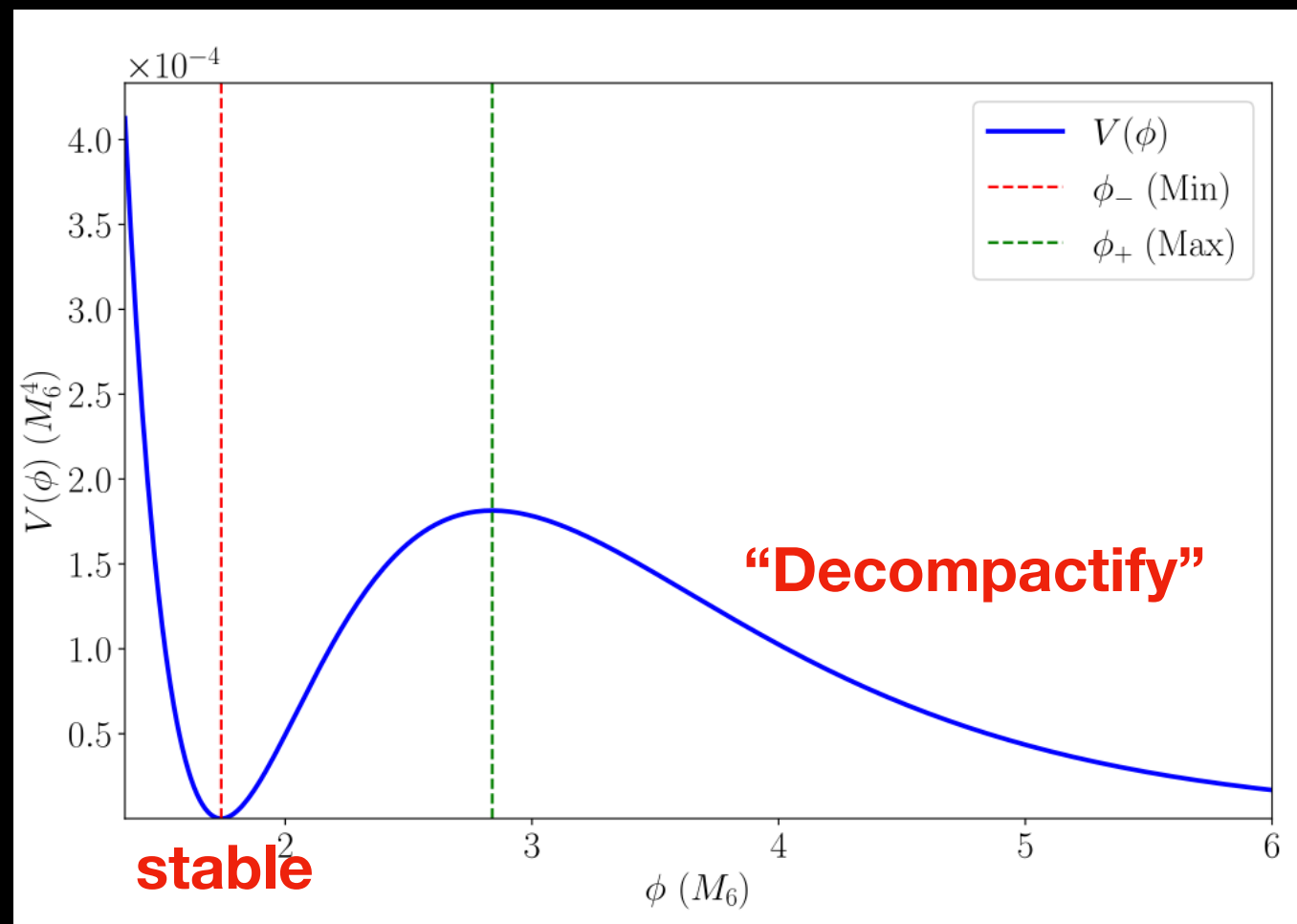
Non-linear stability assumes no FRW and only volume “breathing” compact mode.

Giannadakis, Righi, Flauger, Lim (soon!)

4d Effective Potential

$$S = \frac{M_6^2}{2} \int d^6x \sqrt{-g_6} (R_6 - 2\Lambda_6 - \frac{1}{2} F_{AB} F^{AB}) \rightarrow \frac{M_6^2}{2} \int d^4x \sqrt{-g_4} M_4^2(x) (R_4 + \text{Kinetic terms} + V_{eff})$$

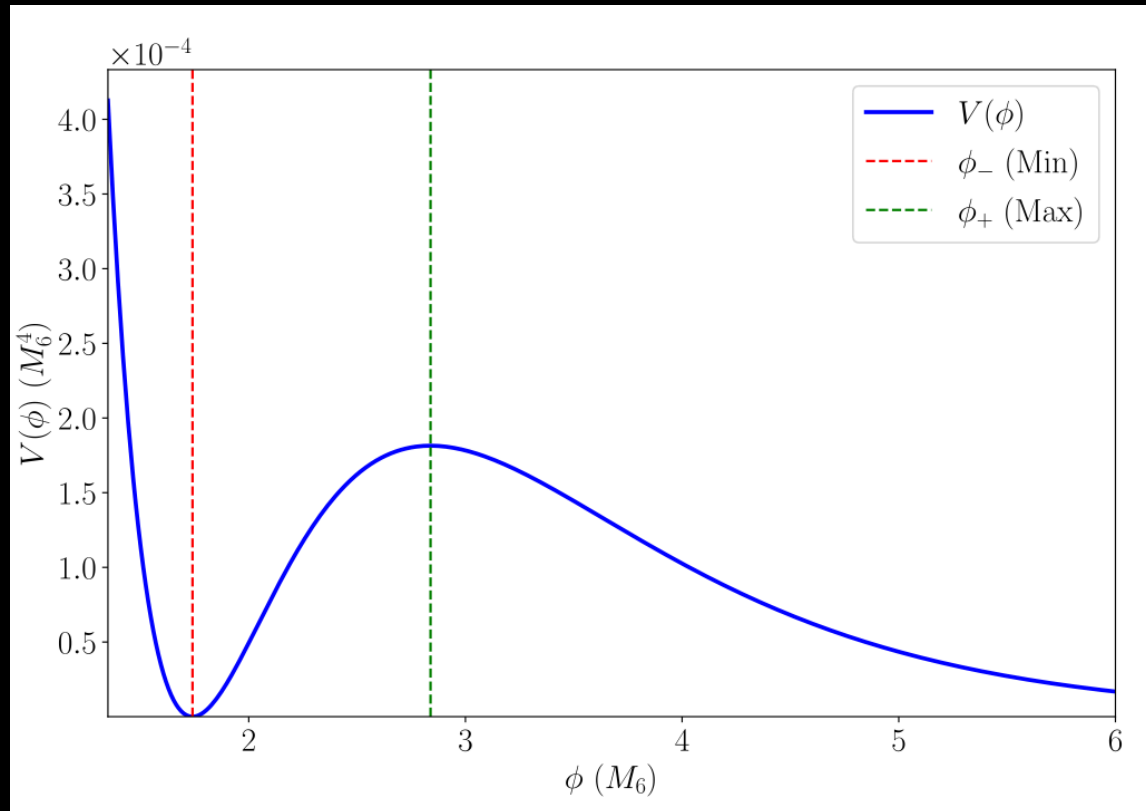
“Energy” felt by Large dimensions



Radius of Sphere

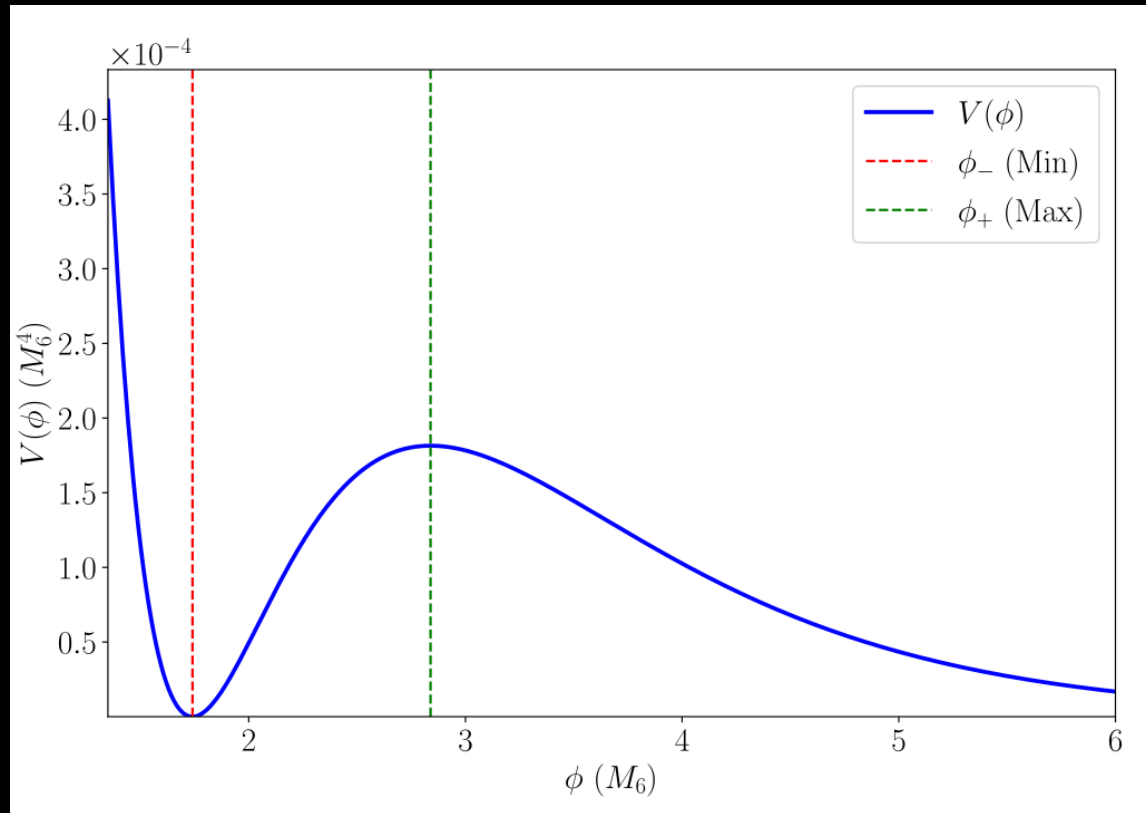
System is a 3+1D non-minimally coupled GR equations :
we can use NR!

Set-up and Results

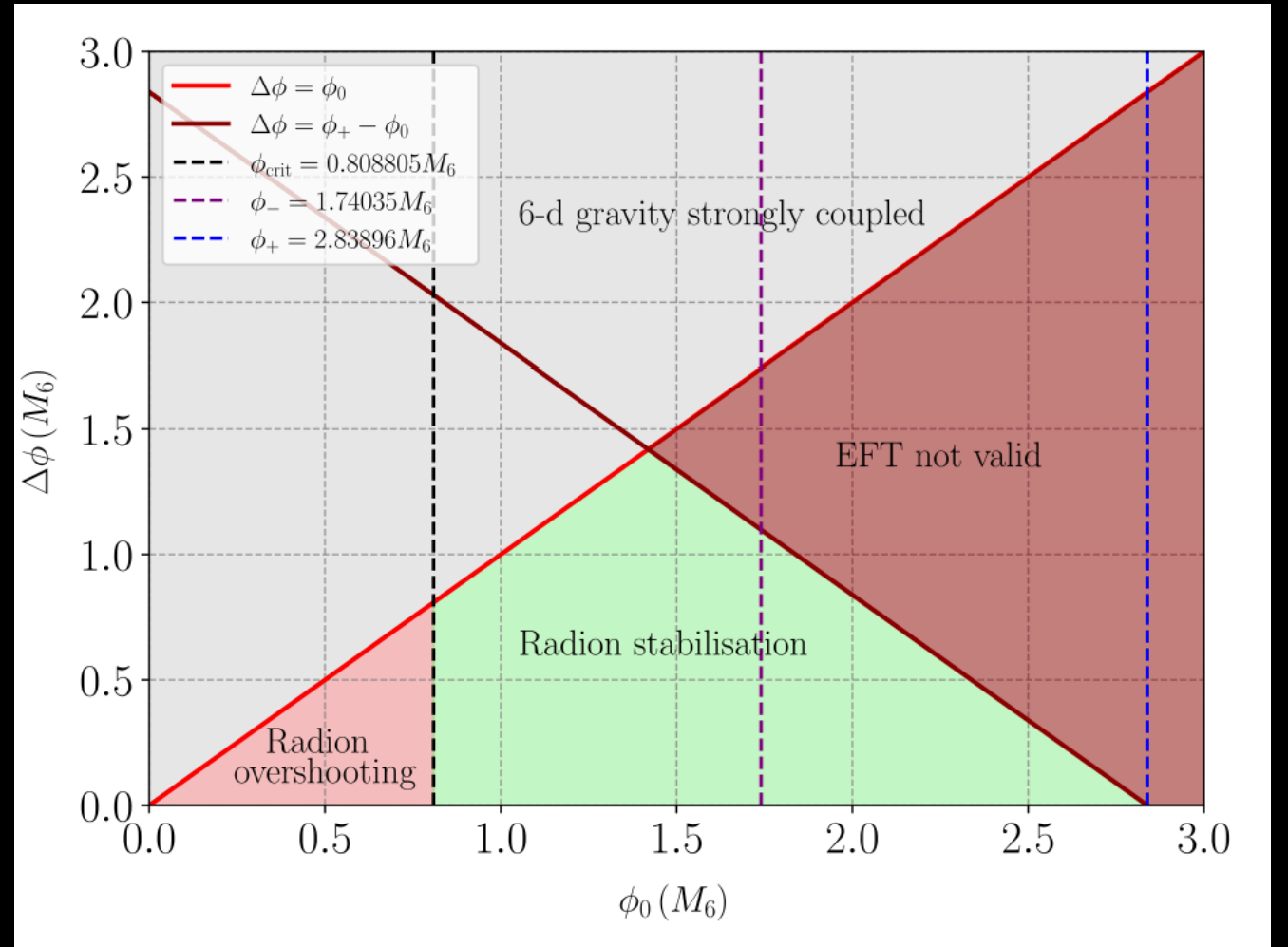


$$\phi_{\text{init}}(\mathbf{x}) = \phi_0 + \frac{\Delta\phi}{3} \sum_{i=1}^3 \cos\left(\frac{2\pi x_i}{L}\right)$$

Set-up and Results



$$\phi_{\text{init}}(\mathbf{x}) = \phi_0 + \frac{\Delta\phi}{3} \sum_{i=1}^3 \cos\left(\frac{2\pi x_i}{L}\right)$$



More to come!