

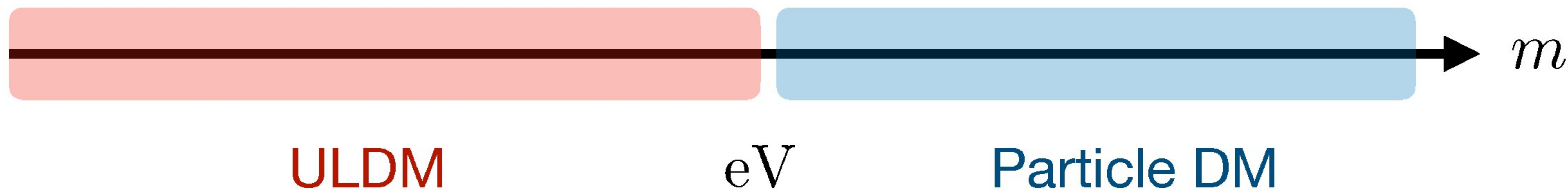
Wave Dark Matter:

Stochastic Fluctuations and Gravitational Probes

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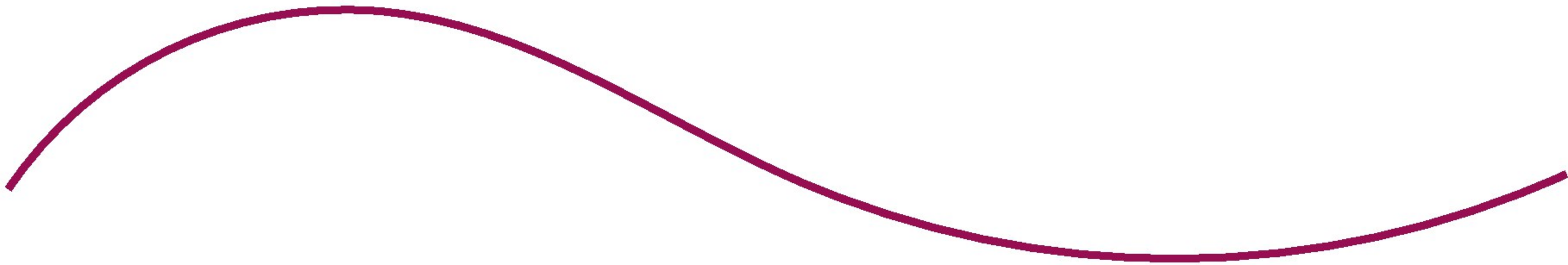
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Ultralight dark matter (ULDM)
as bosonic DM candidates with $m < \text{eV}$



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$$N_{\text{occ}} \sim n_{\text{dm}} \lambda^3 \sim \left(\frac{10 \text{ eV}}{m} \right)^4$$



$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 + h(\phi) \mathcal{O}_{\text{SM}} \right]$$

Gravitational
non-gravitational

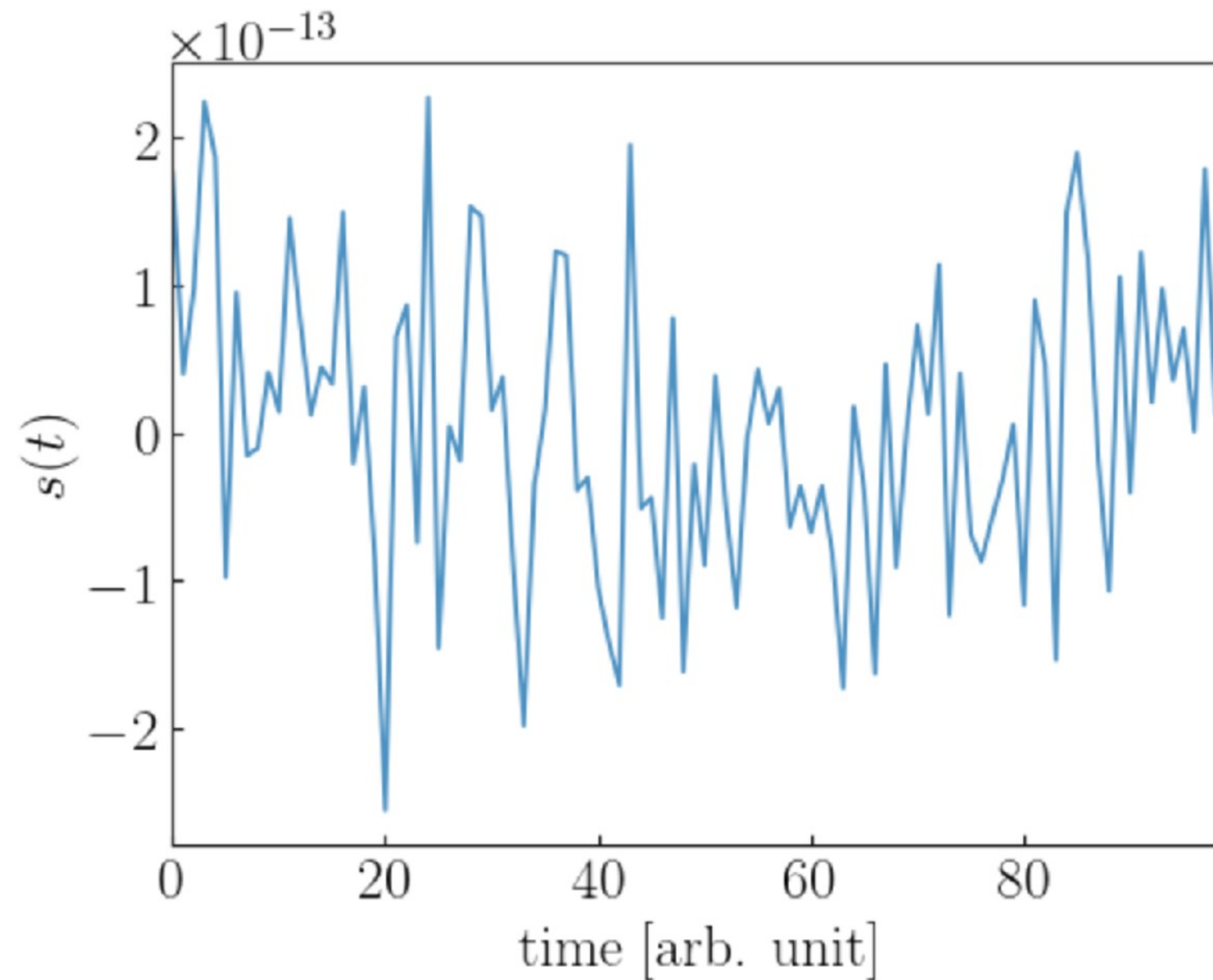
we do not measure dark matter directly but

$$s(t)$$

e.g. electric current

What signal should we expect?

$$s(t) = H[\phi(t)]$$



system response depends on
types of system and of ULDM interaction

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$$\propto \phi \quad \text{linear}$$

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types of system and of ULDM interaction

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$$\propto \phi \quad \text{linear}$$

$$\phi^2 \quad \text{quadratic (nonlinear)}$$

Example 1

$$\mathcal{L} = \frac{1}{4} g \phi F \tilde{F}$$

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$$\nabla \times \mathbf{B} \approx \dot{\mathbf{E}} - g \dot{\phi} \mathbf{B}$$

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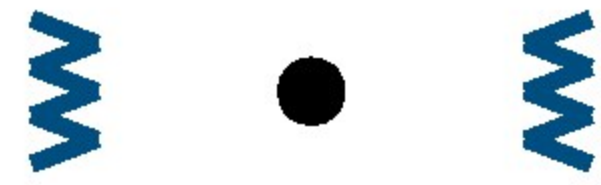
$$\mathcal{L} = \frac{1}{4} g \phi F \tilde{F}$$

$$\nabla \times \mathbf{B} \approx \dot{\mathbf{E}} - g \dot{\phi} \mathbf{B}$$

$$s(t) \propto \dot{\phi}$$

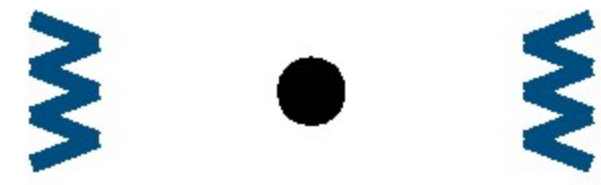
An example of **linear** signal

Example 2



$$\ddot{x} = -\nabla\Phi$$

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$$\rho = \frac{1}{2} \left[\dot{\phi}^2 + (\nabla \phi)^2 + m^2 \phi^2 \right]$$

Example 2

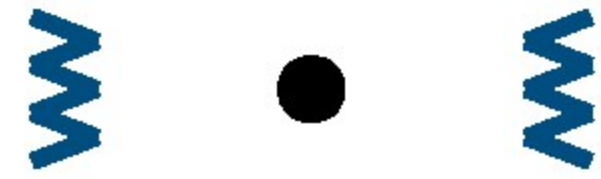


$$\ddot{x} = -\nabla \Phi$$

$$\nabla^2 \Phi = 4\pi G \delta \rho$$

$$\rho = \frac{1}{2} \left[\dot{\phi}^2 + (\nabla \phi)^2 + m^2 \phi^2 \right] \sim \phi^2$$

Example 2



An example of **quadratic** signal

$$s(t) = H[\phi(t)] \propto \phi^2$$

regardless
the signal will be a function of ULDM field

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the signal will **inherit**
properties of underlying ULDM field

to understand what signal one should expect
we first need to understand

$$\phi(t)$$

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$$\phi(t)$$

how do we describe ULDM field?

the description

$$\phi(t, \boldsymbol{x}) = \sum_{\boldsymbol{k}_i} \frac{1}{\sqrt{2mV}} \left[\alpha_{\boldsymbol{k}_i} e^{-i\boldsymbol{k}_i \cdot \boldsymbol{x}} + \alpha_{\boldsymbol{k}_i}^* e^{i\boldsymbol{k}_i \cdot \boldsymbol{x}} \right]$$

the description

random variables

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$$\alpha_{\mathbf{k}_i} = r_{\mathbf{k}_i} e^{i\theta_{\mathbf{k}_i}}$$

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$$\begin{aligned} \phi(t, \mathbf{x}) &= \sum_{\mathbf{k}_i} \frac{1}{\sqrt{2mV}} \left[\alpha_{\mathbf{k}_i} e^{-i\mathbf{k}_i \cdot \mathbf{x}} + \alpha_{\mathbf{k}_i}^* e^{i\mathbf{k}_i \cdot \mathbf{x}} \right] \\ &= \sum_{\mathbf{k}_i} \sqrt{\frac{2}{mV}} r_{\mathbf{k}_i} \cos(\omega_{\mathbf{k}_i} t - \mathbf{k}_i \cdot \mathbf{x} + \theta_{\mathbf{k}_i}) \end{aligned}$$

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this expression is nothing but sum of cosine waves

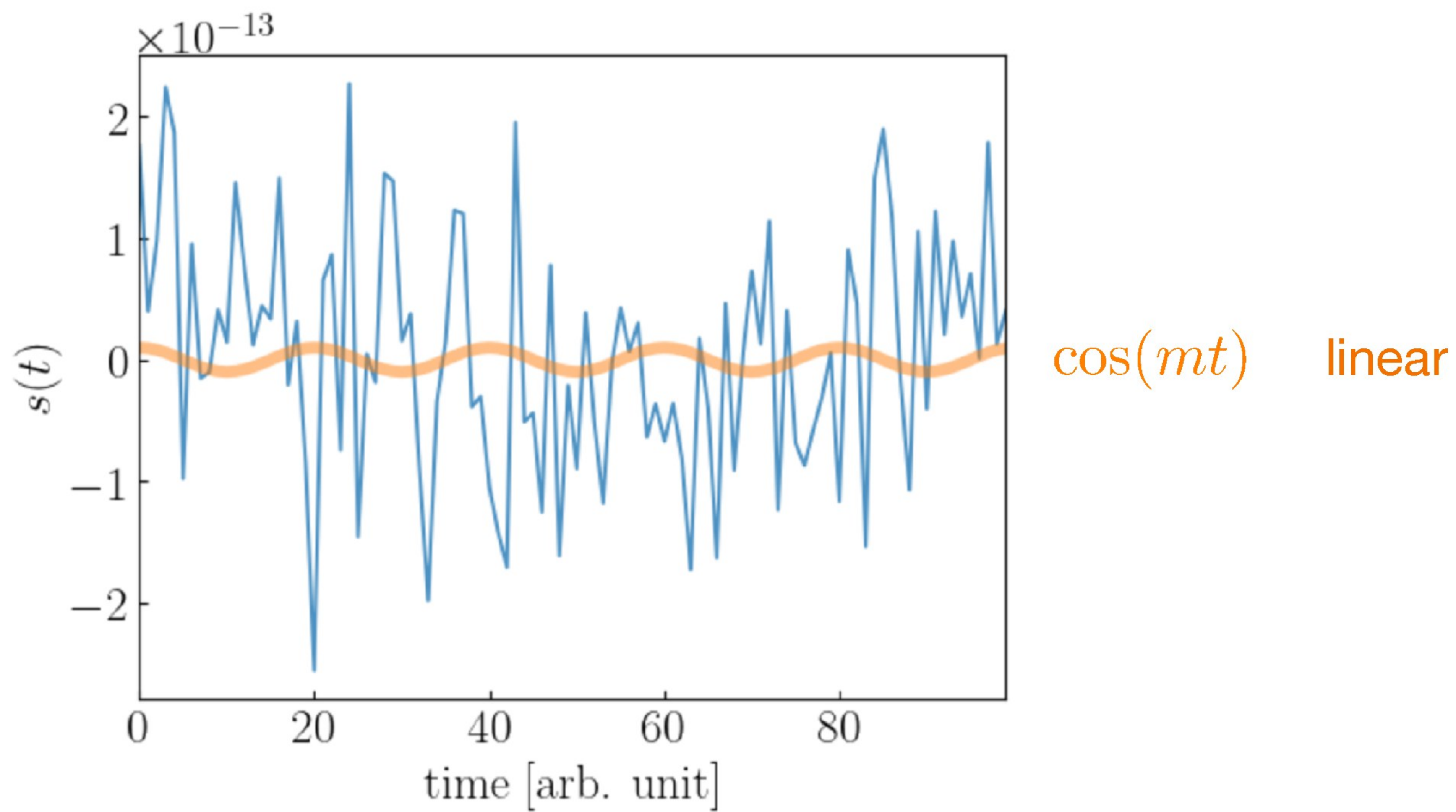
for **linear signals**

$$s(t) \propto \phi(t) \propto \sum_{\mathbf{k}_i} r_{\mathbf{k}_i} \cos(mt + \theta_{\mathbf{k}_i})$$

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we look for **coherent oscillations**



noise is often larger than signal
coherency of the signal allows a potential detection

$$\cos(mt)$$

$$\cos(2mt)$$

Fourier transform of a detector output

$$\tilde{d}(\omega) = \int dt e^{i\omega t} [s(t) + n(t)]$$

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$$\cos(2mt)$$

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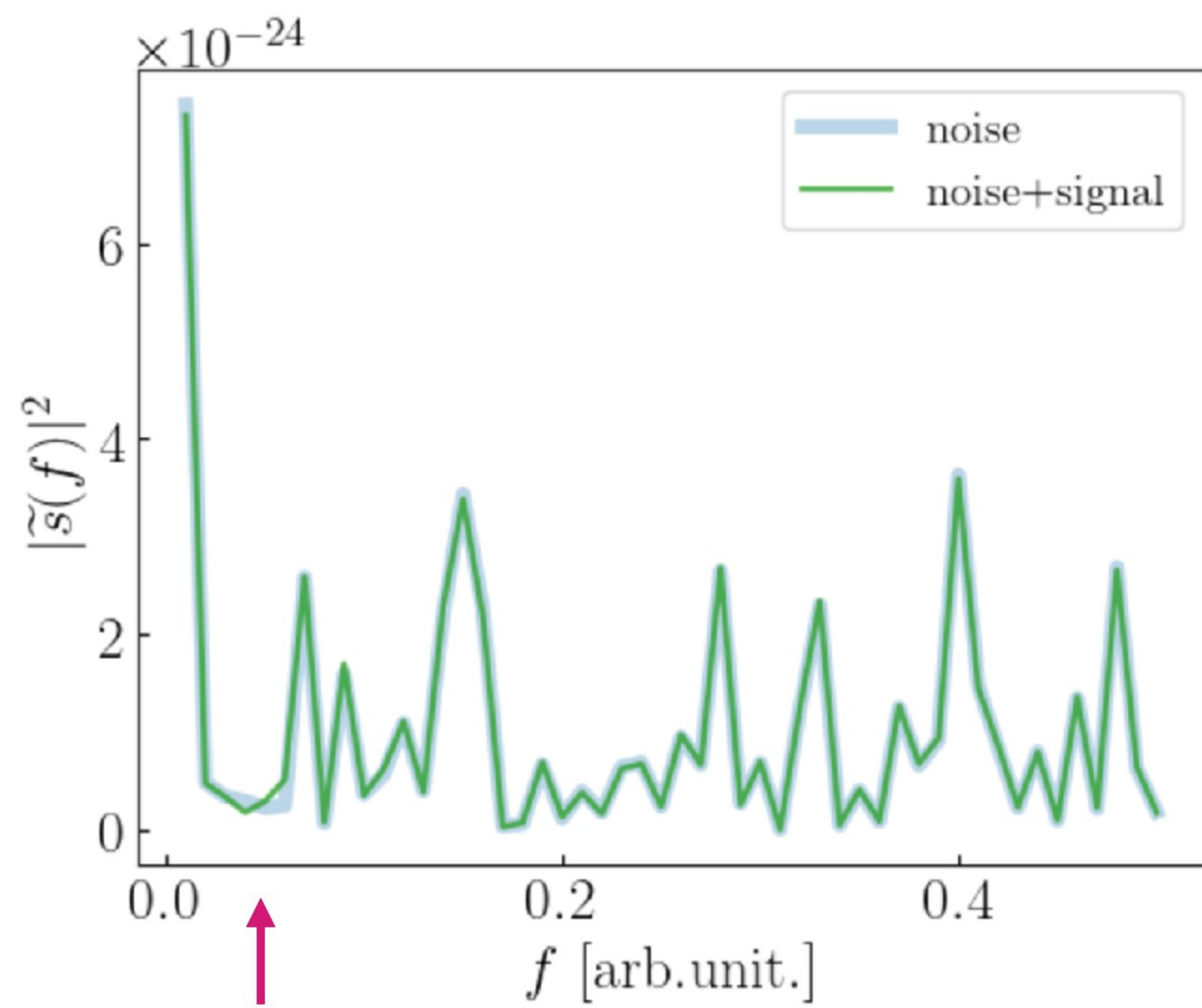
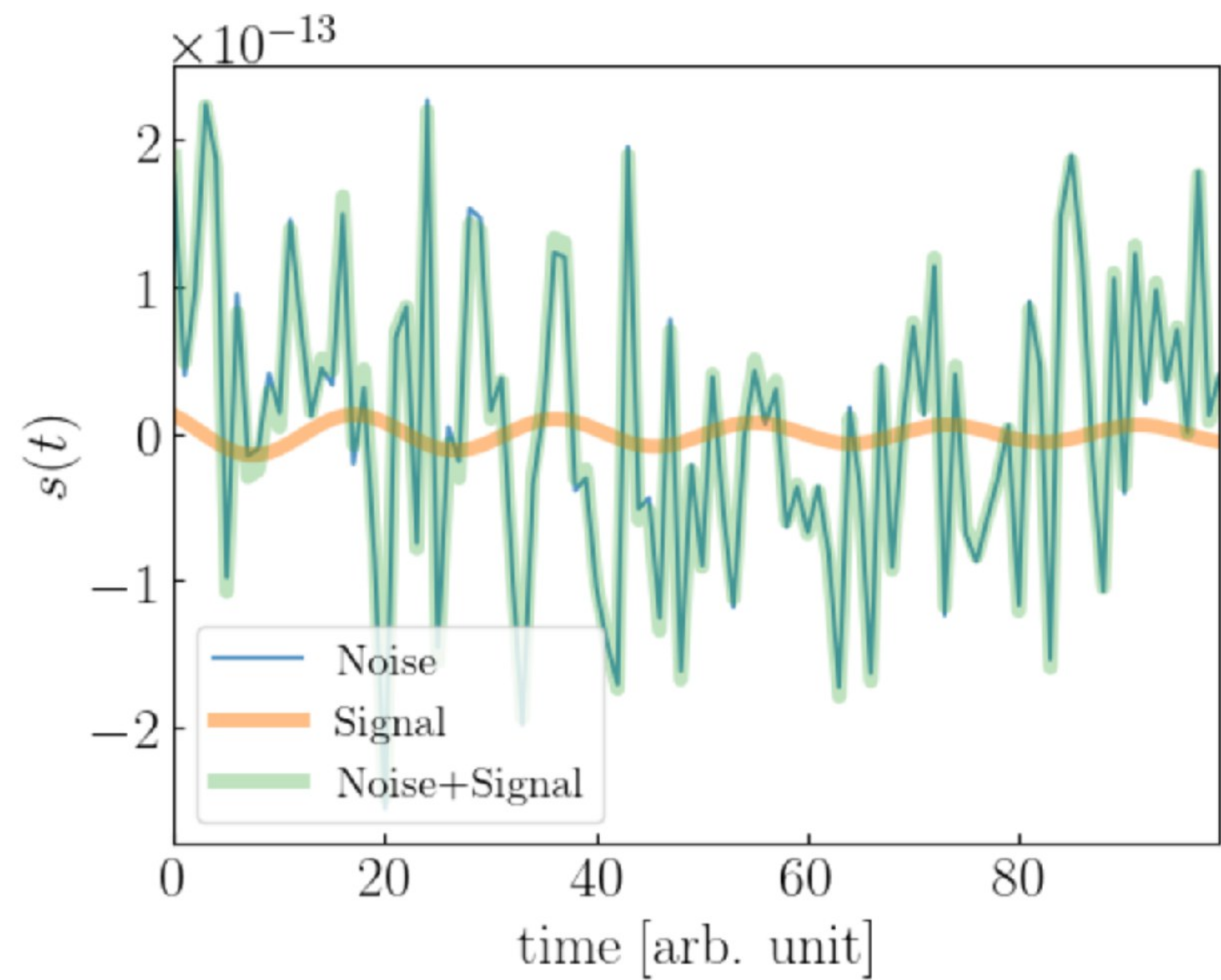
$\propto T$

$$\cos(mt)$$

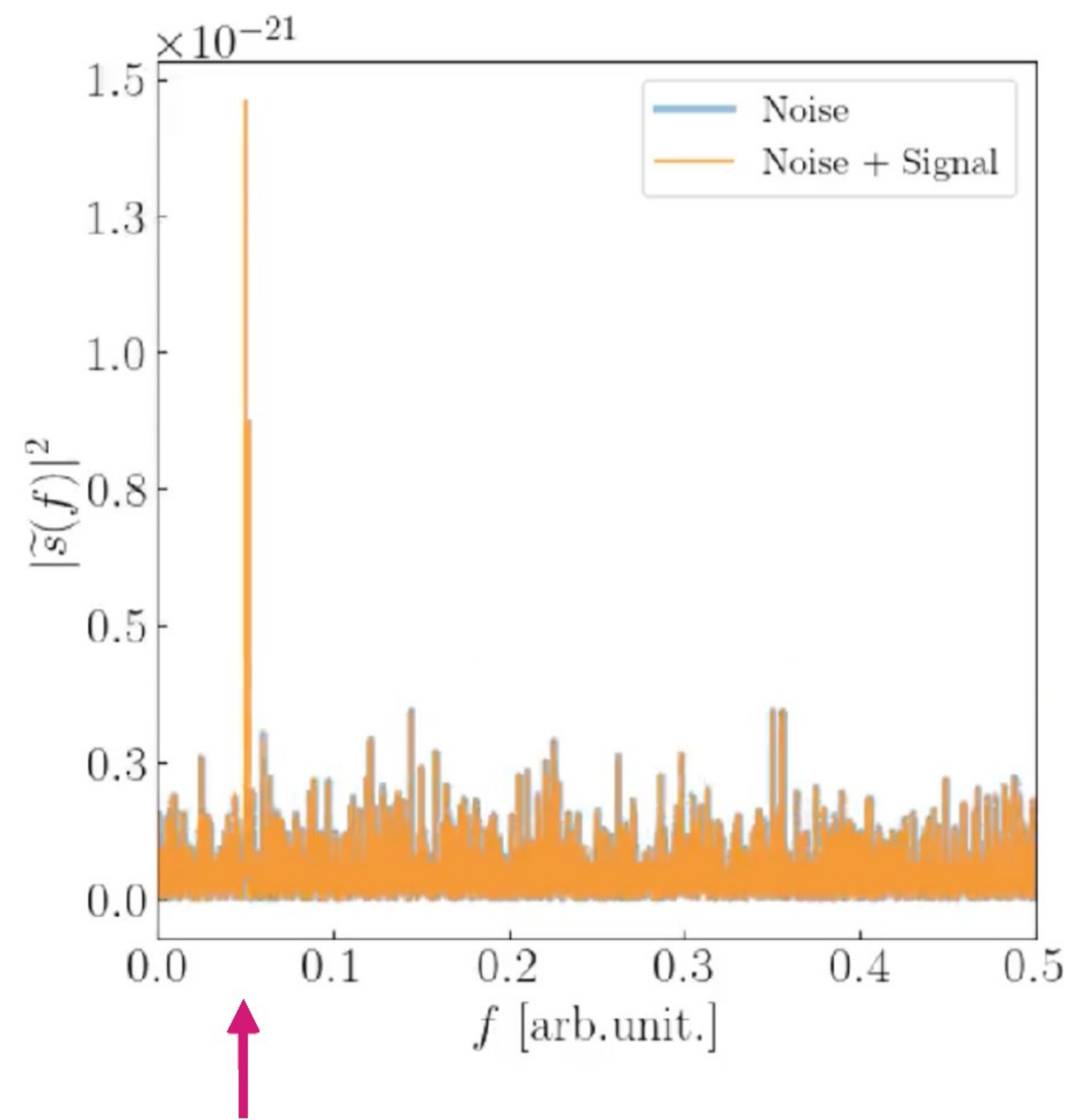
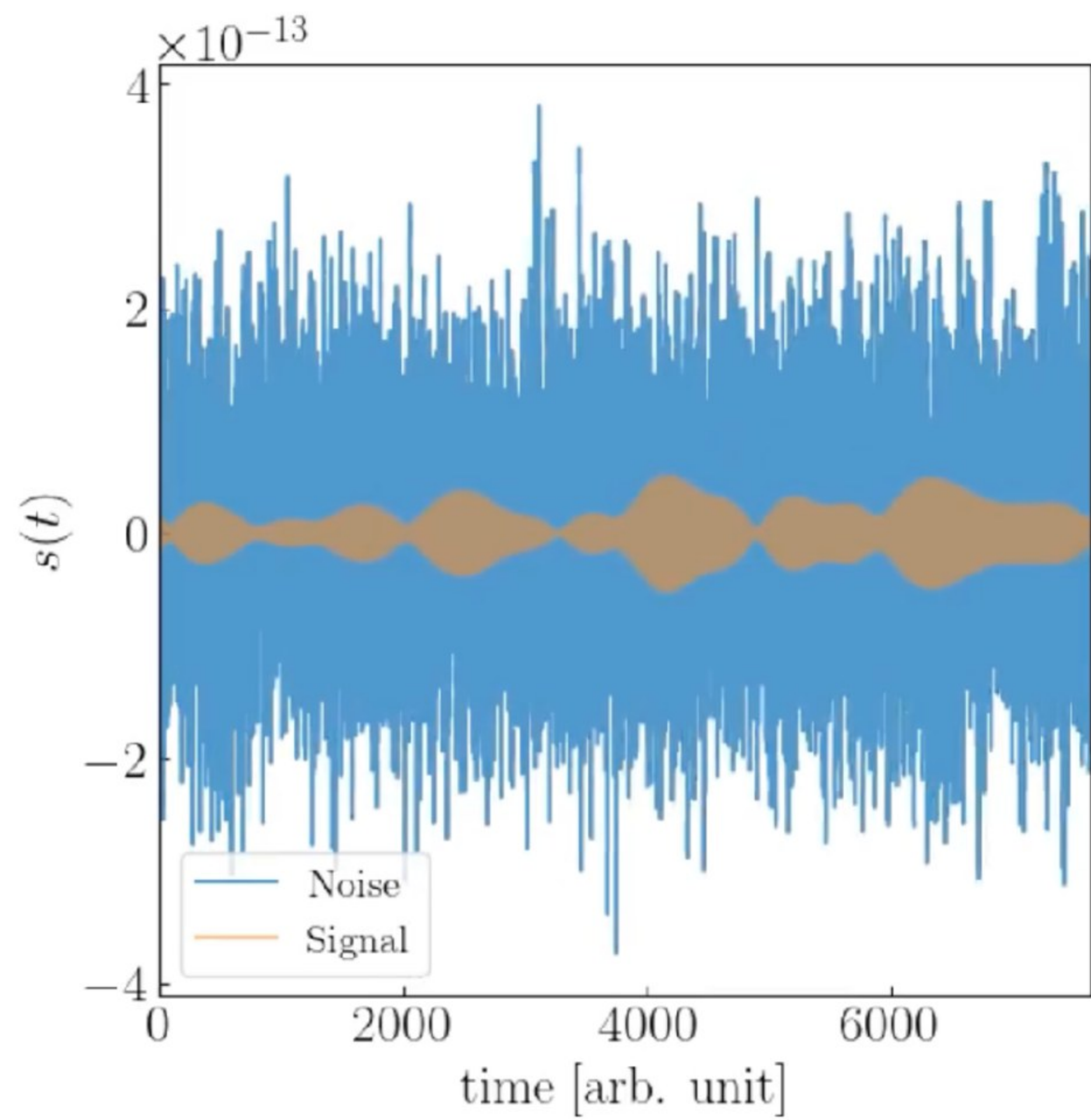
$$\cos(2mt)$$

Fourier transform of a detector output

$$\tilde{d}(\omega) = \int dt \underbrace{e^{i\omega t}}_{\propto T} \underbrace{[s(t) + n(t)]}_{\propto \sqrt{T}}$$



Coherent signal expected to appear here
 $\omega = m$



for **quadratic signals**
stochastic fluctuation emerges

$$s(t) \propto \phi^2(t)$$

$$\propto \sum_{\mathbf{k}_i, \mathbf{k}_j} r_{\mathbf{k}_i} r_{\mathbf{k}_j} \cos(\omega_{\mathbf{k}_i} t + \theta_{\mathbf{k}_i}) \cos(\omega_{\mathbf{k}_j} t + \theta_{\mathbf{k}_j})$$

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$$\propto \sum_{\mathbf{k}_i, \mathbf{k}_j} r_{\mathbf{k}_i} r_{\mathbf{k}_j} \left[\cos [(\omega_{\mathbf{k}_i} - \omega_{\mathbf{k}_j})t + (\theta_{\mathbf{k}_i} - \theta_{\mathbf{k}_j})] + \cos [(\omega_{\mathbf{k}_i} + \omega_{\mathbf{k}_j})t + (\theta_{\mathbf{k}_i} + \theta_{\mathbf{k}_j})] \right]$$

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$$\omega \approx 2m$$

coherent fluctuation

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$$\omega \lesssim mv^2$$

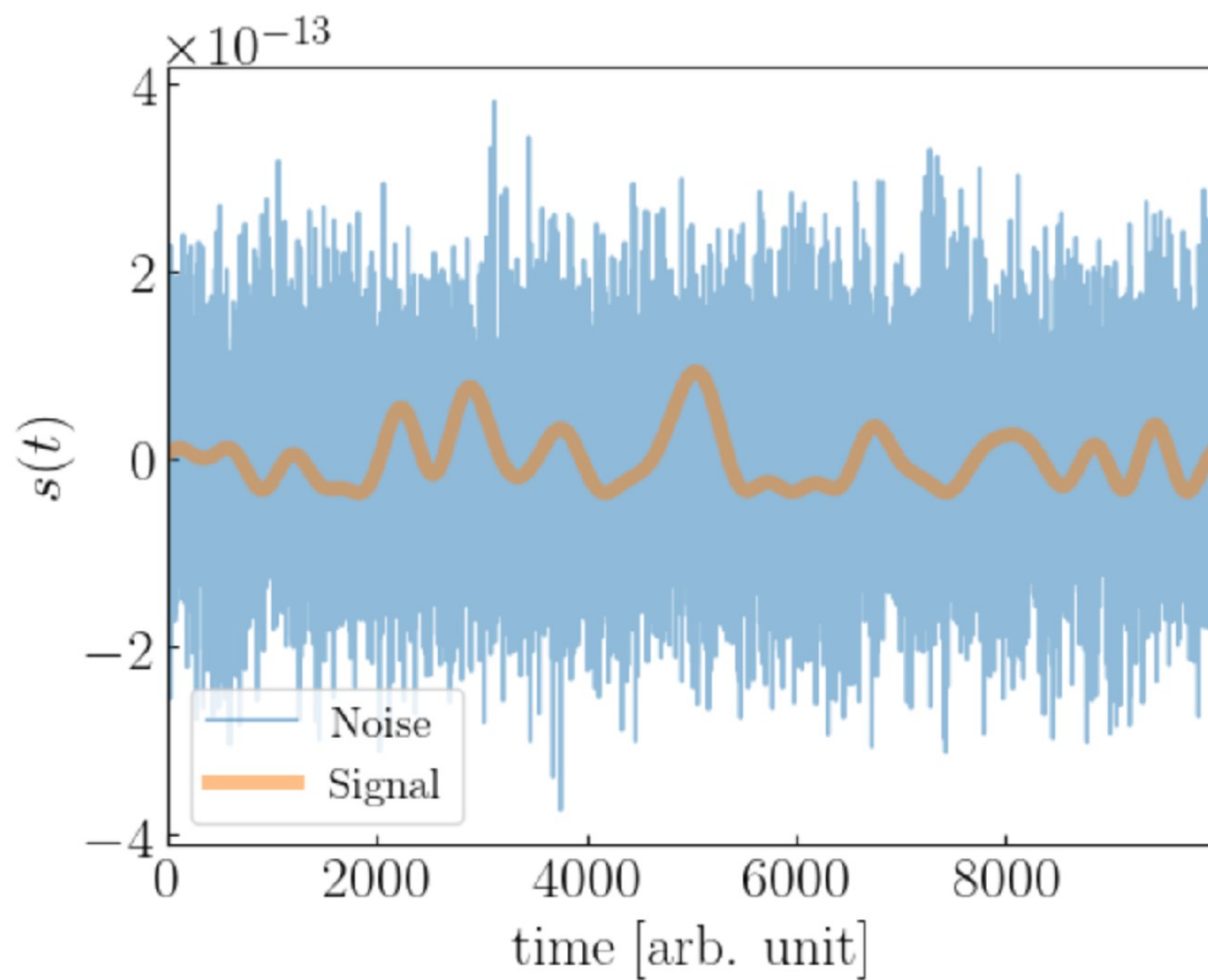
stochastic fluctuation

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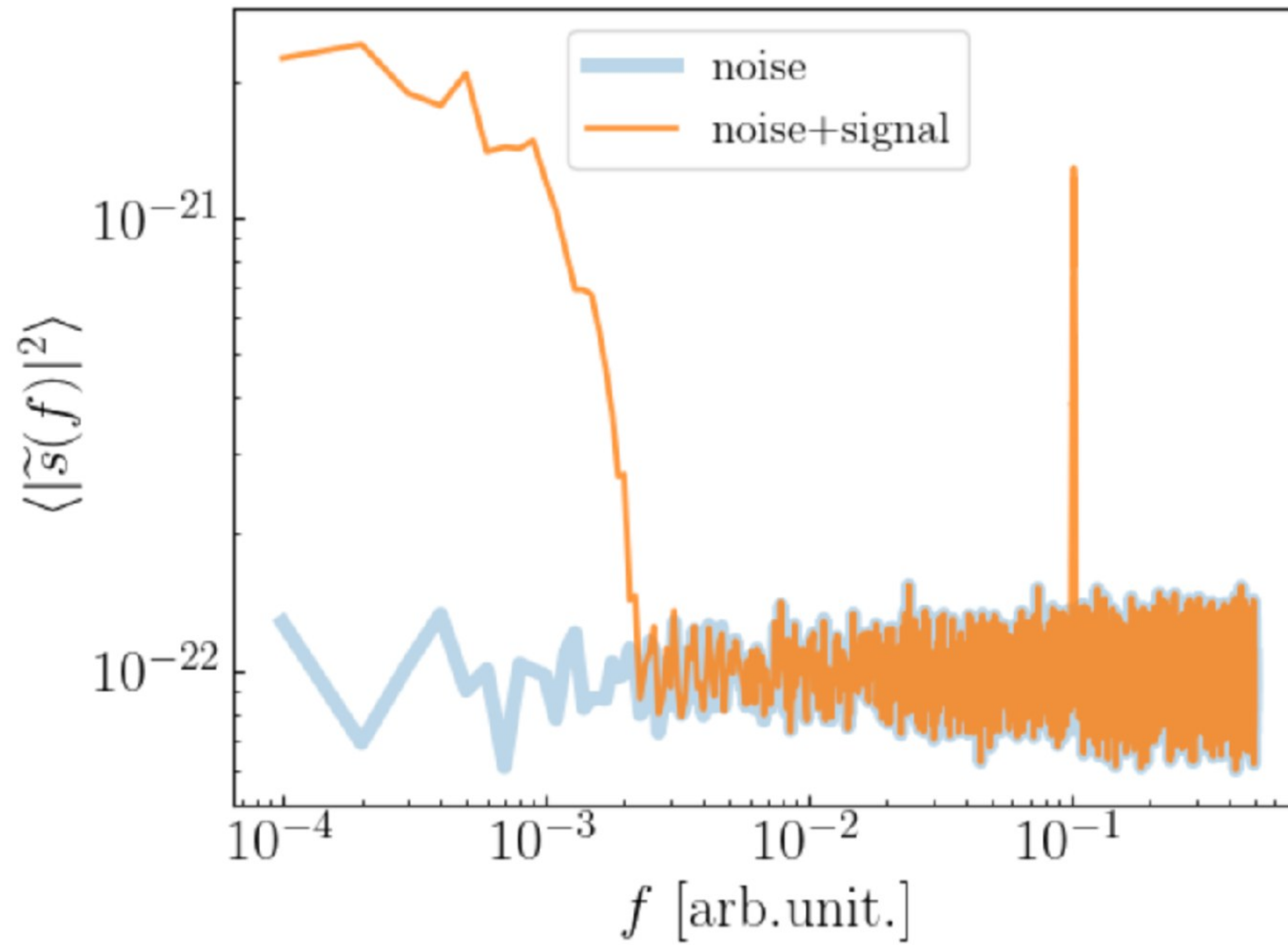
coherent fluctuation

how would stochastic fluctuations appear in the output?

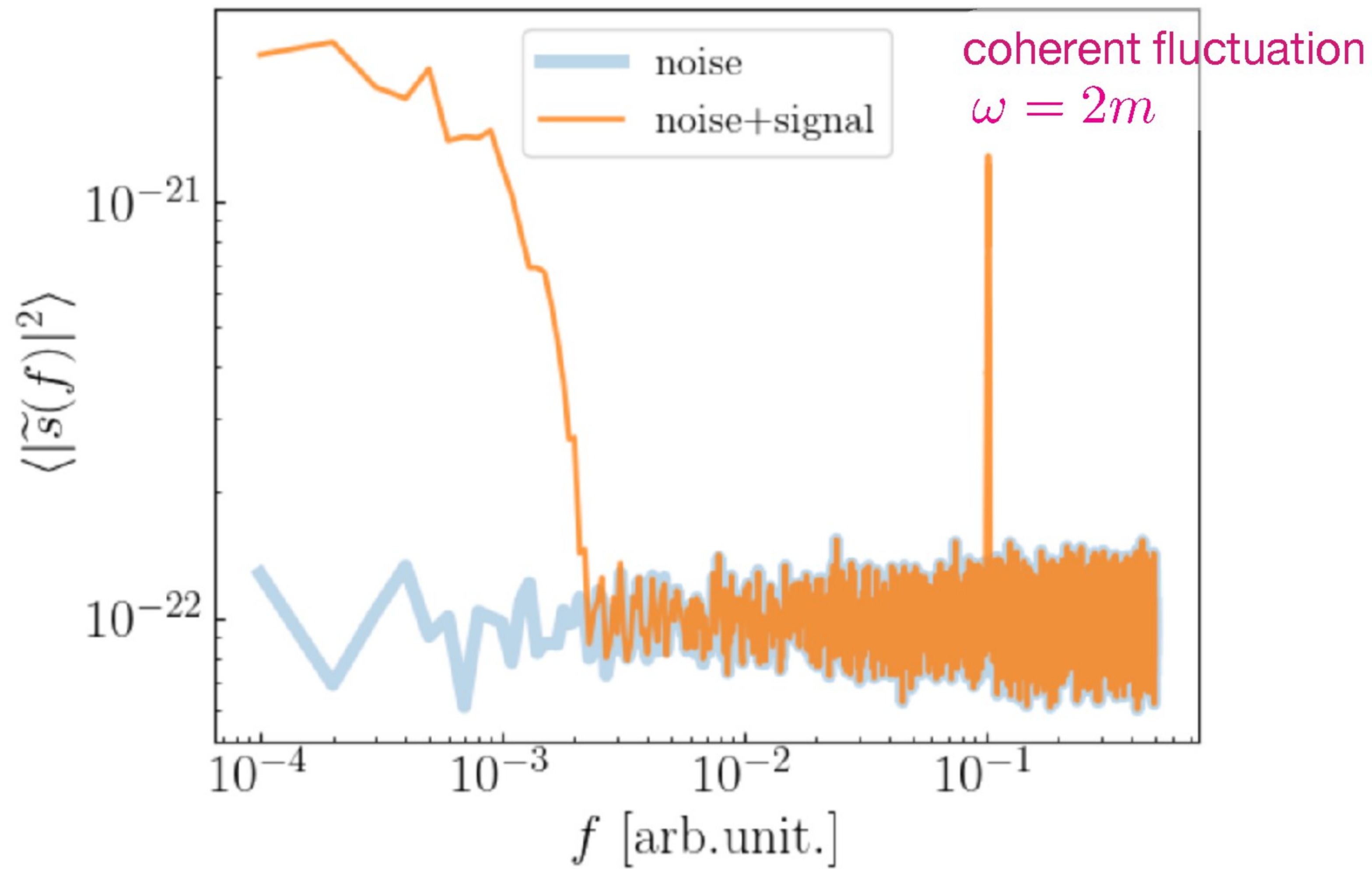
stochastic part of signal will appear



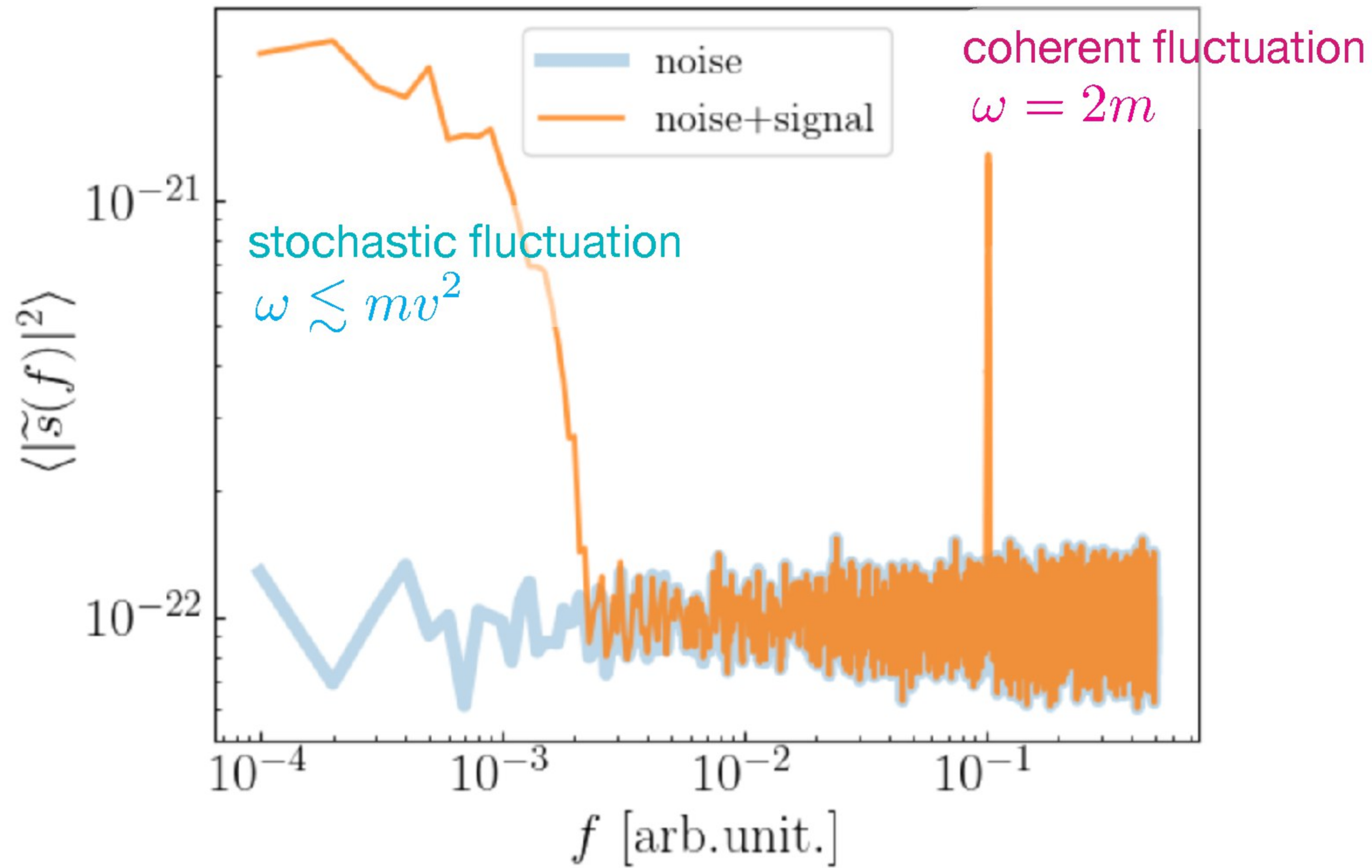
the power spectrum now would look like



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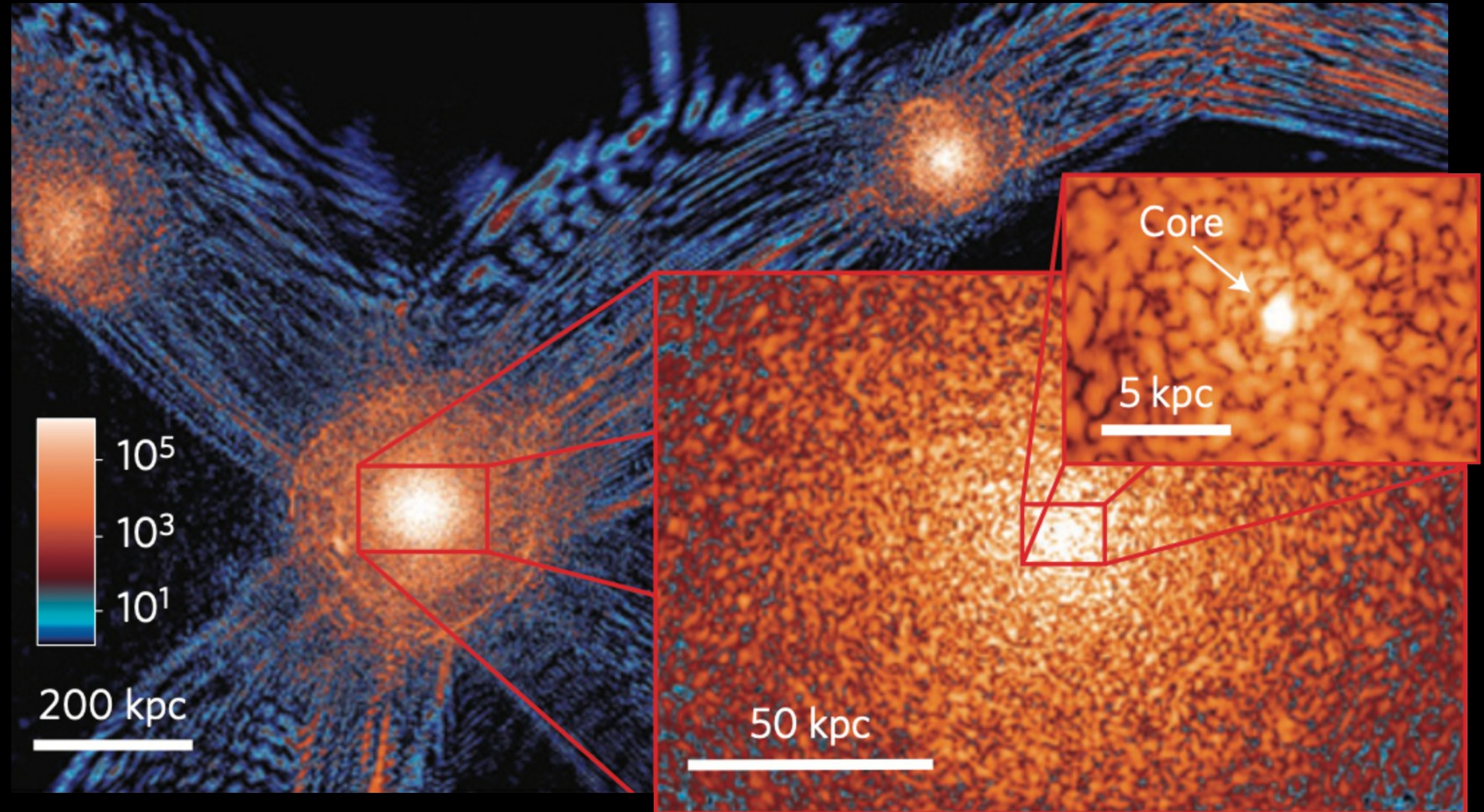


the power spectrum now would look like



Applications

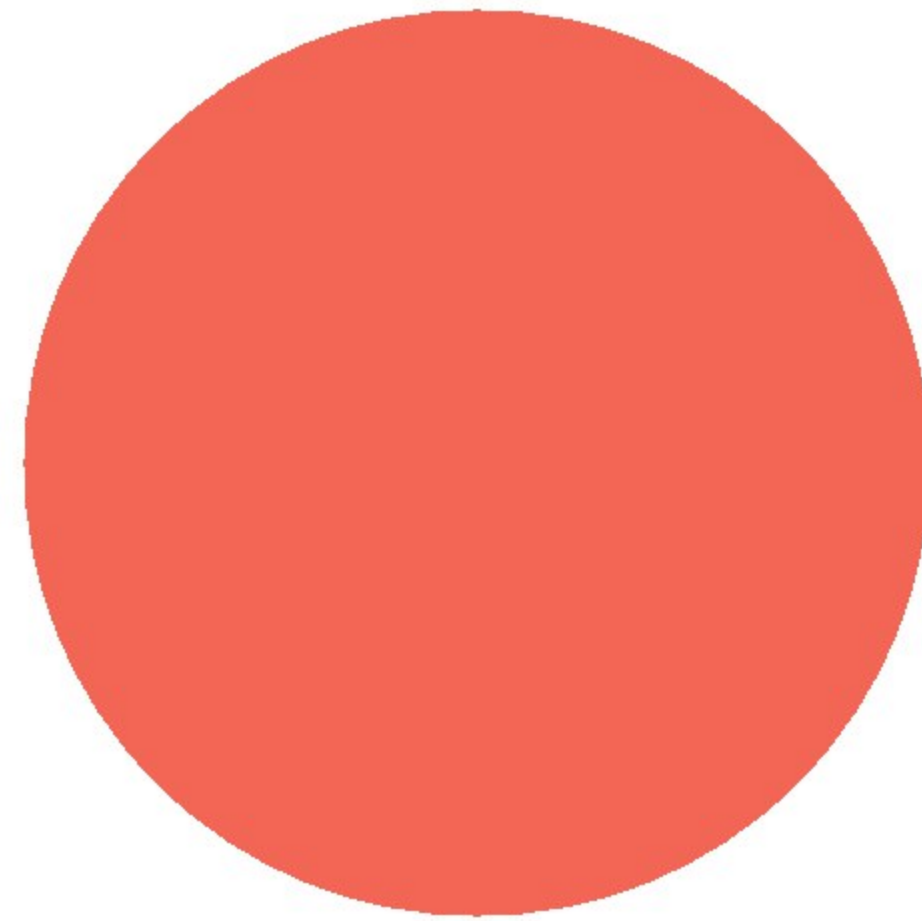
focusing on gravitational probes



An intuitive understanding of the granule structure:

Quasiparticle

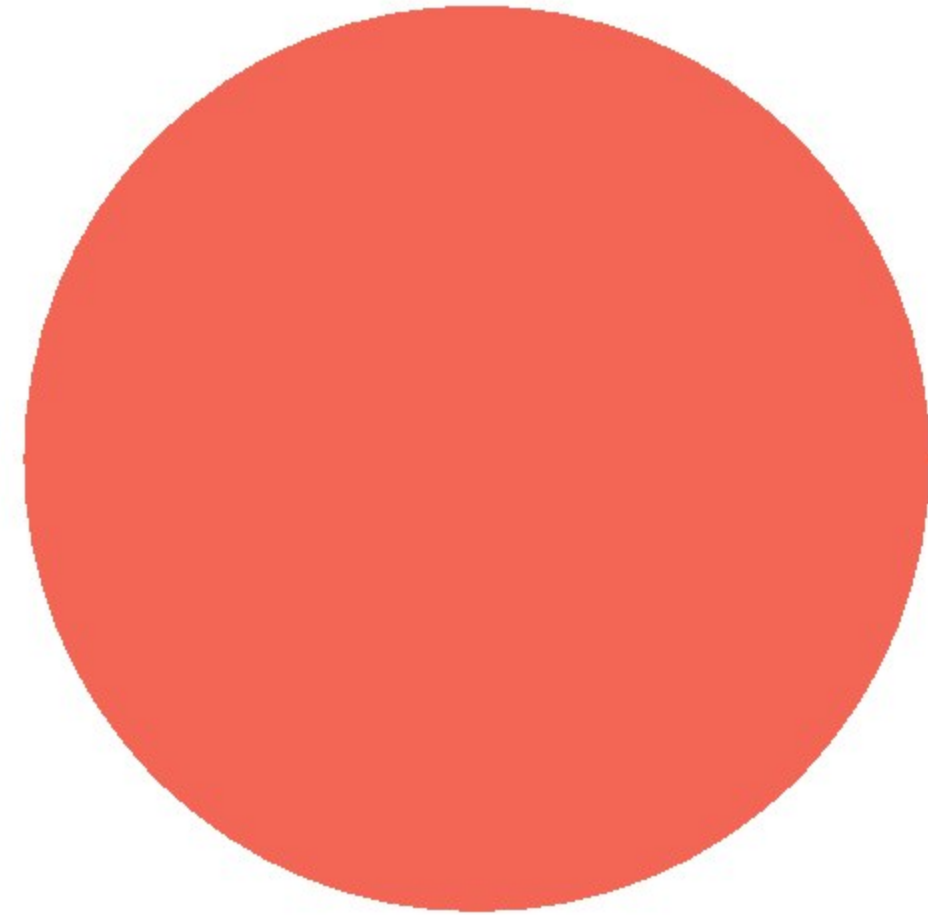
[Hui et al 17]



$$\ell \sim \lambda = \frac{1}{mv}$$

$$m_{\text{eff}} \sim \rho_{\text{DM}} \ell^3$$

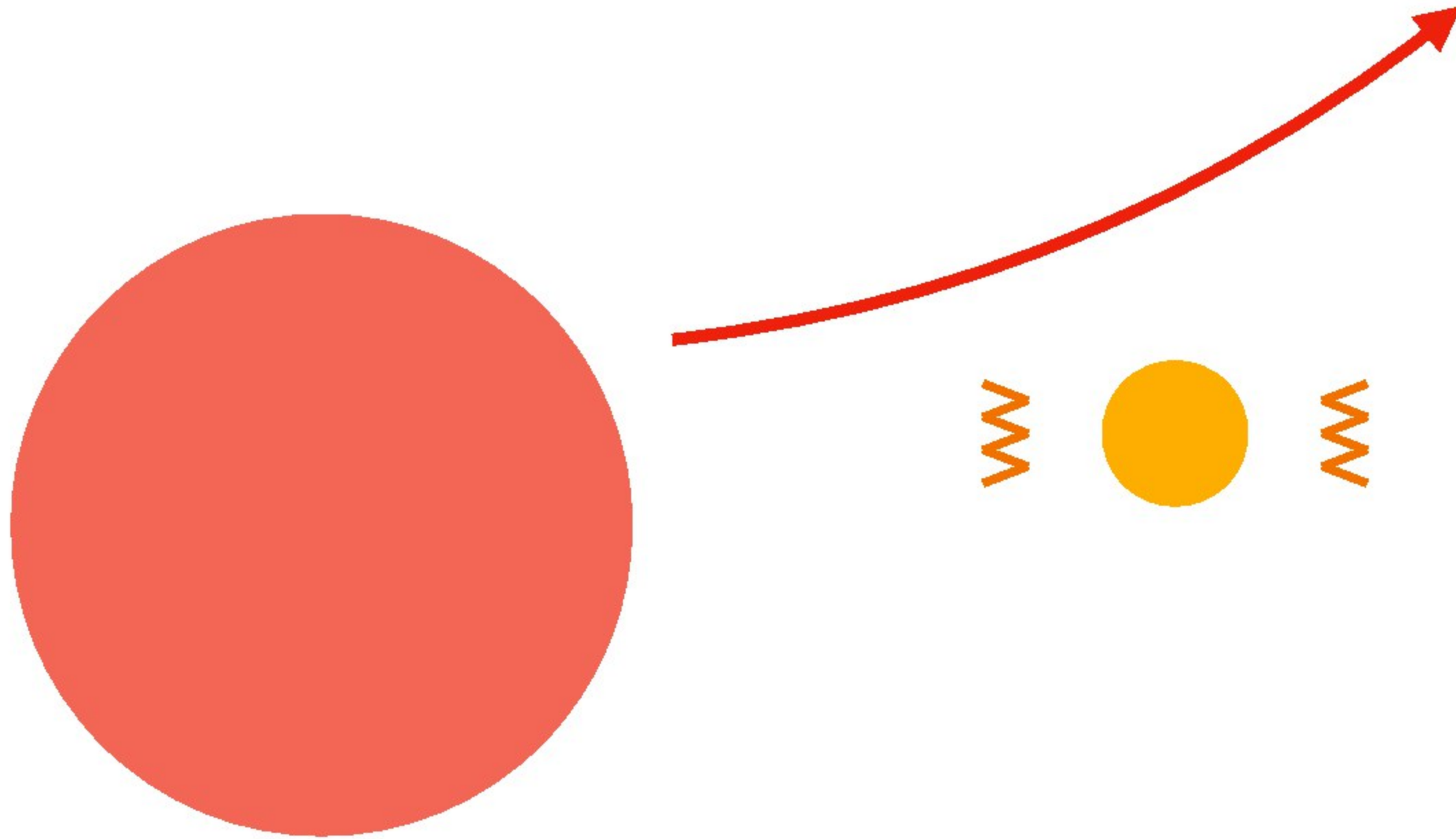
the size and mass of them could be astronomical



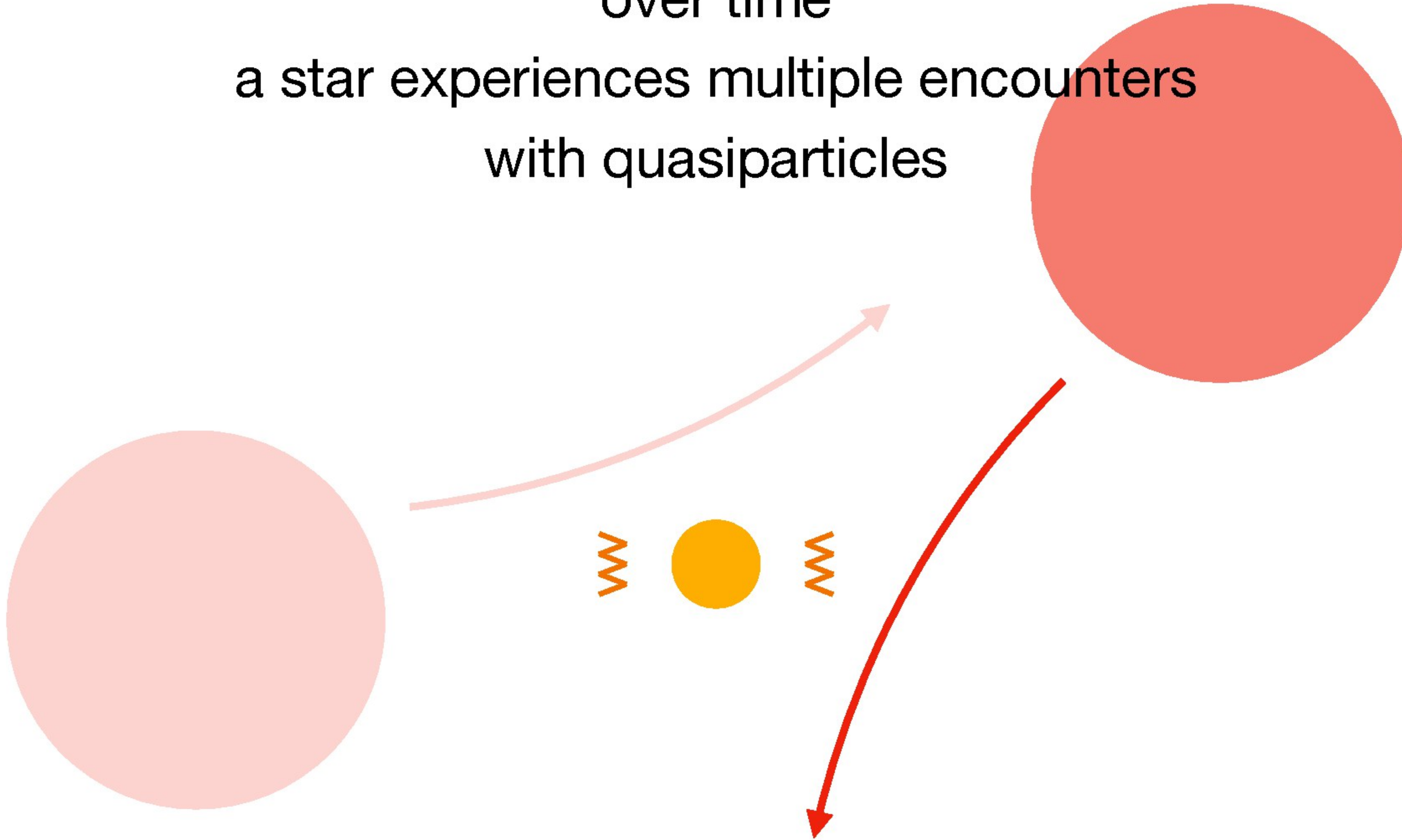
$$\ell \sim \lambda = \frac{1}{mv} \sim 10 \text{ AU} \times \left(\frac{10^{-16} \text{ eV}}{m} \right)$$

$$m_{\text{eff}} \sim \rho_{\text{DM}} \ell^3 \sim 10^{15} \text{ kg} \times \left(\frac{10^{-16} \text{ eV}}{m} \right)^3$$

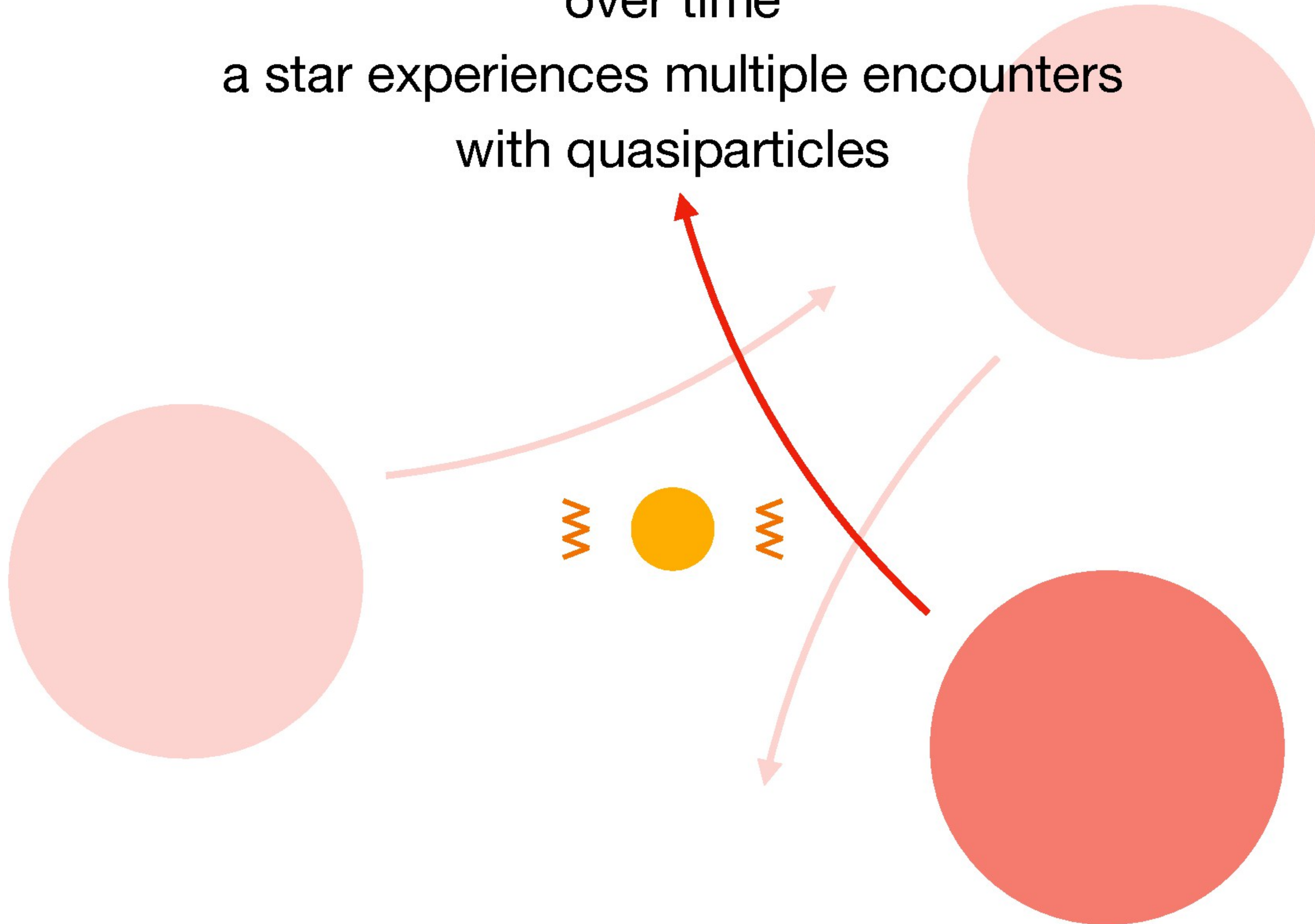
being that massive
it may engage in interaction with stars
and significantly perturb the motion of them



over time
a star experiences multiple encounters
with quasiparticles



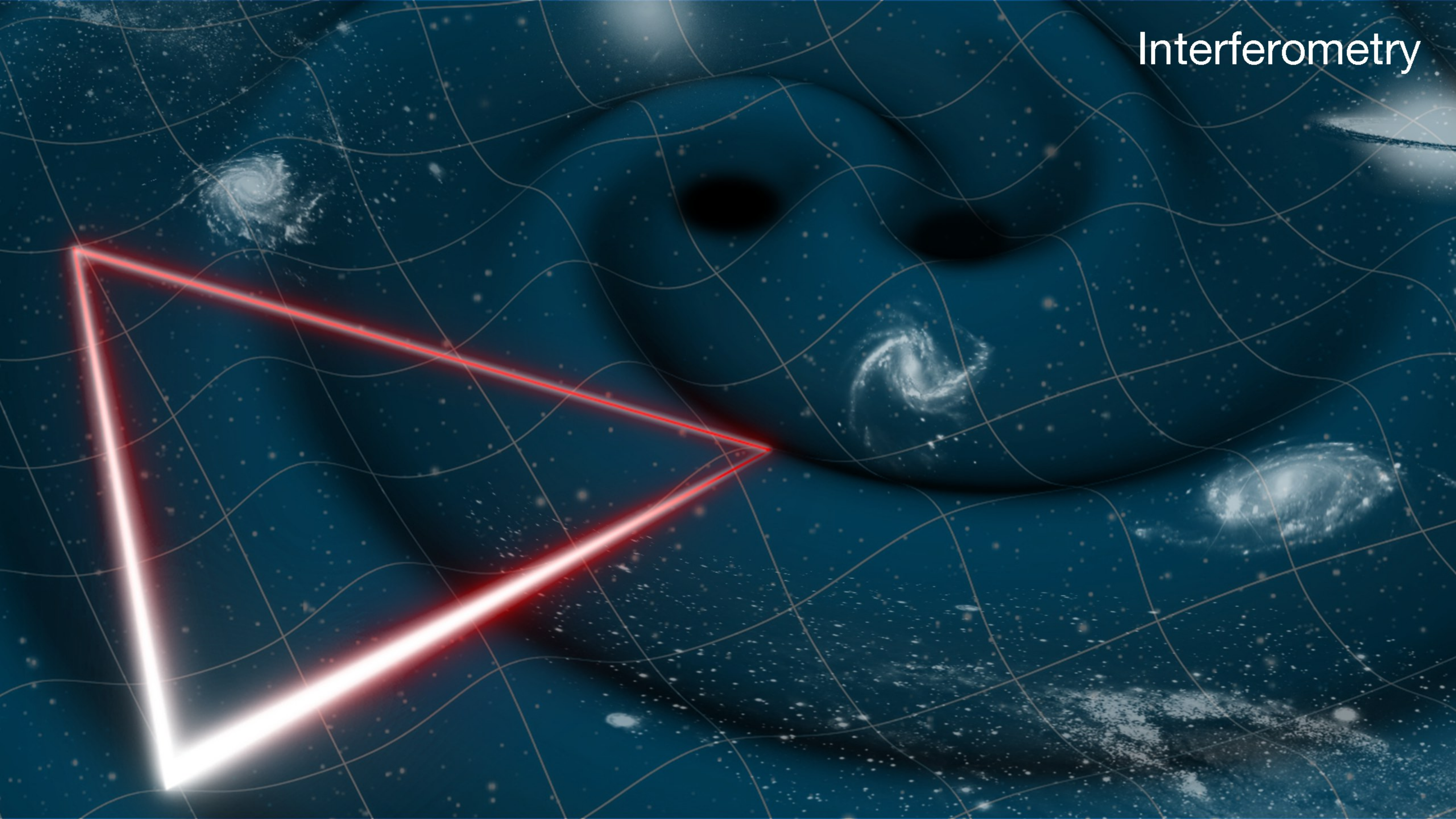
over time
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so what?

quasiparticles *bombards*
normal matters, leaving *distinctive stochastic signals*
in *gravitational wave detectors*

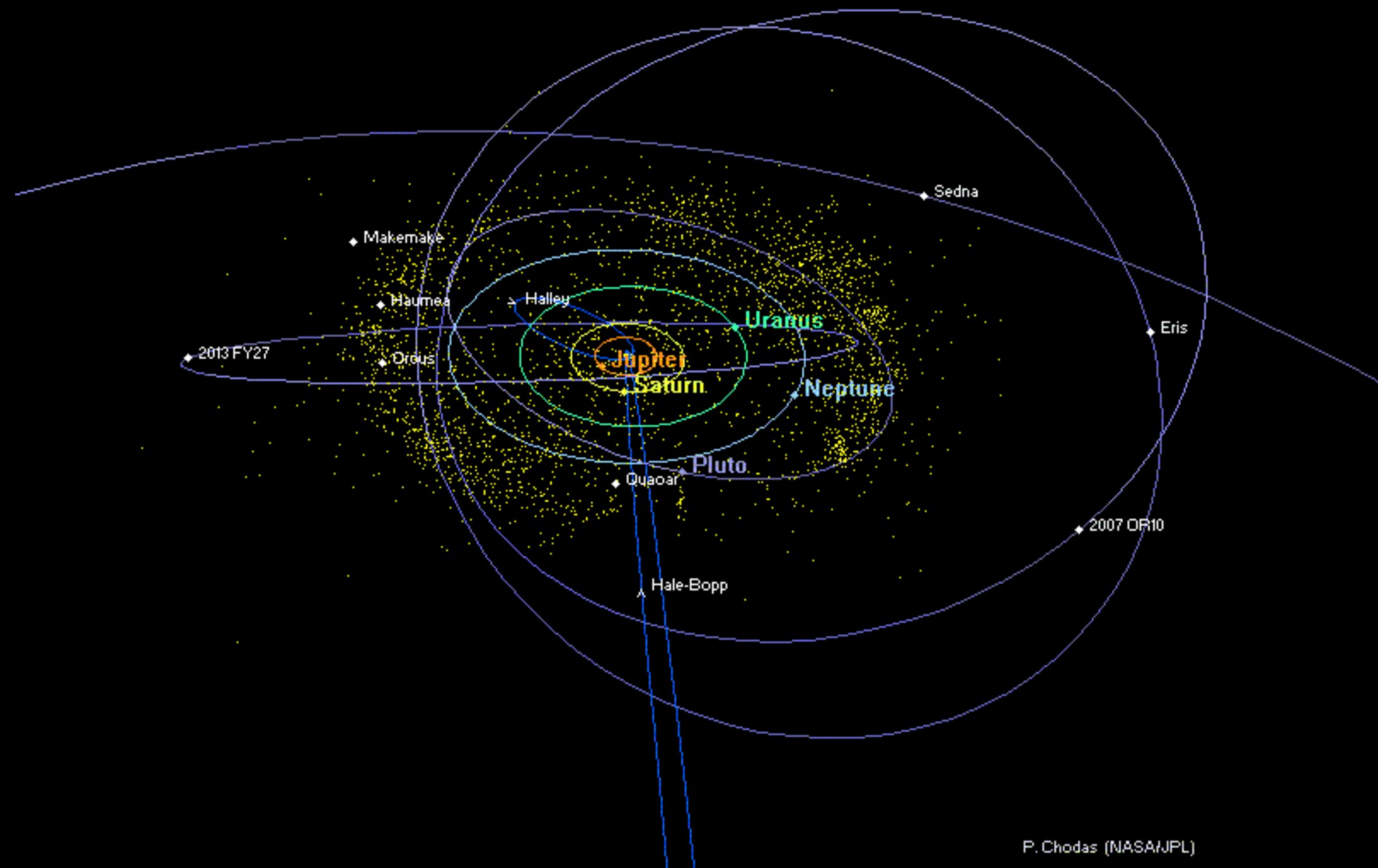
Interferometry

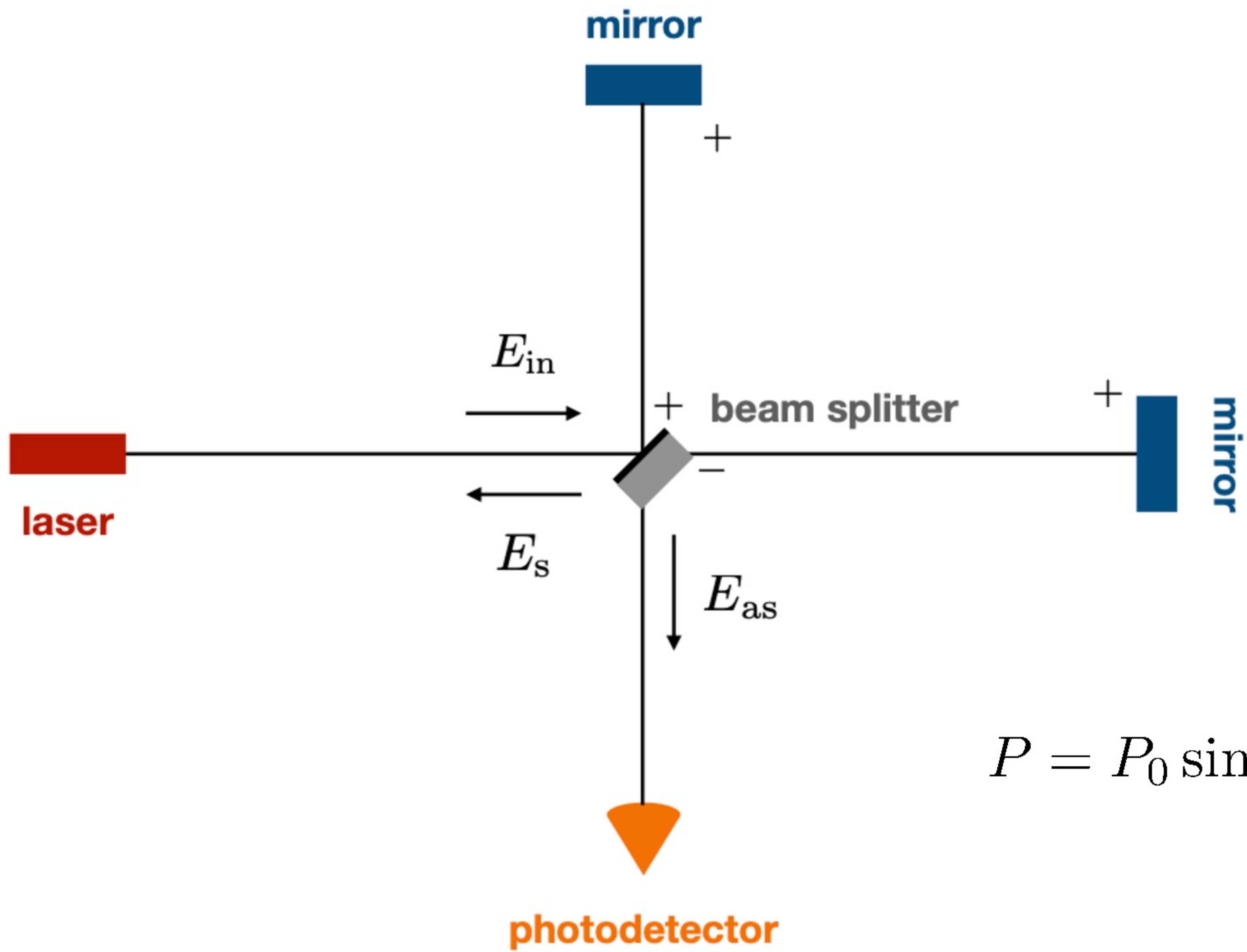


Pulsar Timing



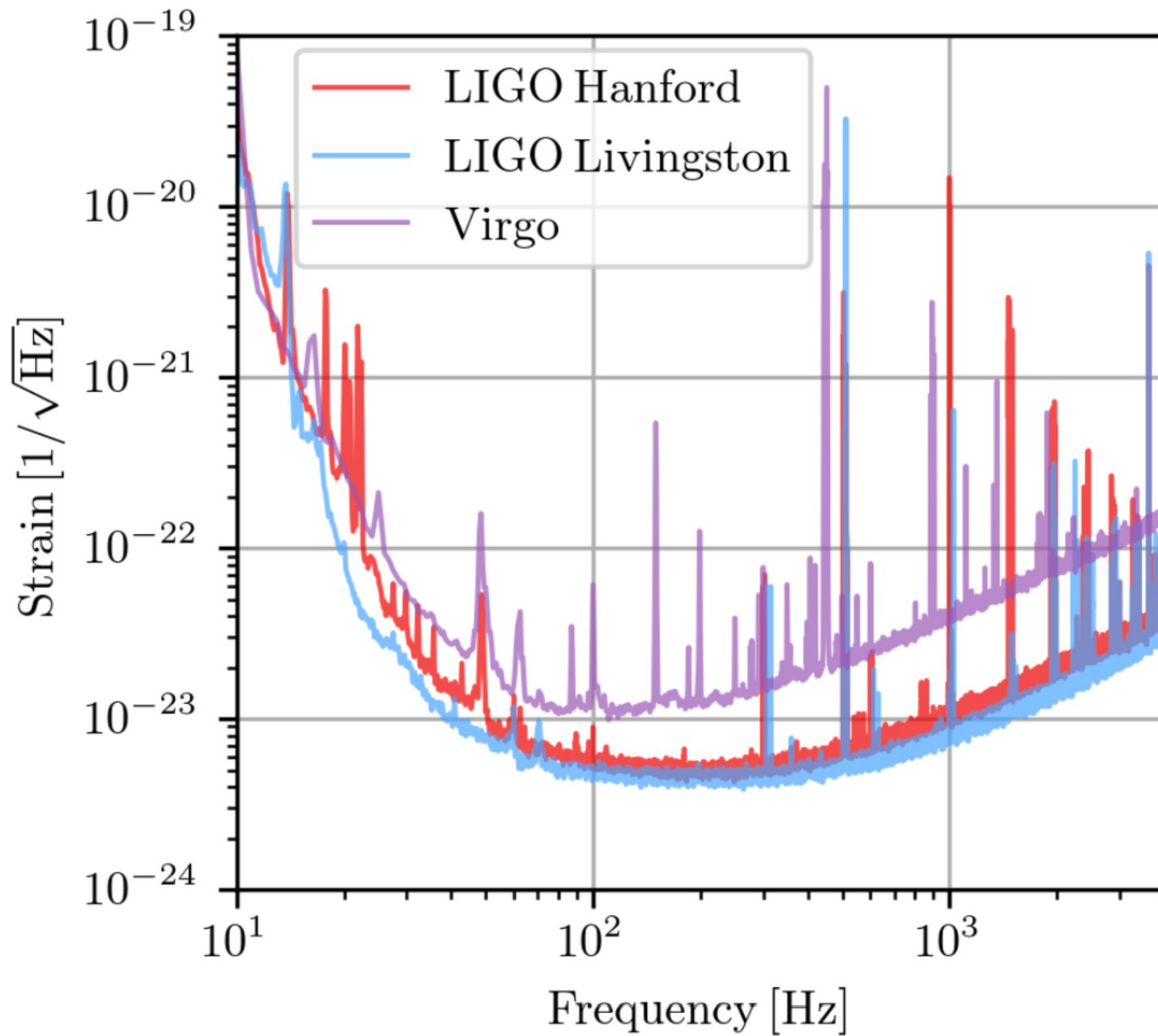
Solar system ephemerides

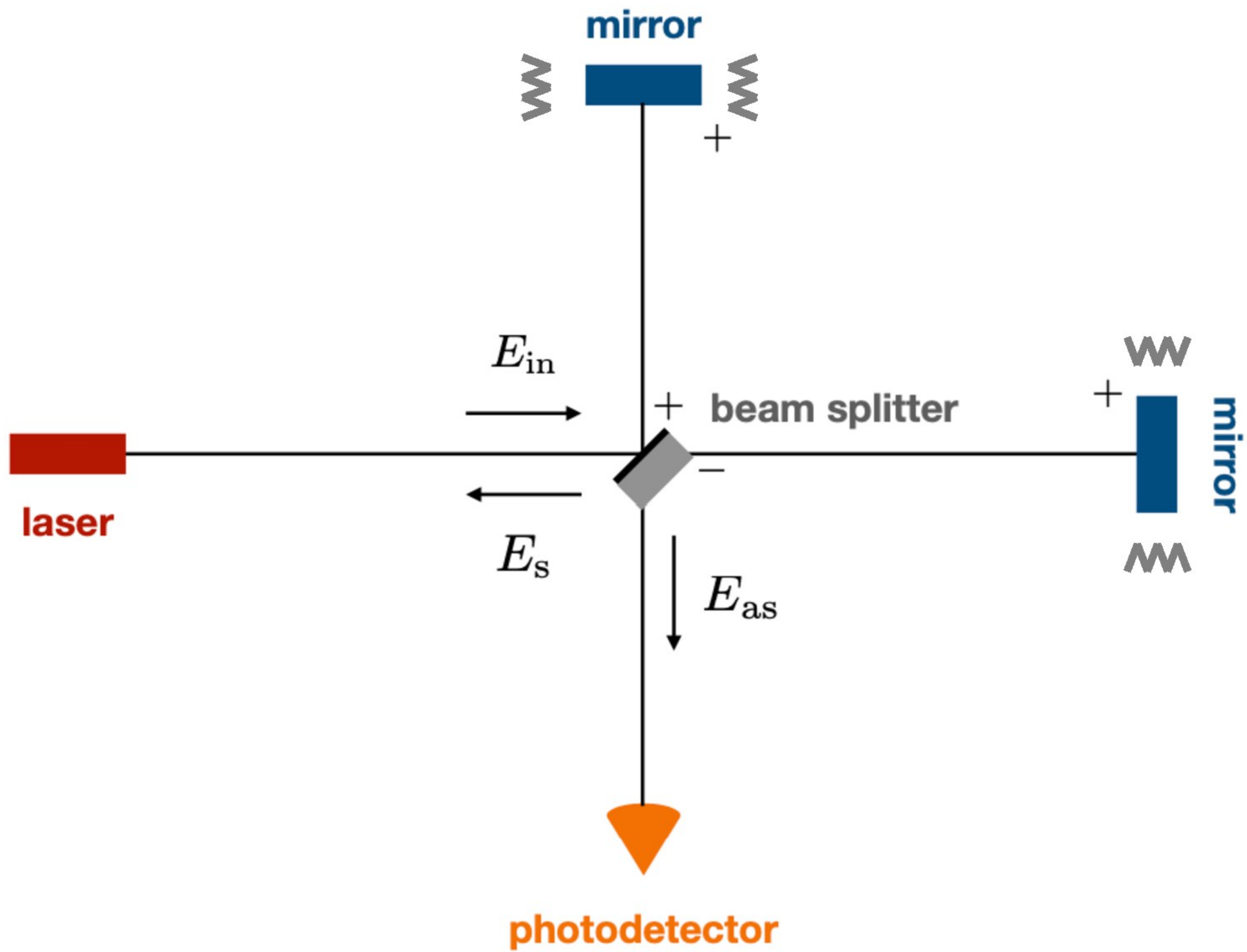




$$P = P_0 \sin^2(k\Delta L)$$







can we actually measure it?

$$\ddot{x} = -\nabla\Phi$$



$$\ddot{x} = -\nabla\Phi$$

$$\nabla^2\Phi = 4\pi G\rho$$

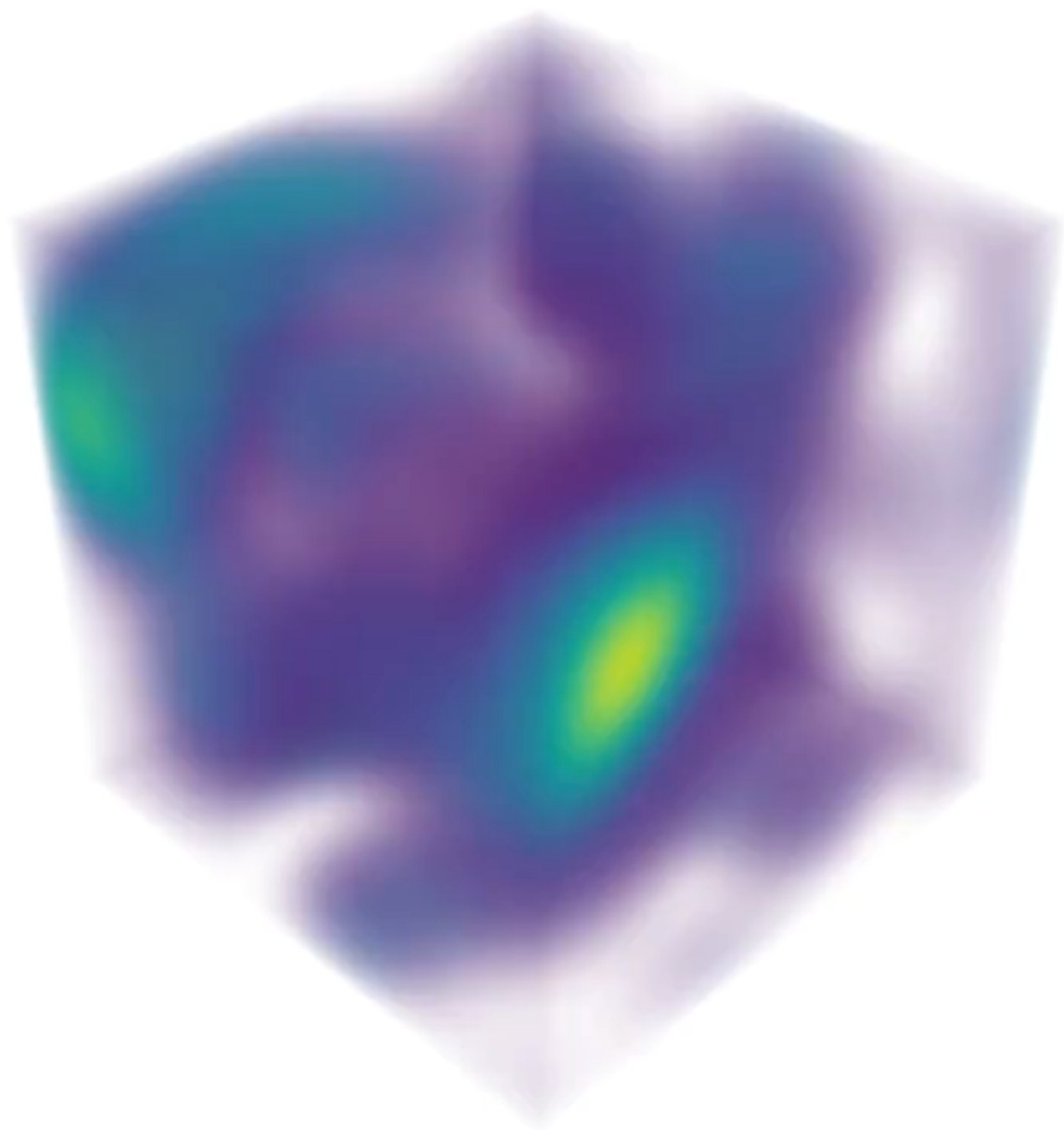


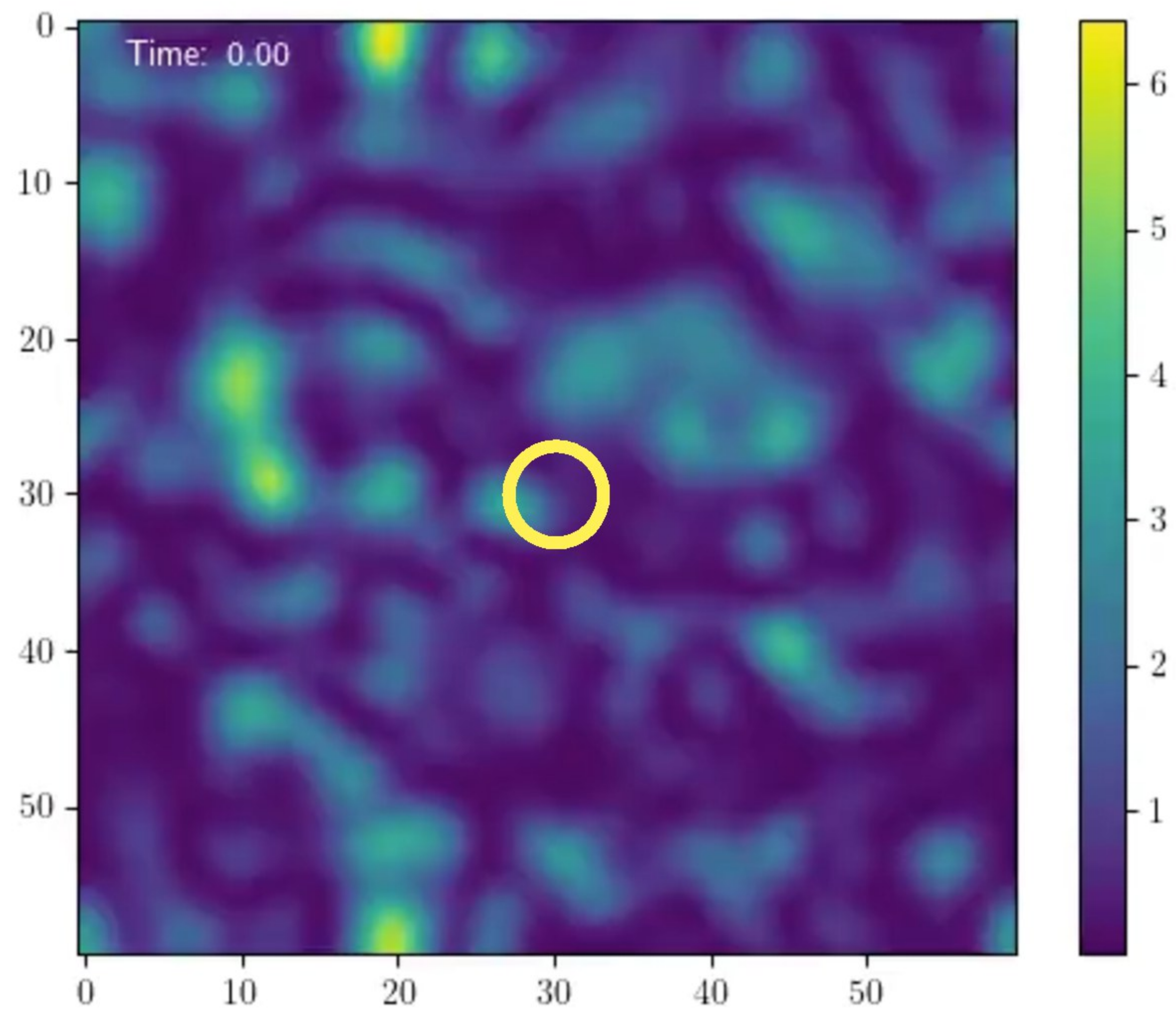
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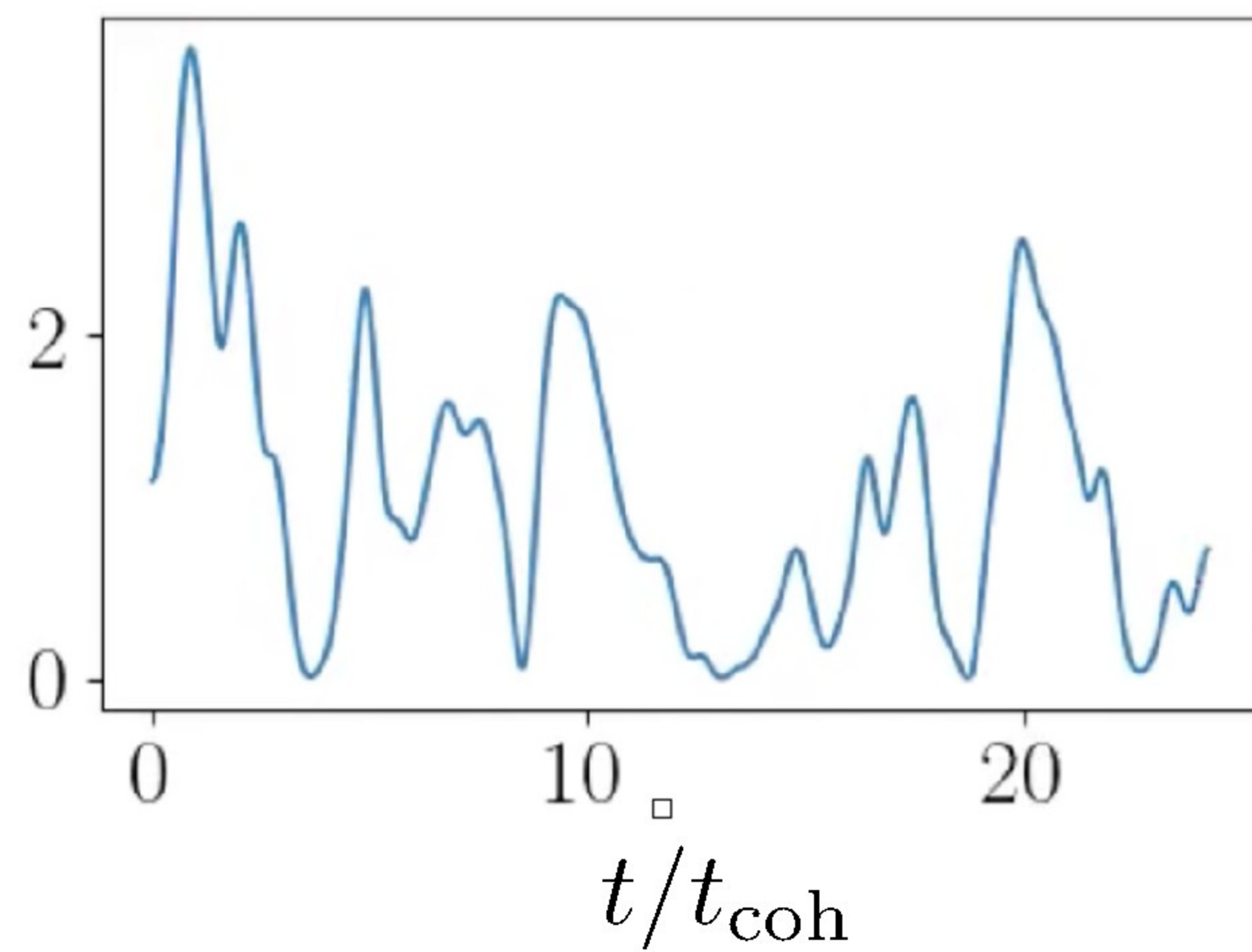


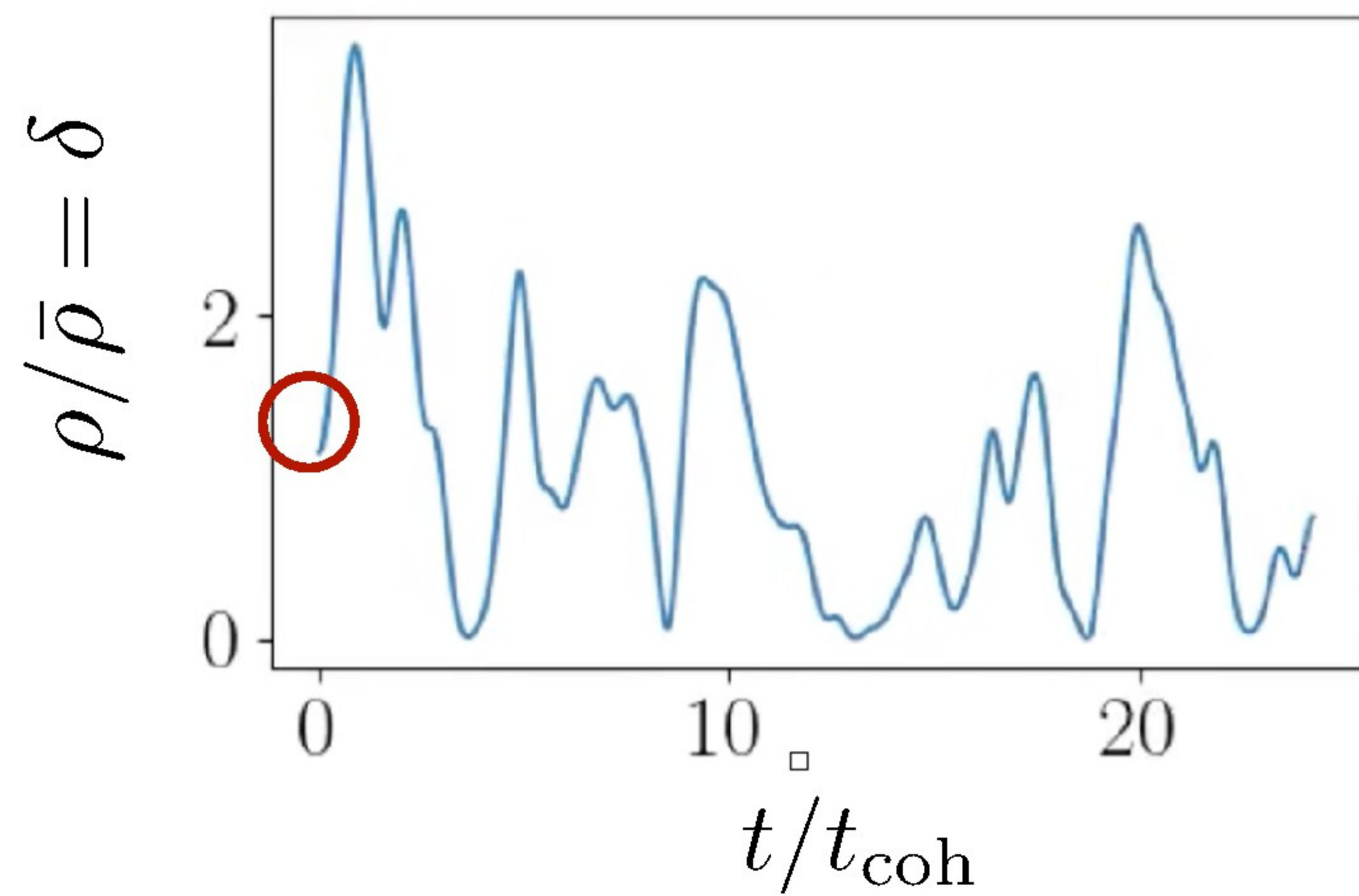
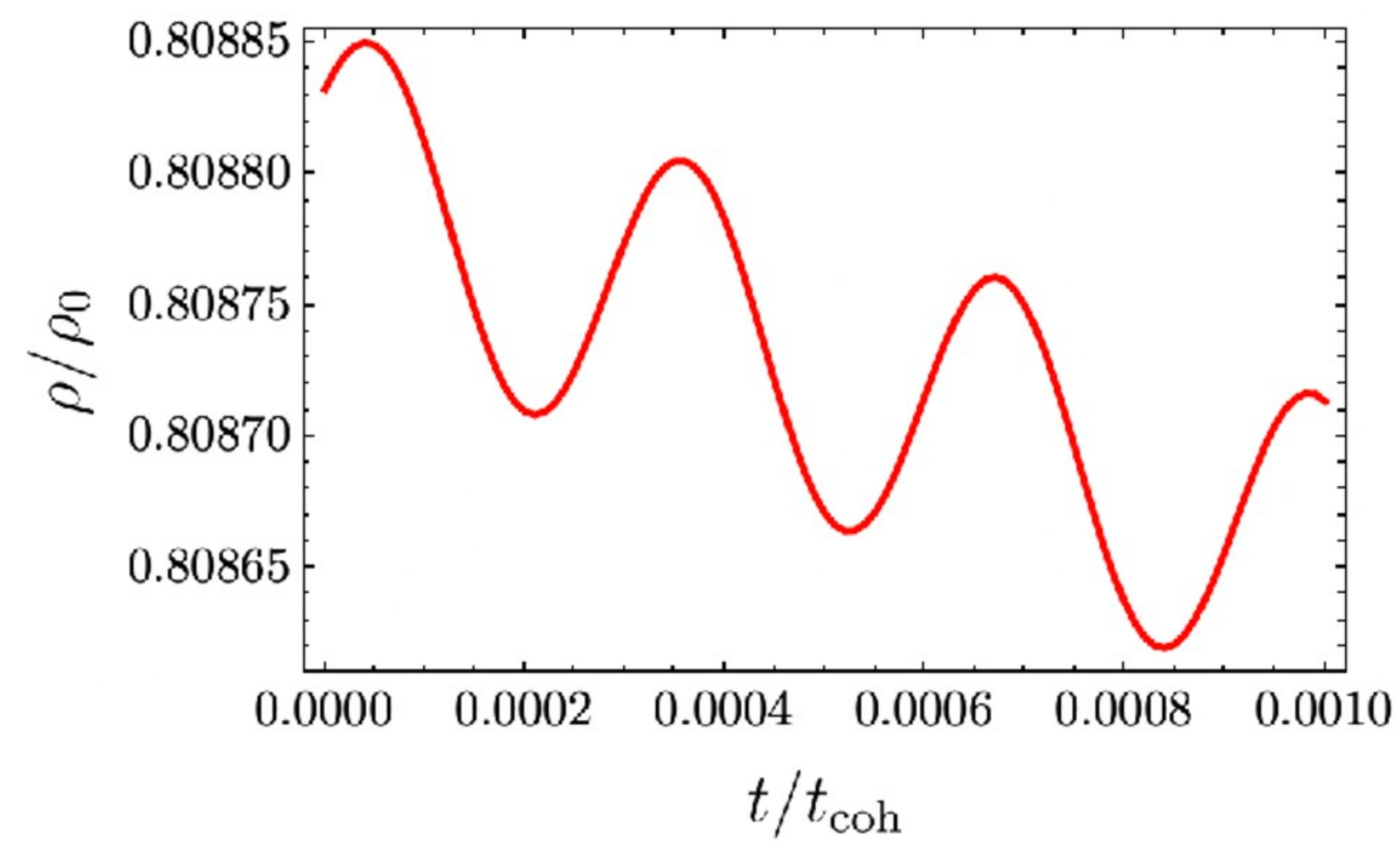
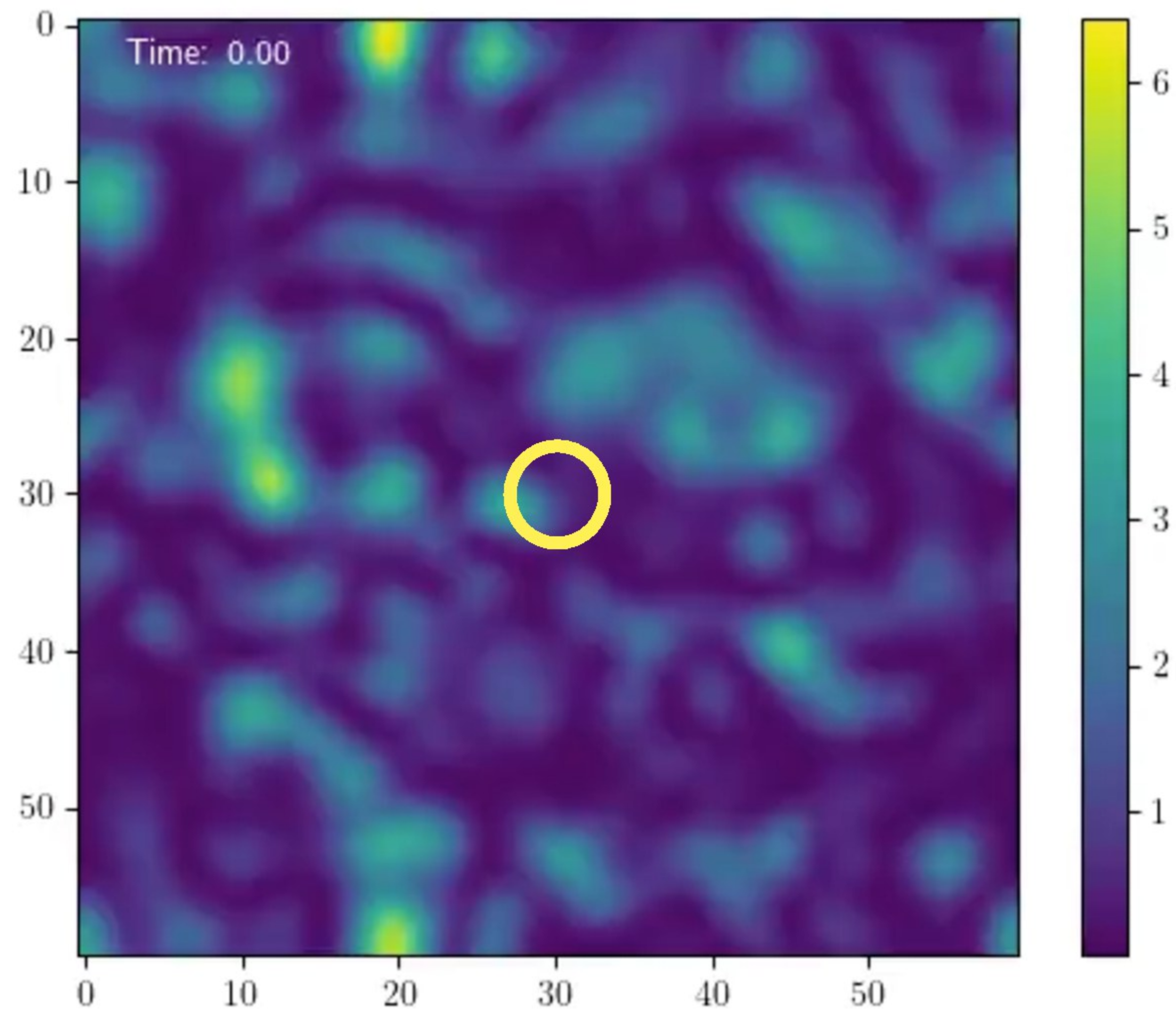
what is reflected in *detector observables*
 is the *statistical properties* of
density fluctuations of ULDM

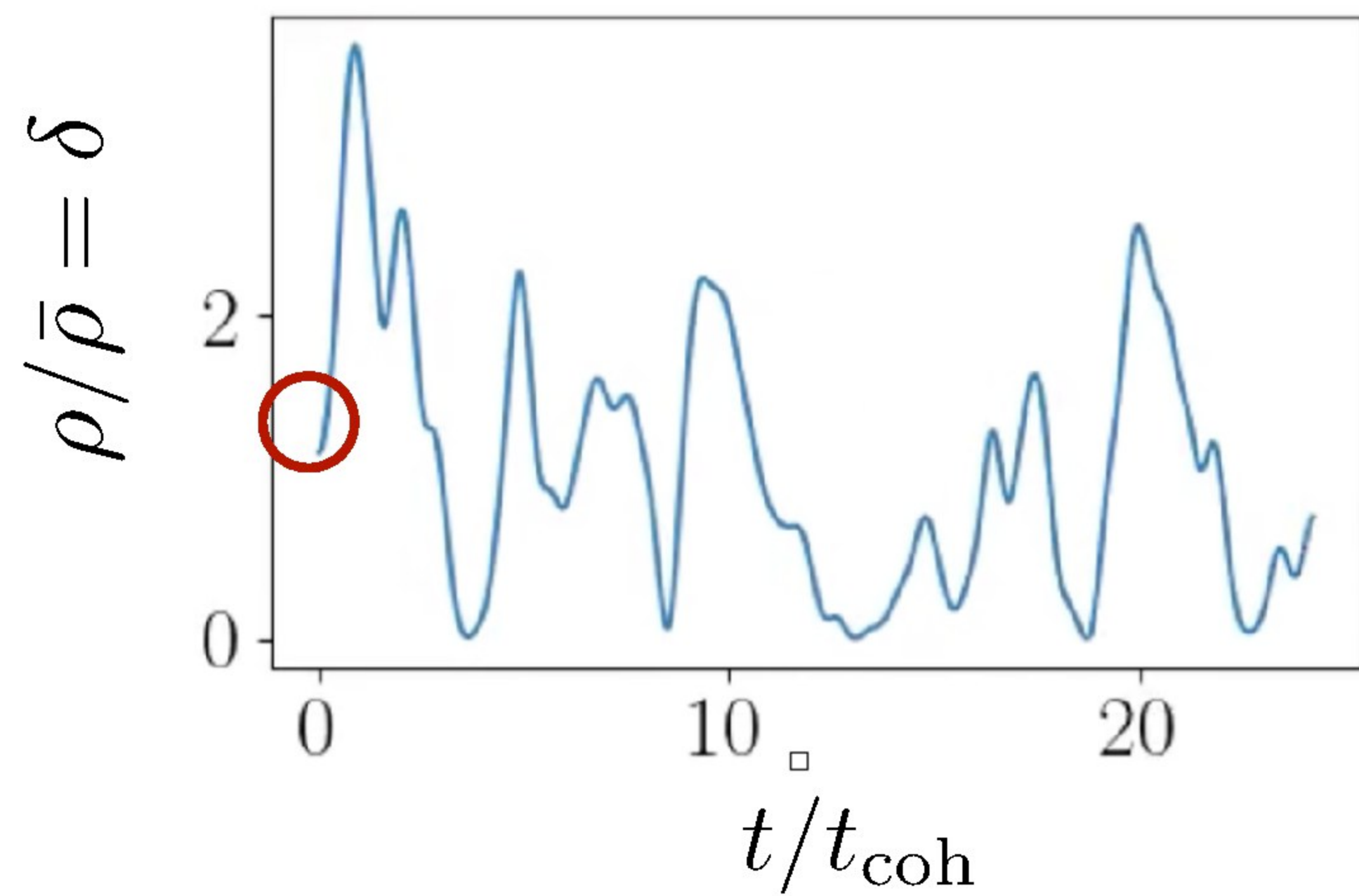
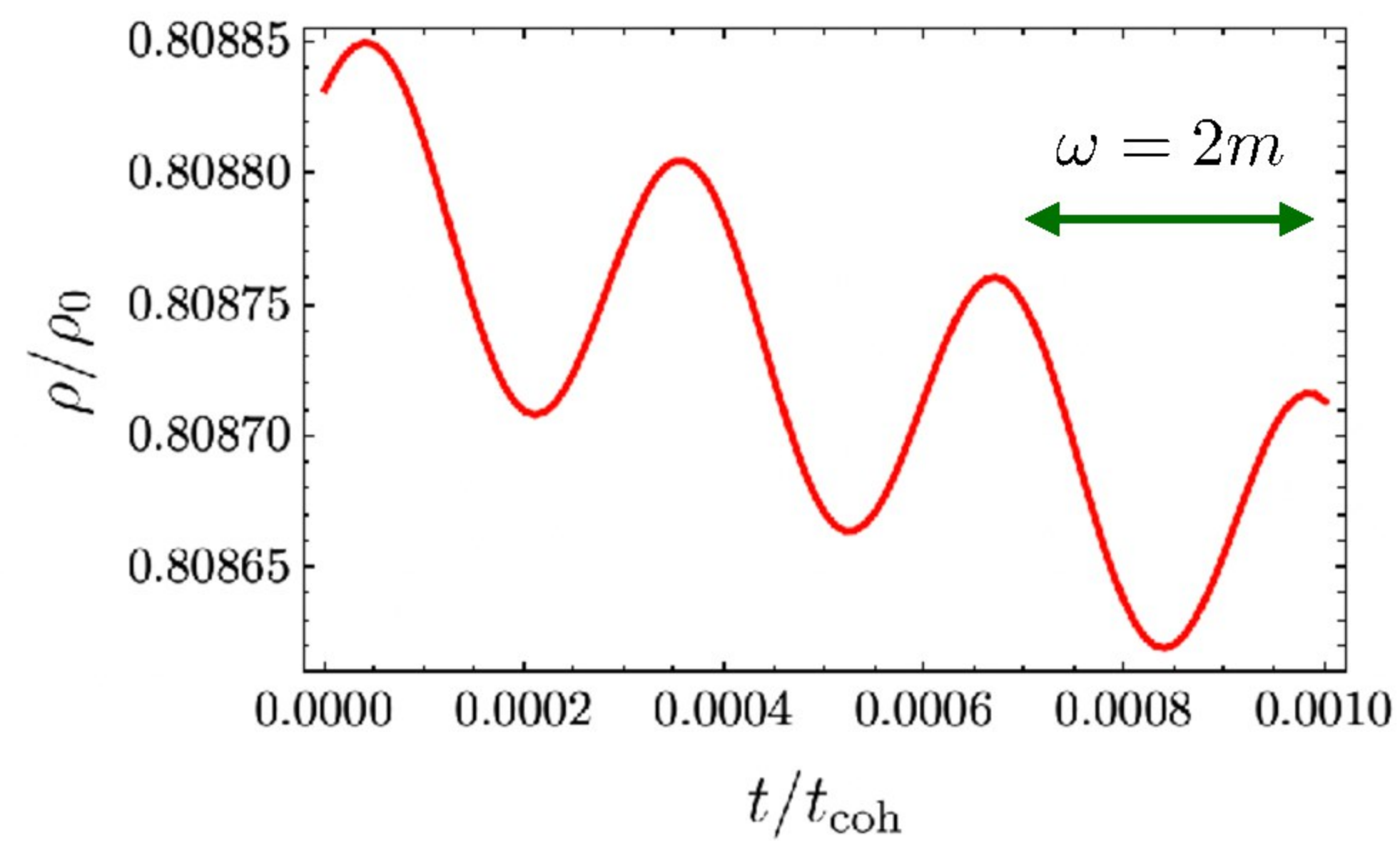
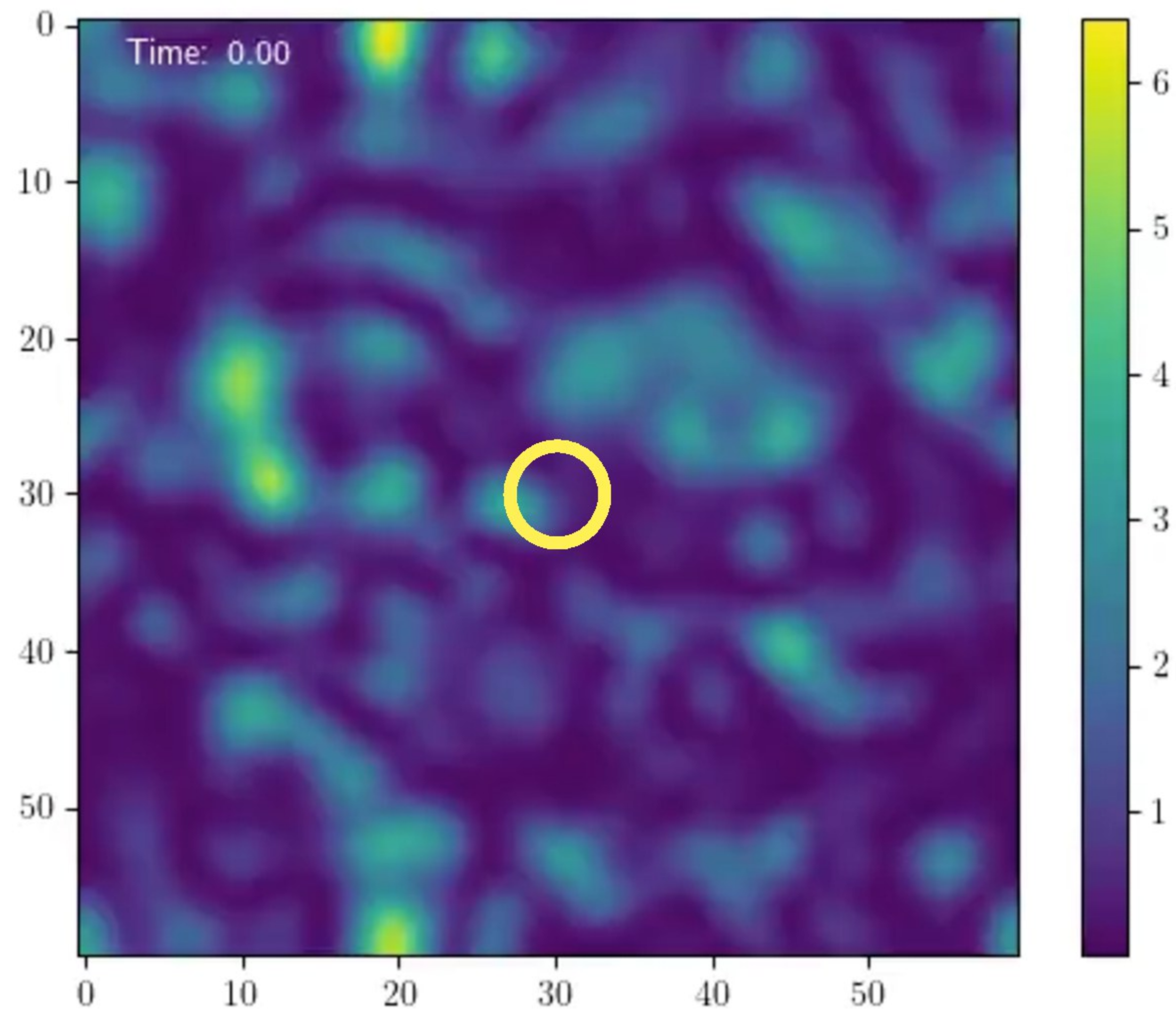


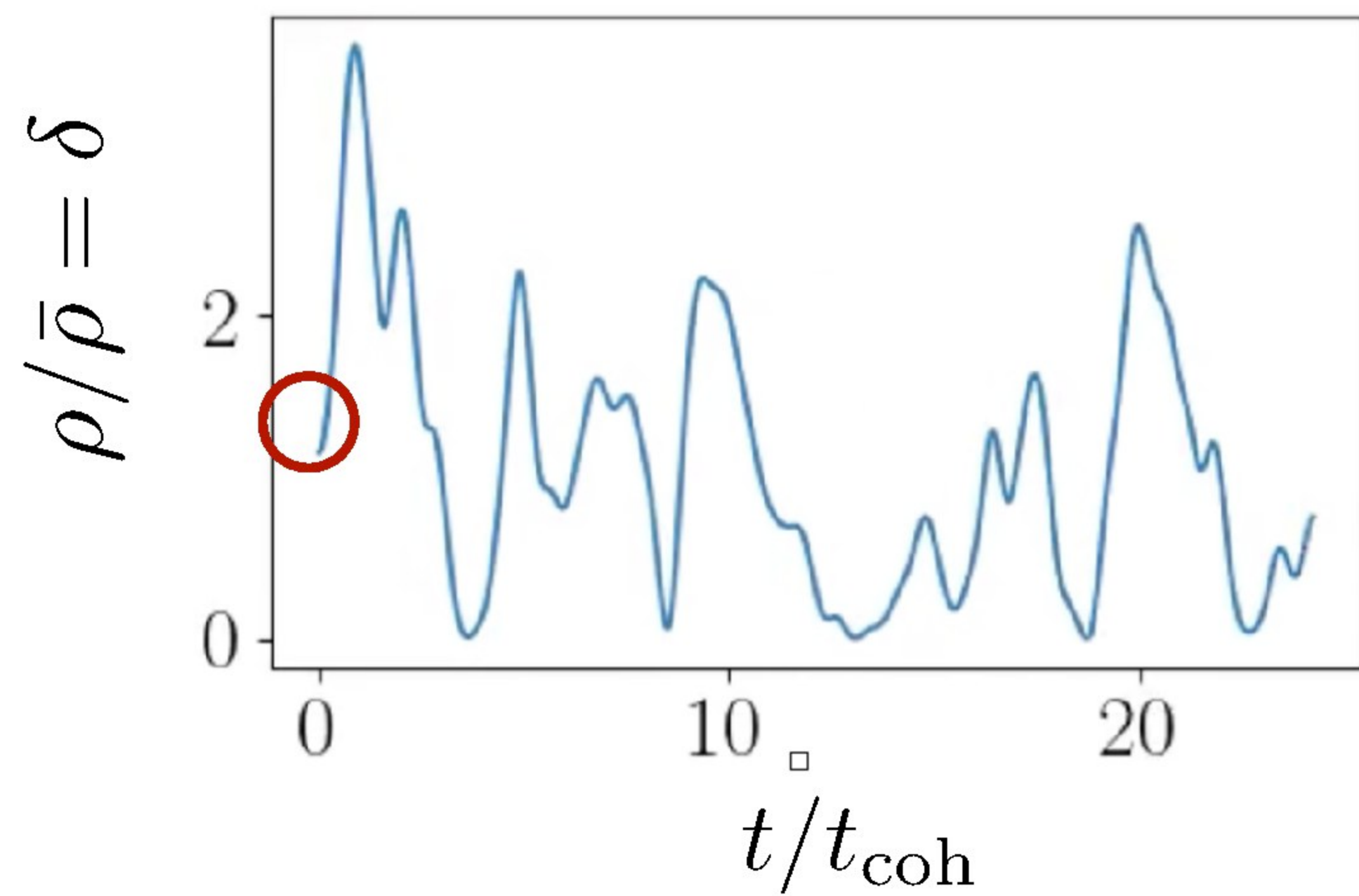
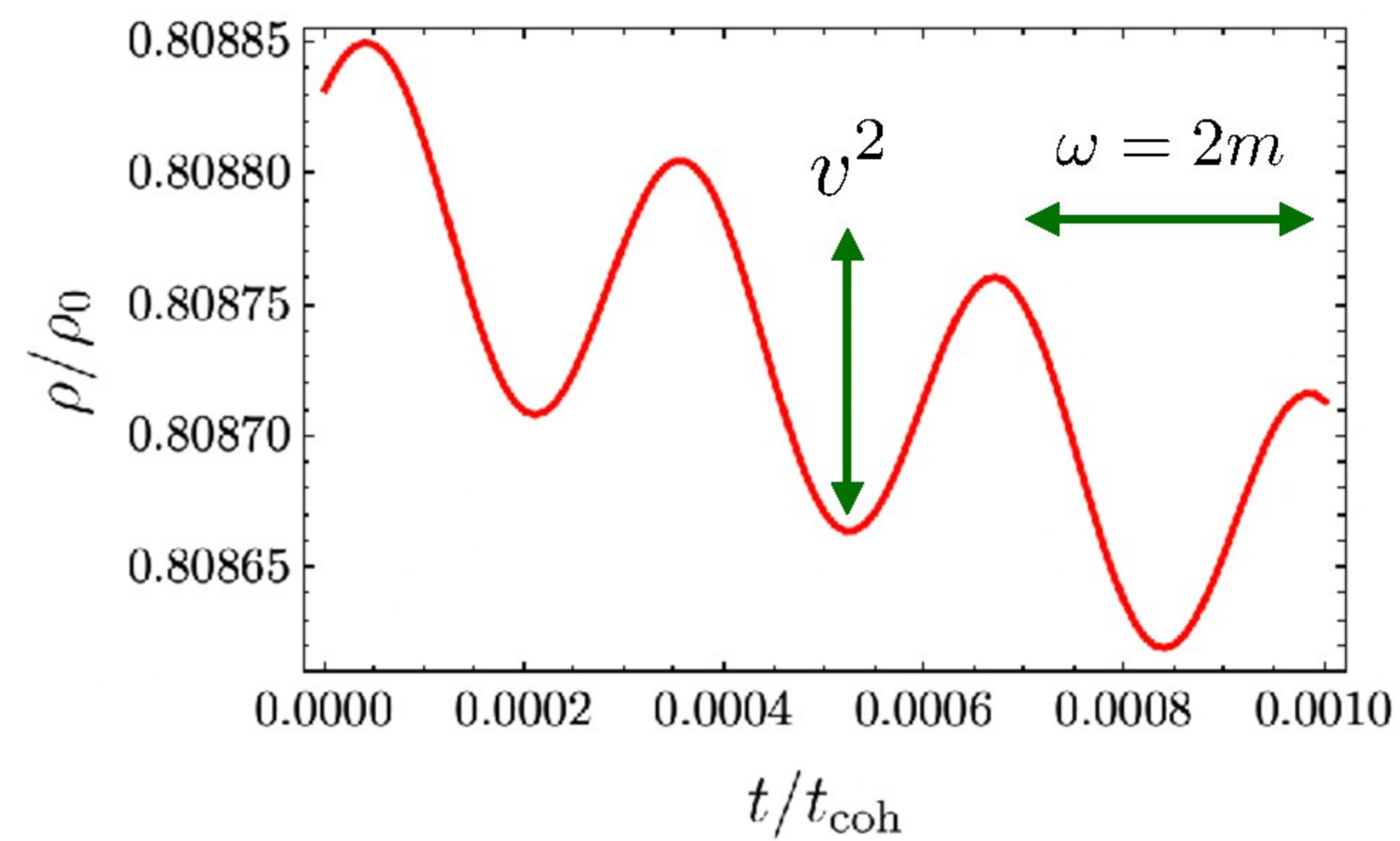
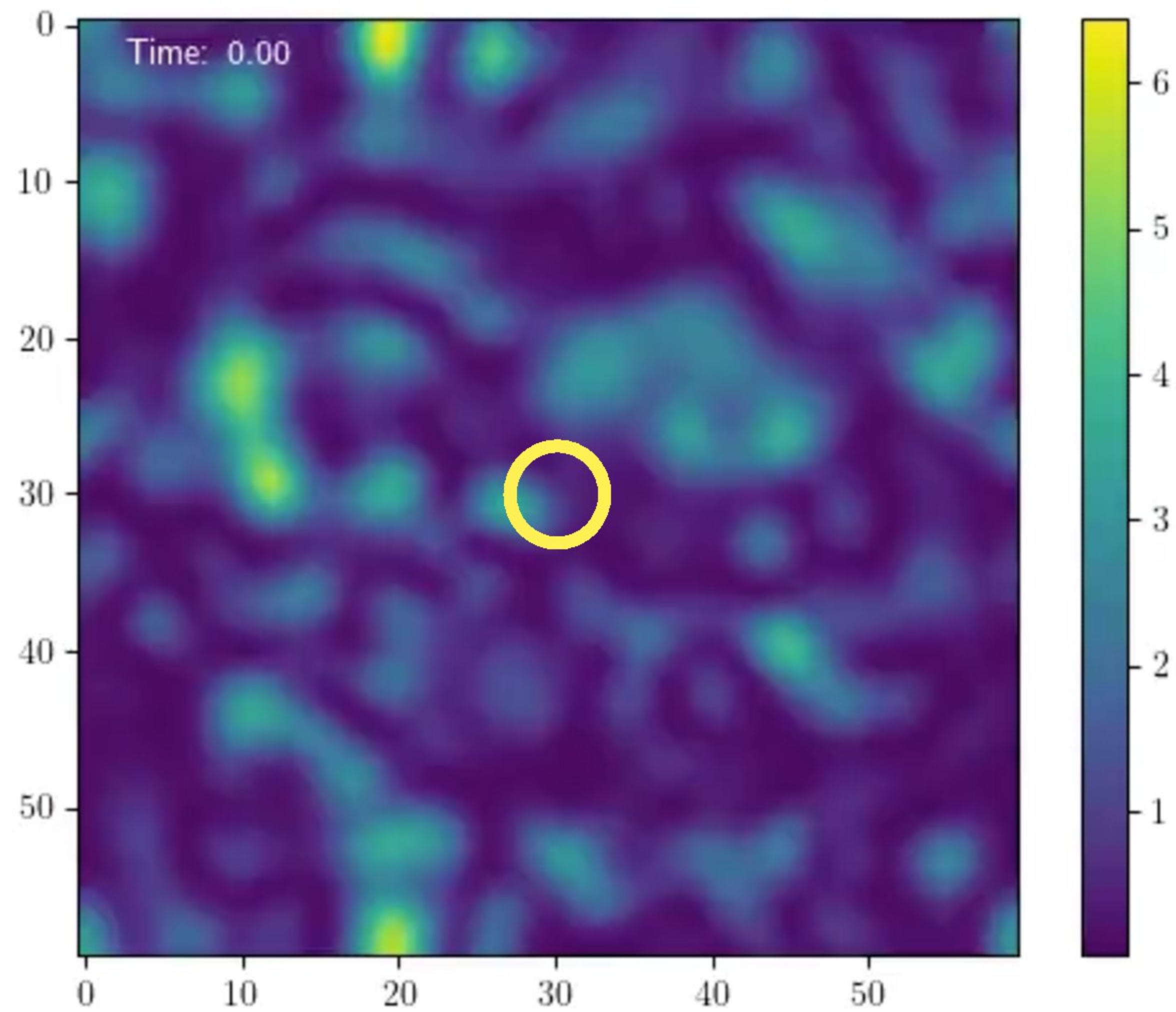


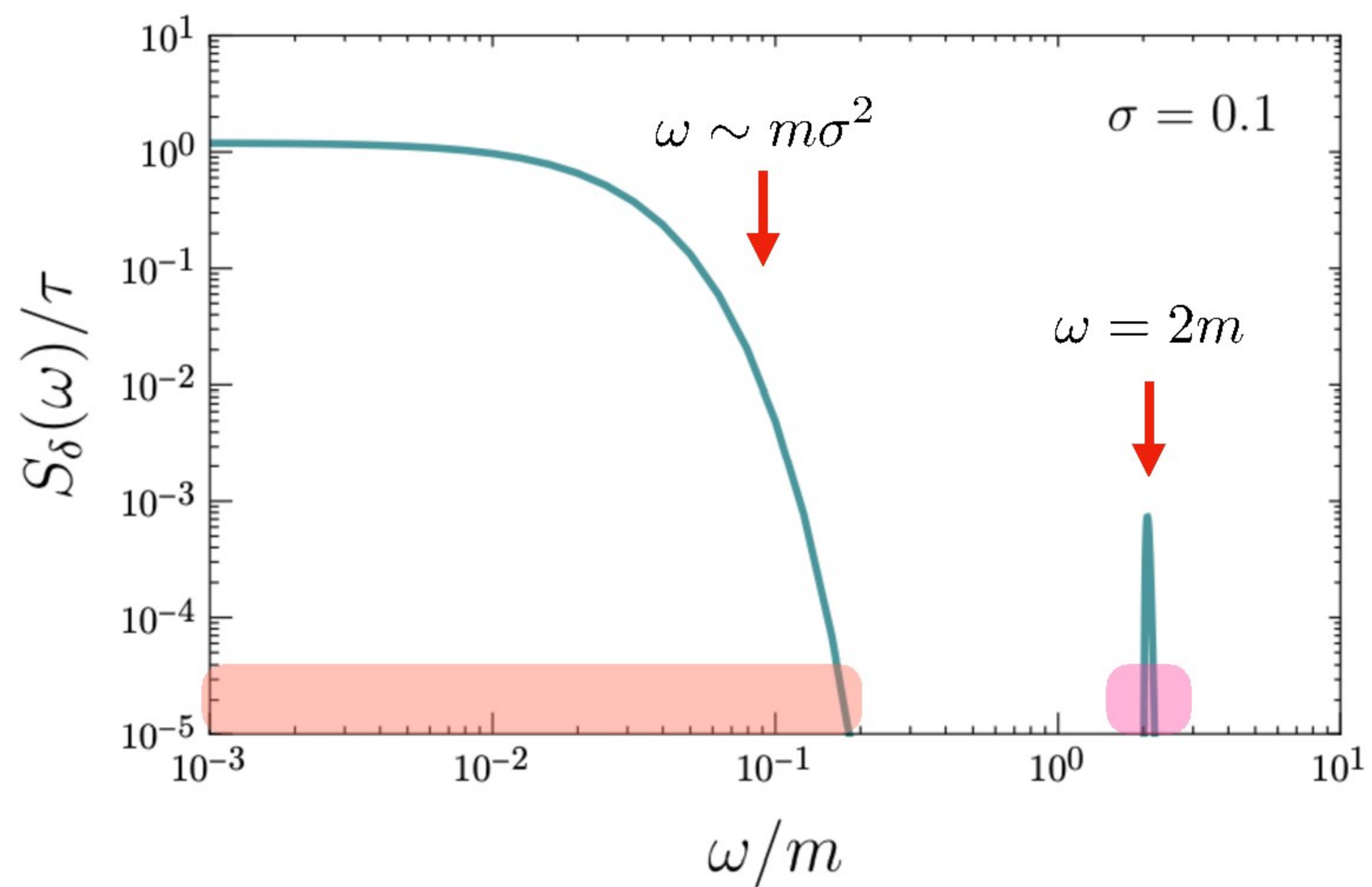
$$\rho/\bar{\rho} = \delta$$





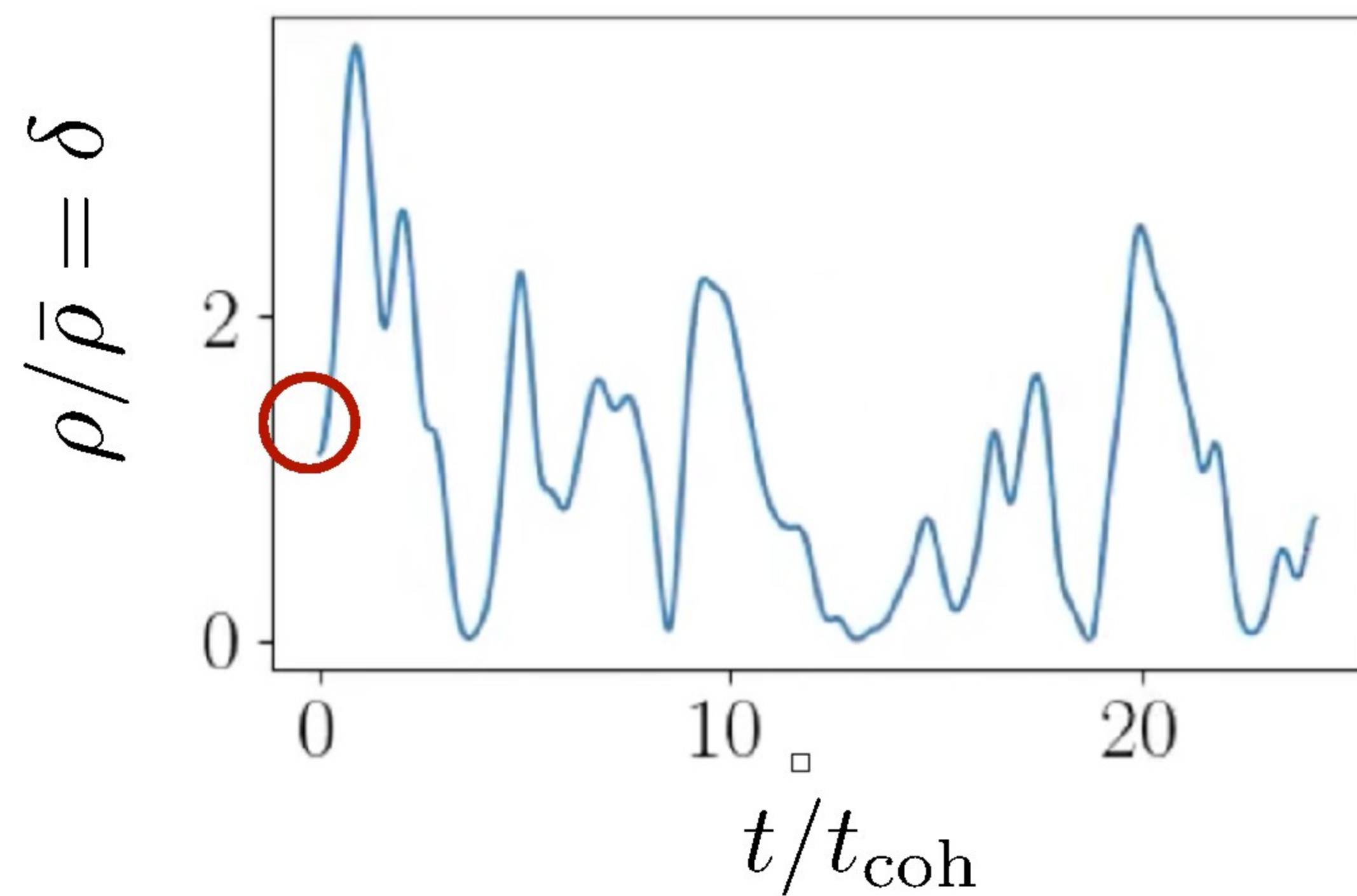
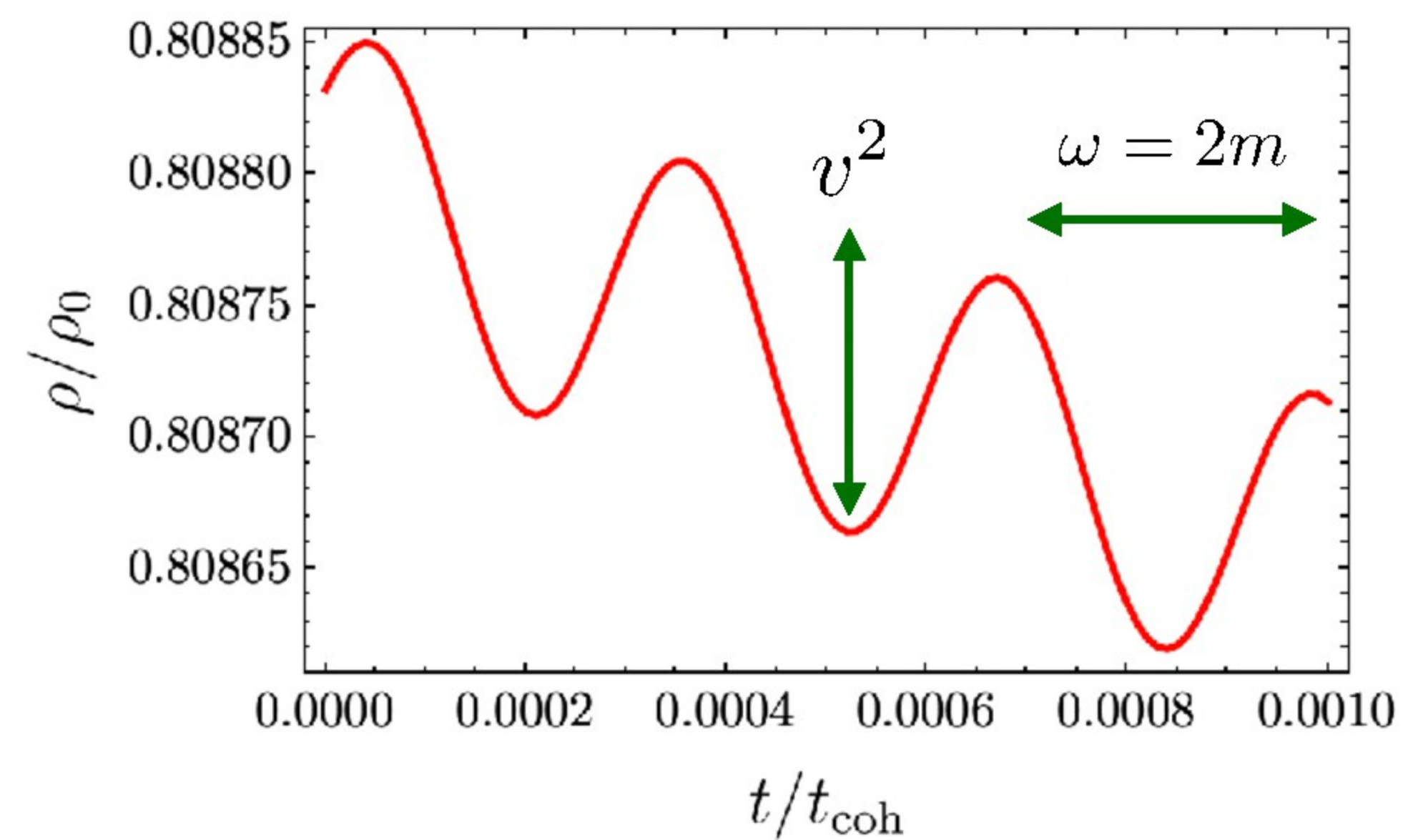


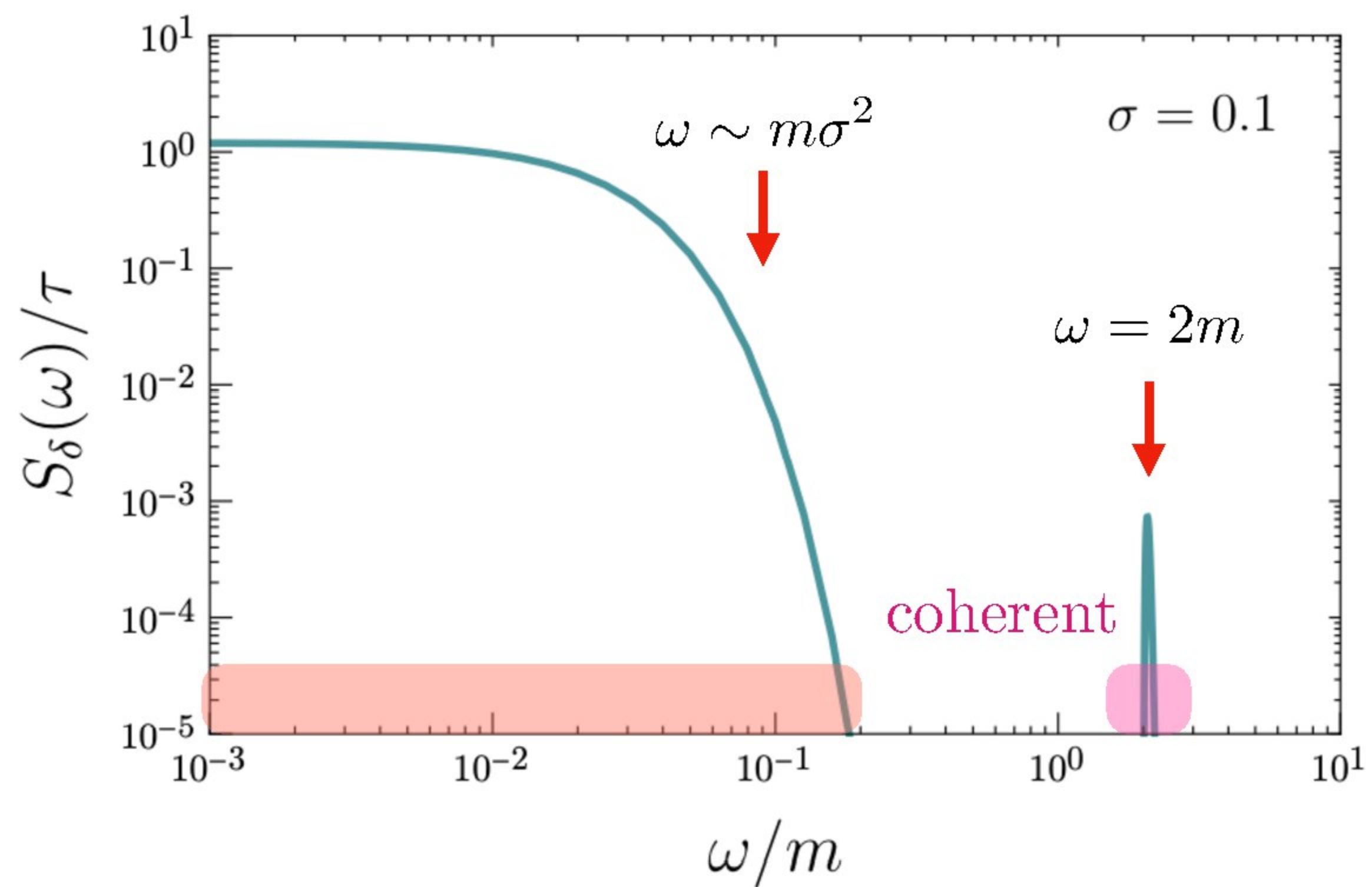




[Kim 2306.13348]

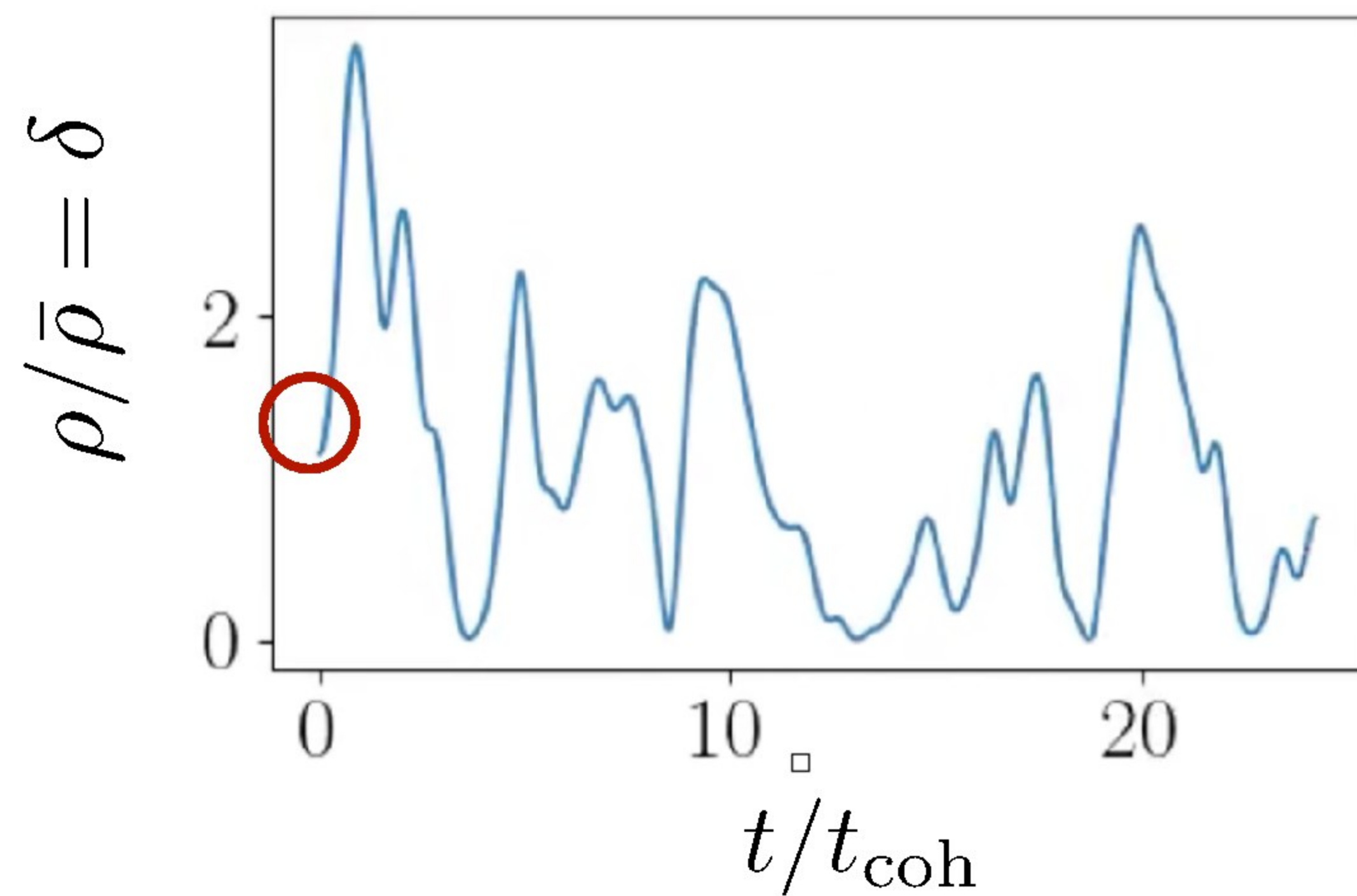
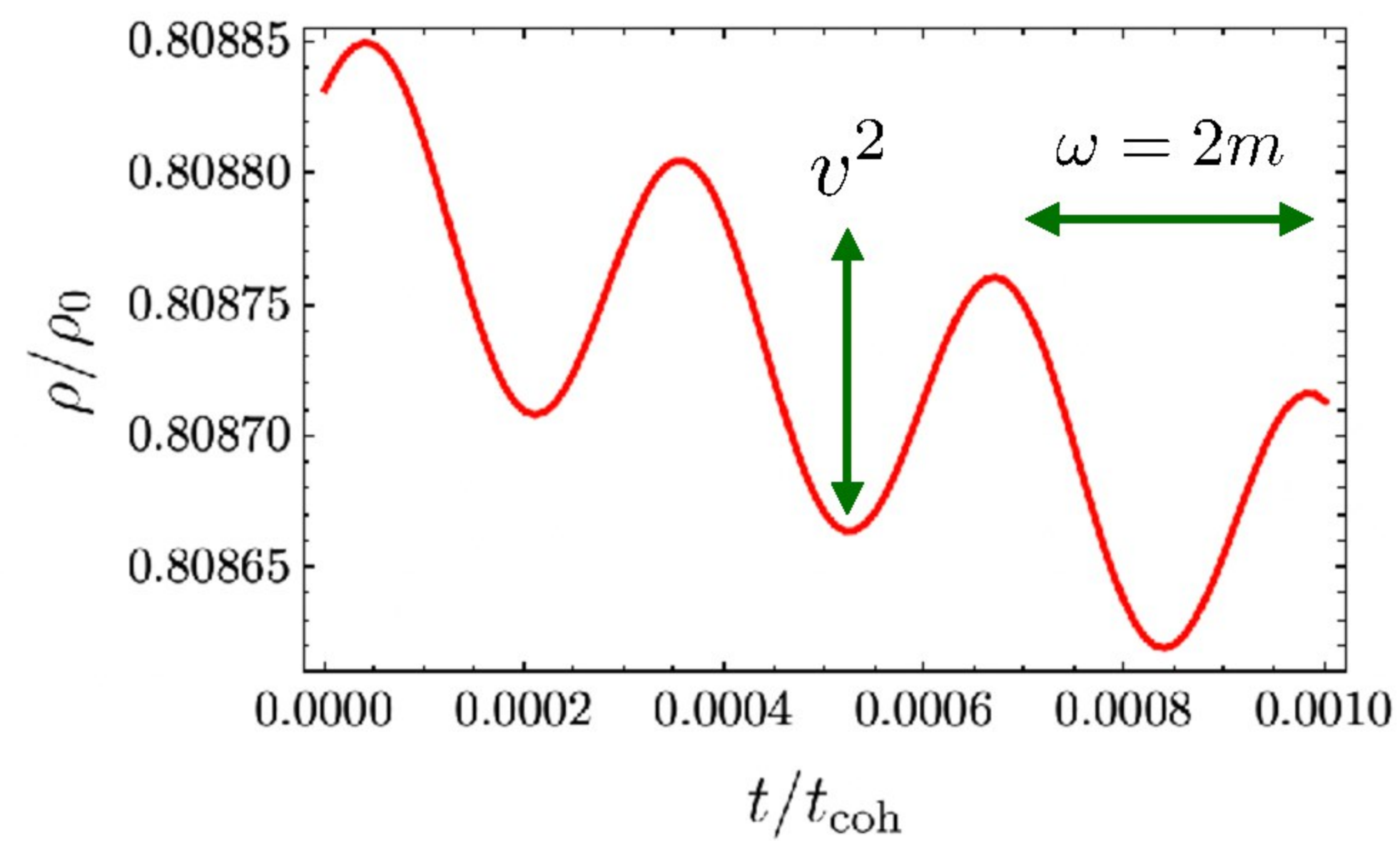
[Kim, Lenoci, Perez, Ratzinger, 2307.14962]

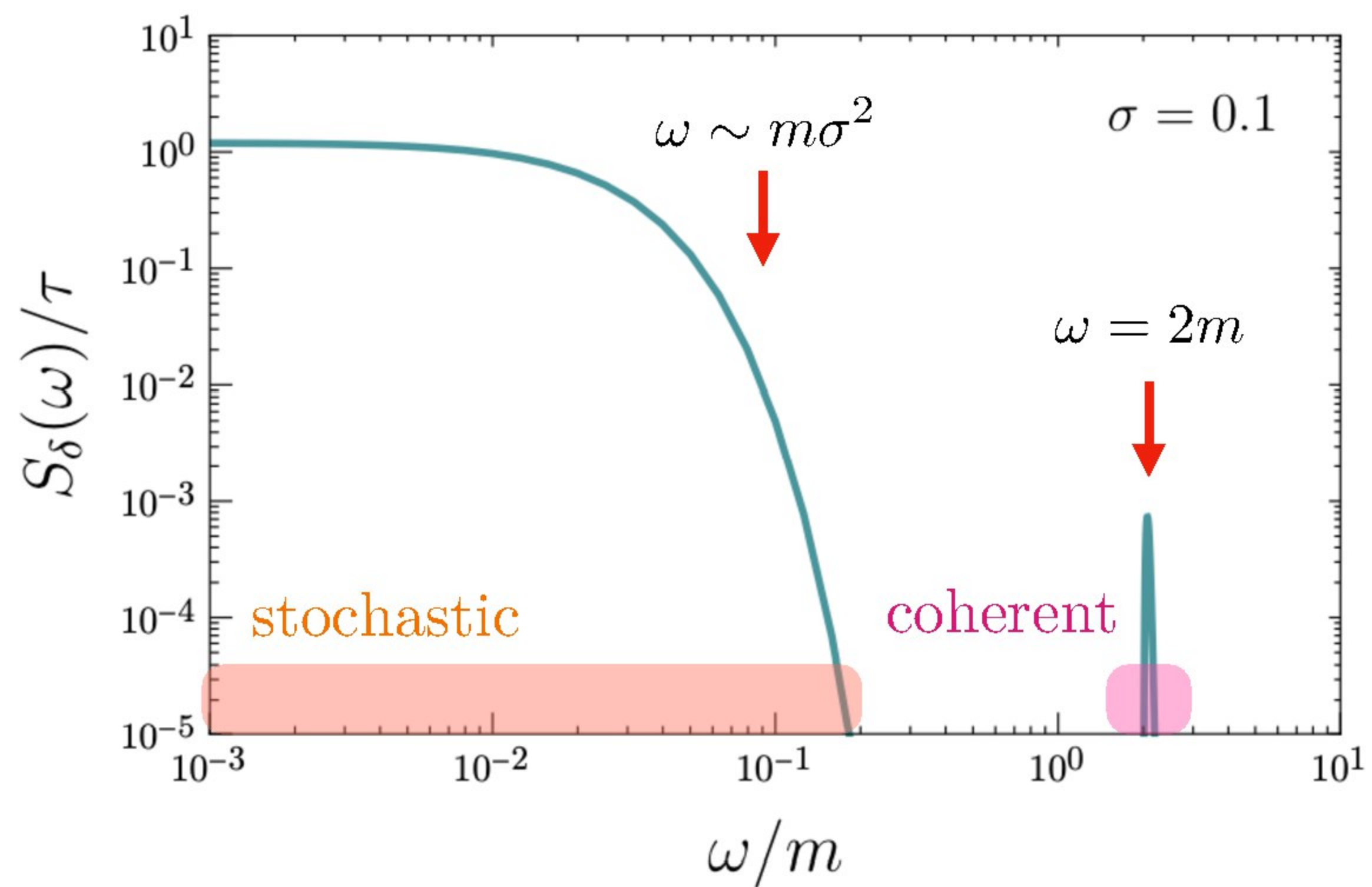




[Kim 2306.13348]

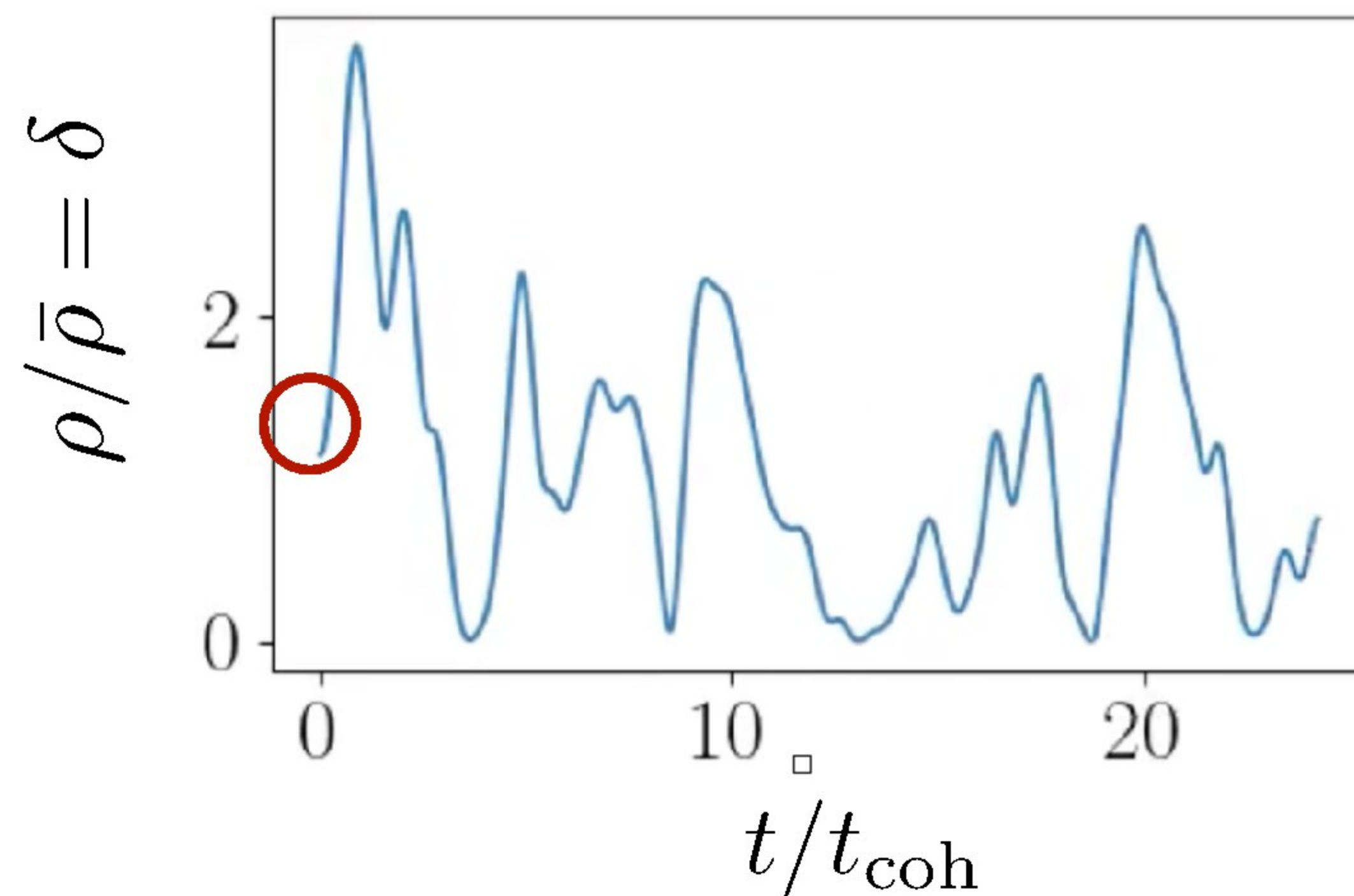
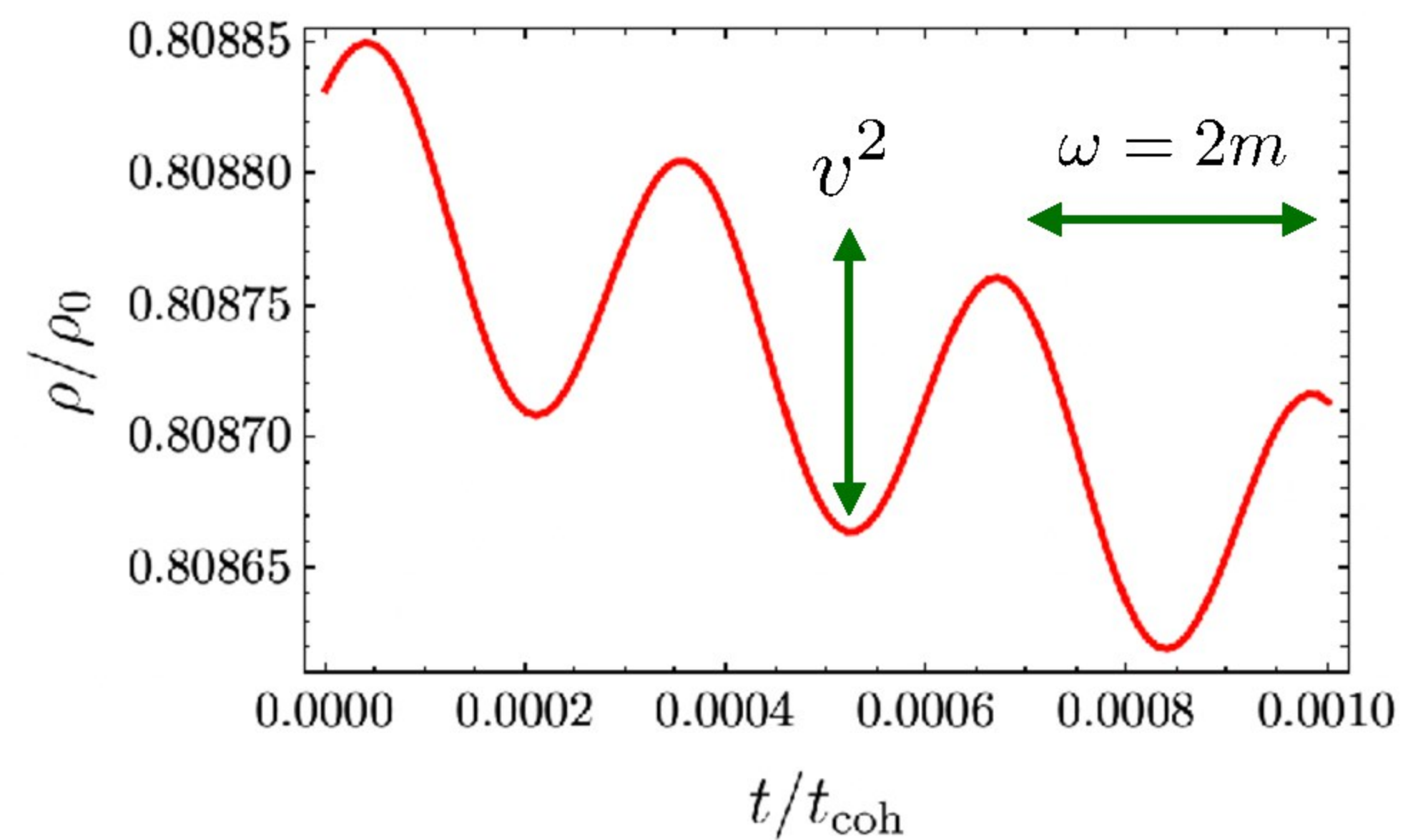
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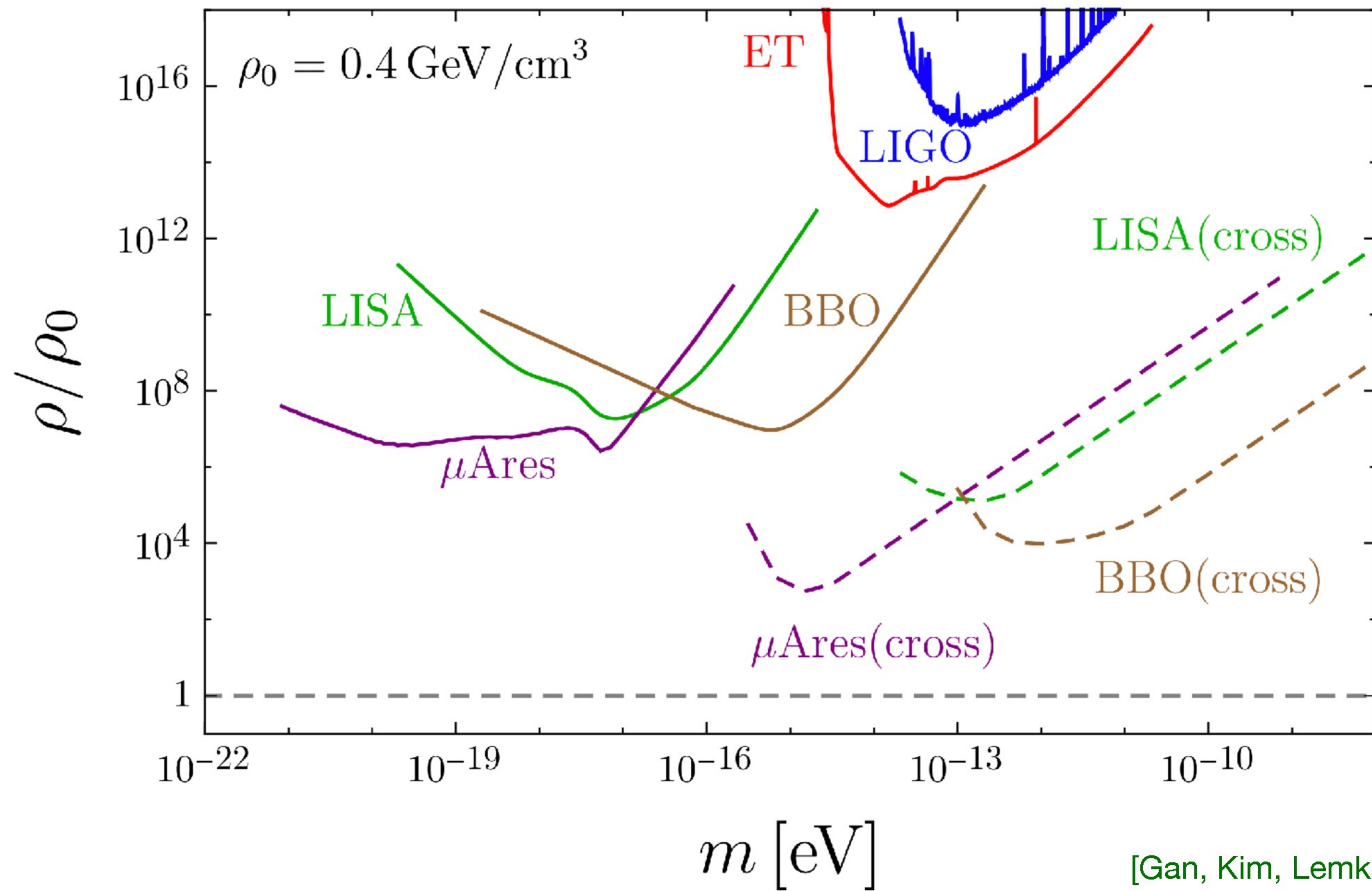


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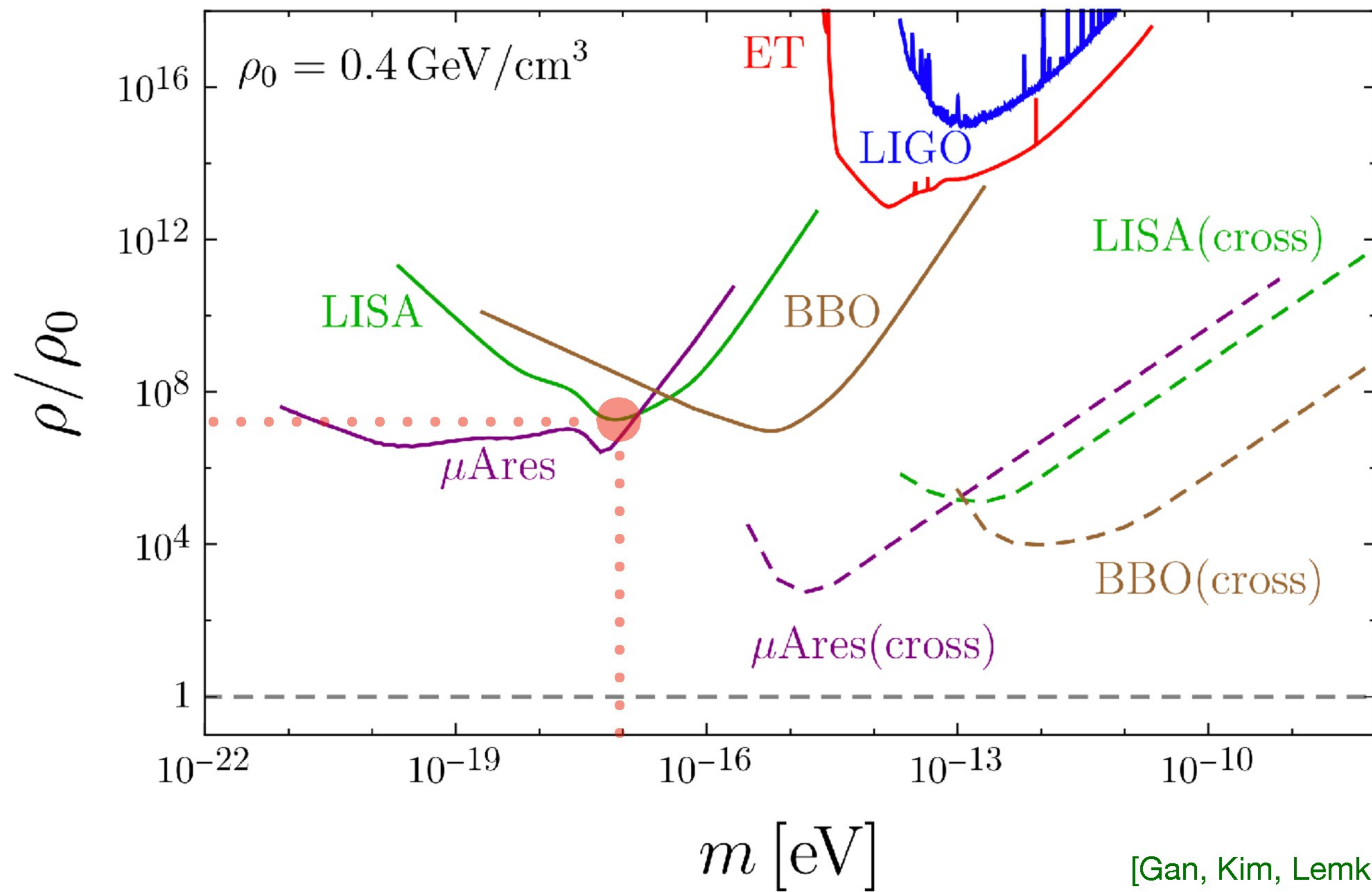
[Aoki, Soda (16); Yu et al (24); Badurina et al (25)]
for coherent signals with interferometers



[Kim (23)]

[Gan, Kim, Lemke, Mitridate (26)]

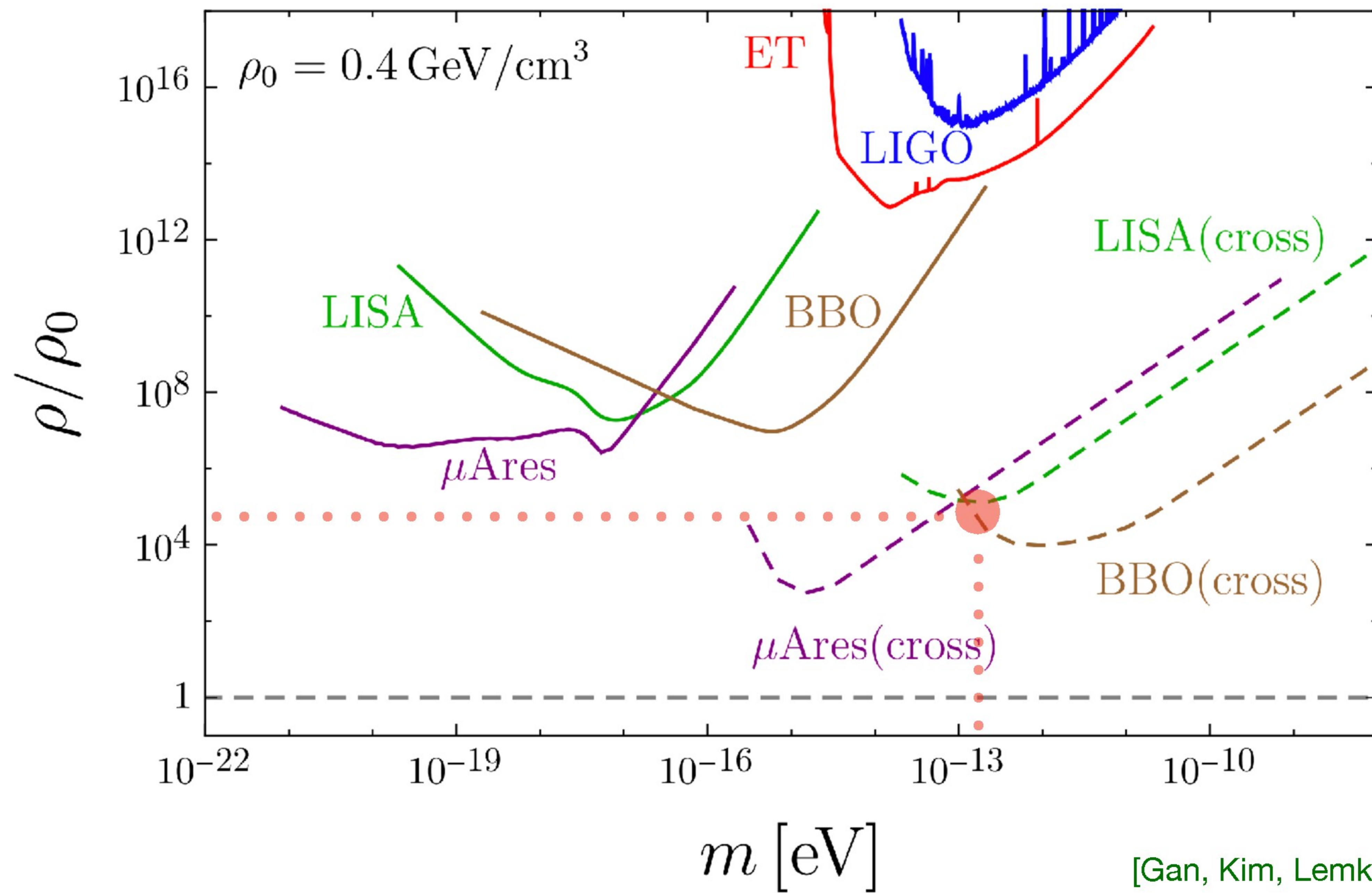
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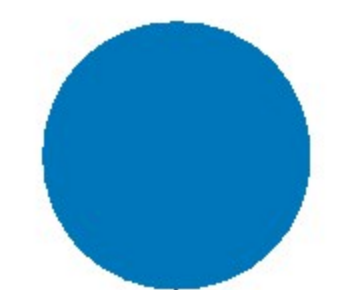


[Kim (23)]

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another example:

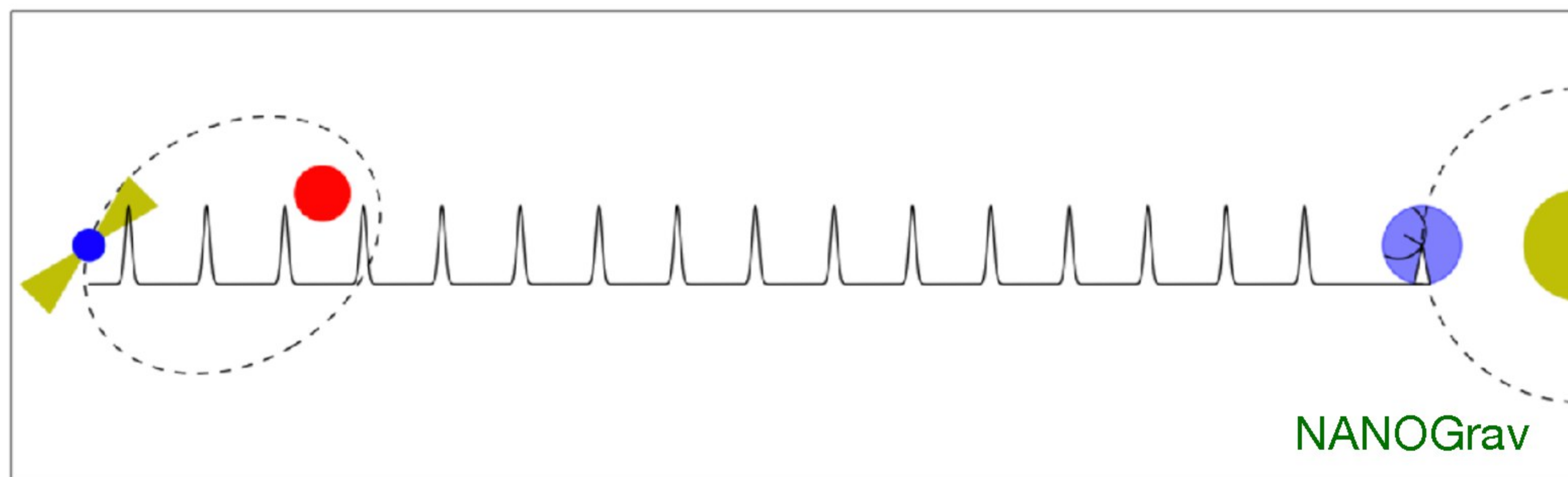
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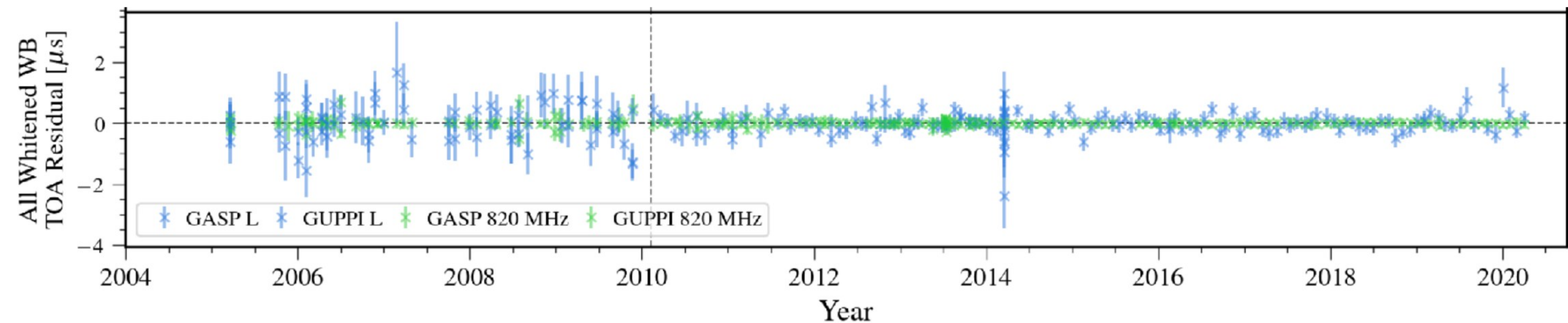
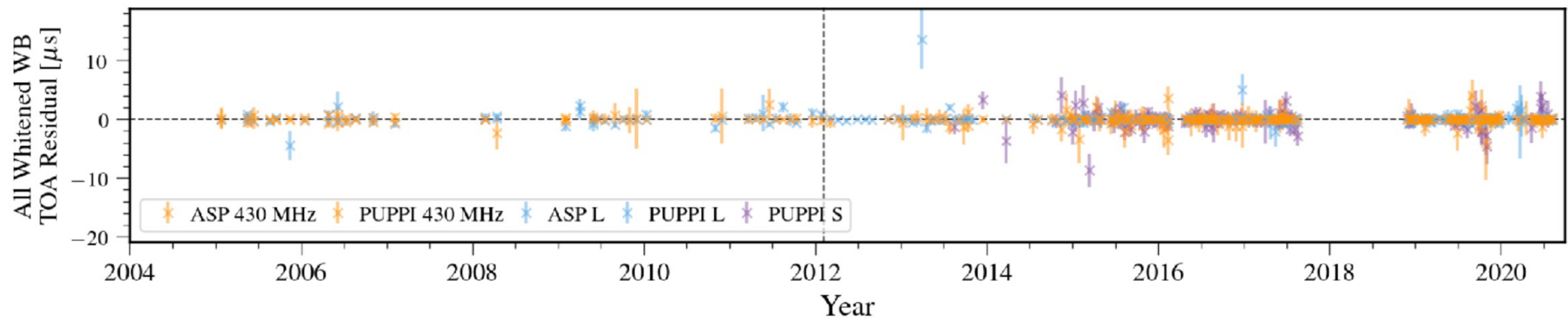
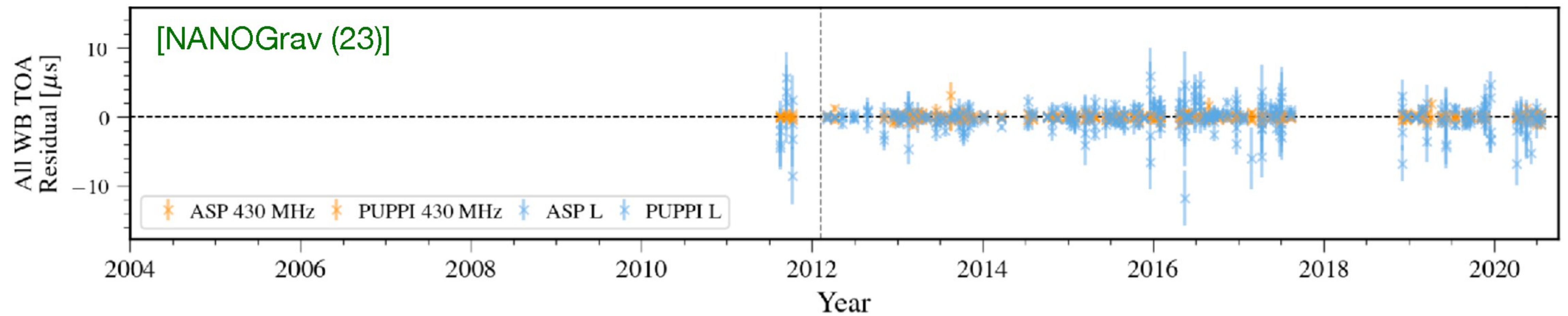
Earth

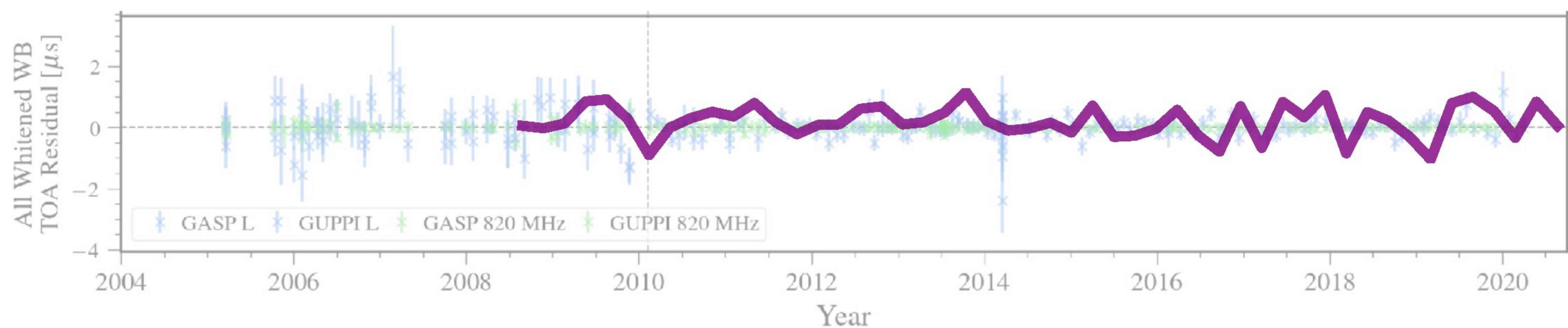
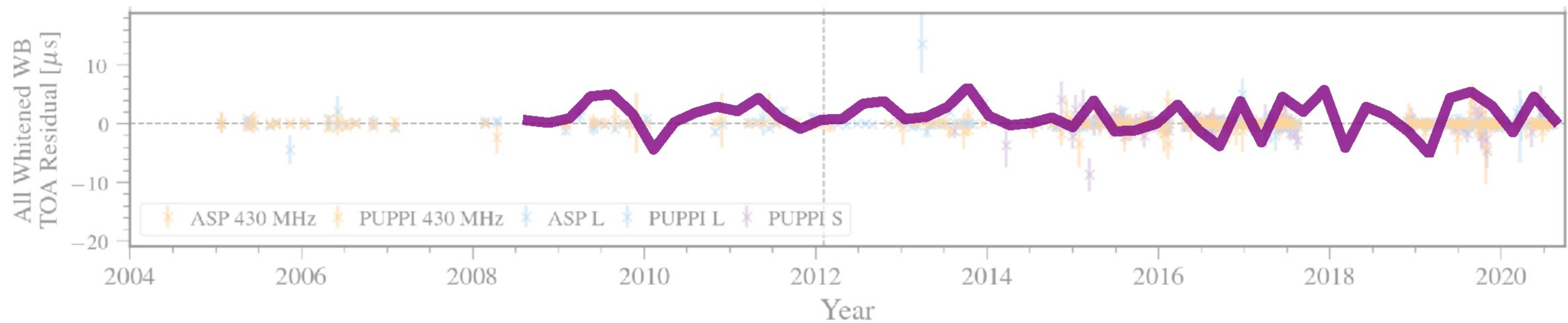
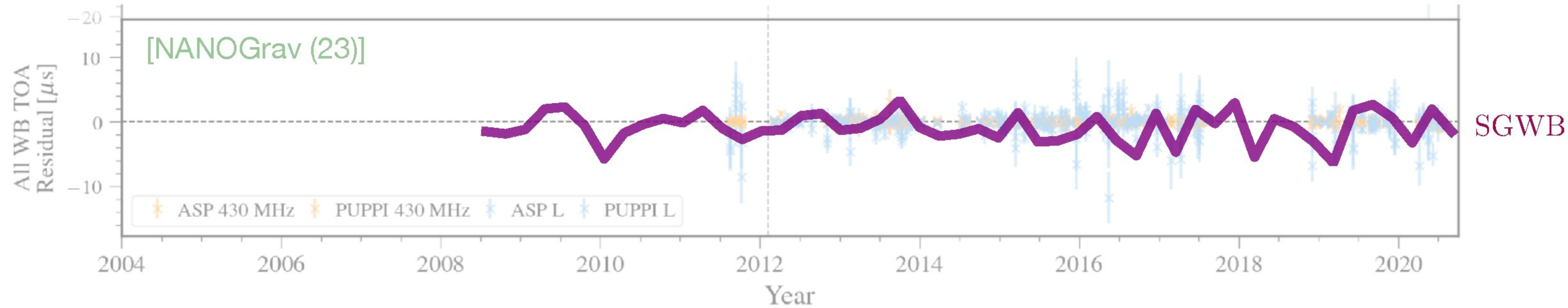


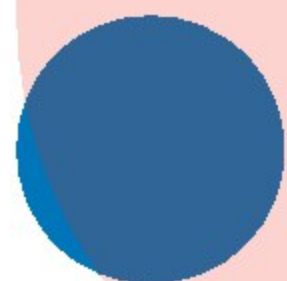
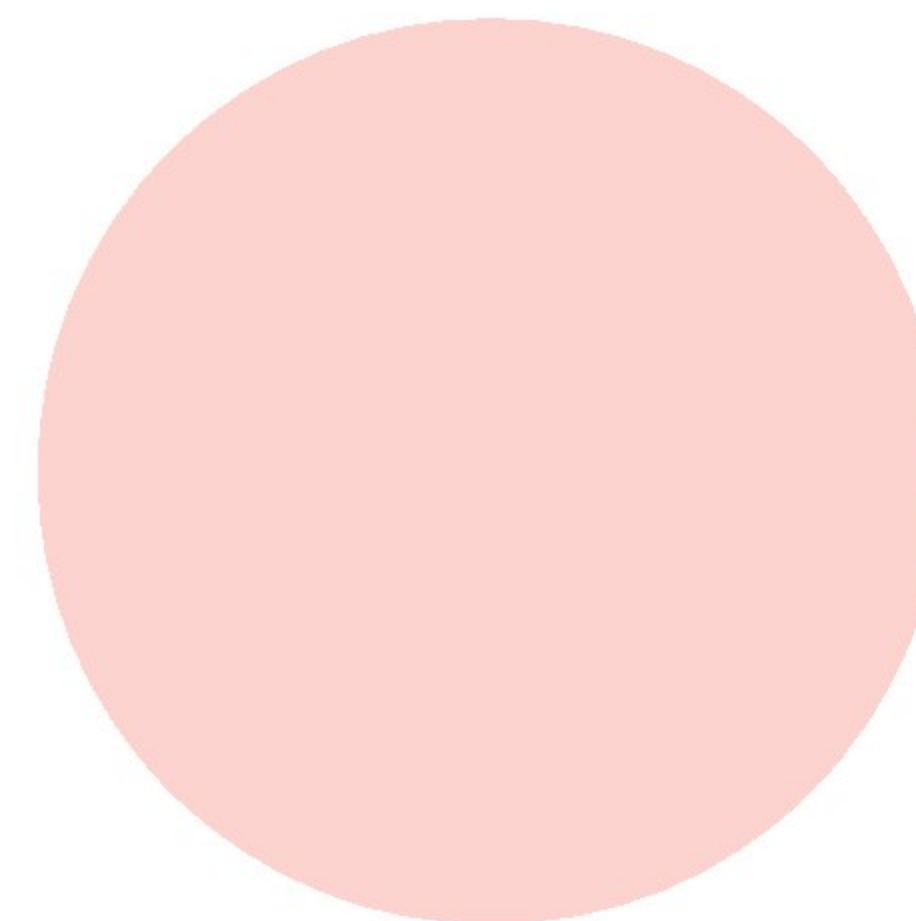
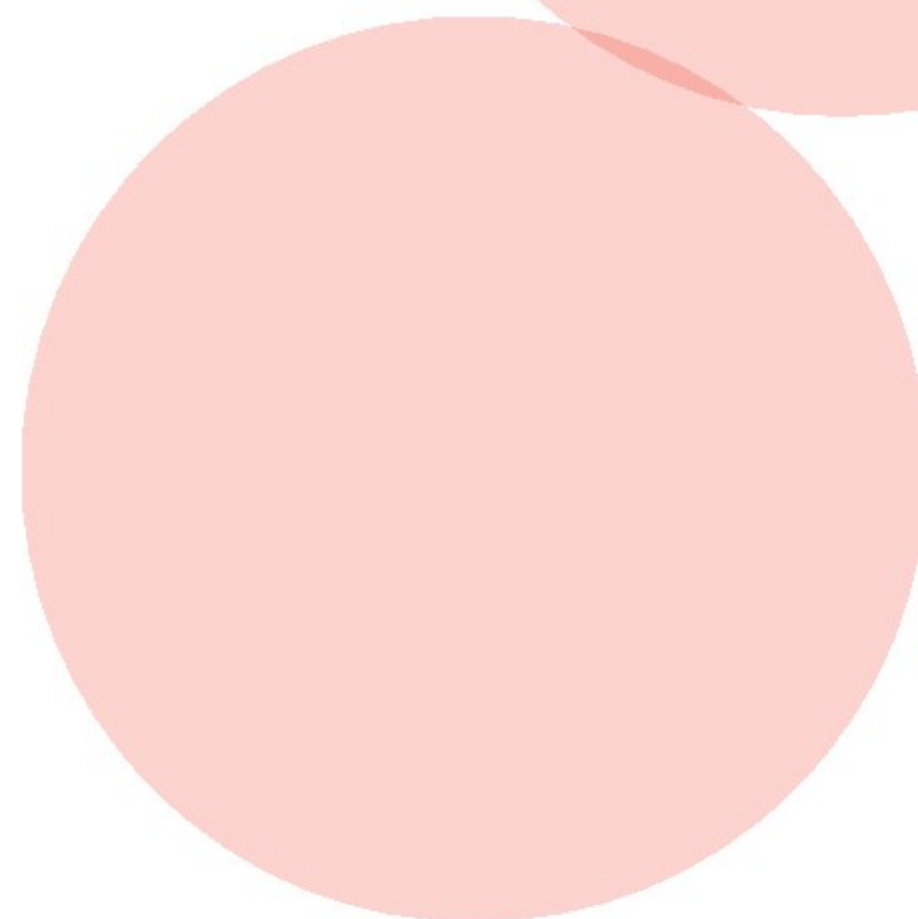
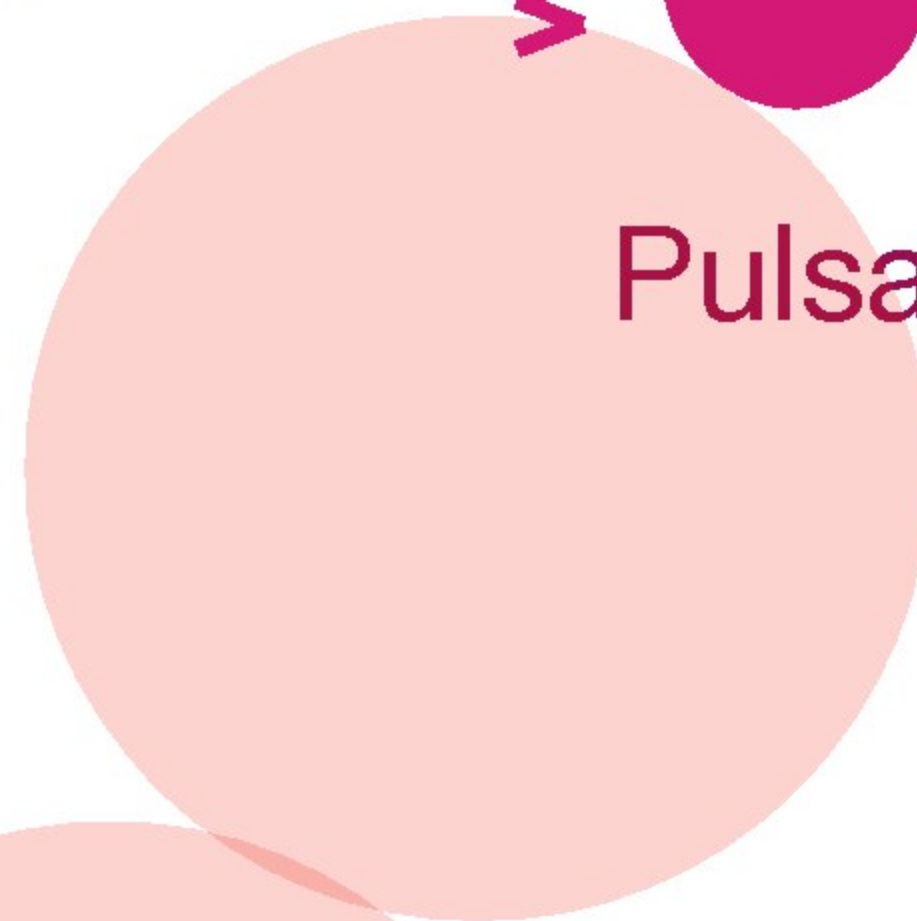
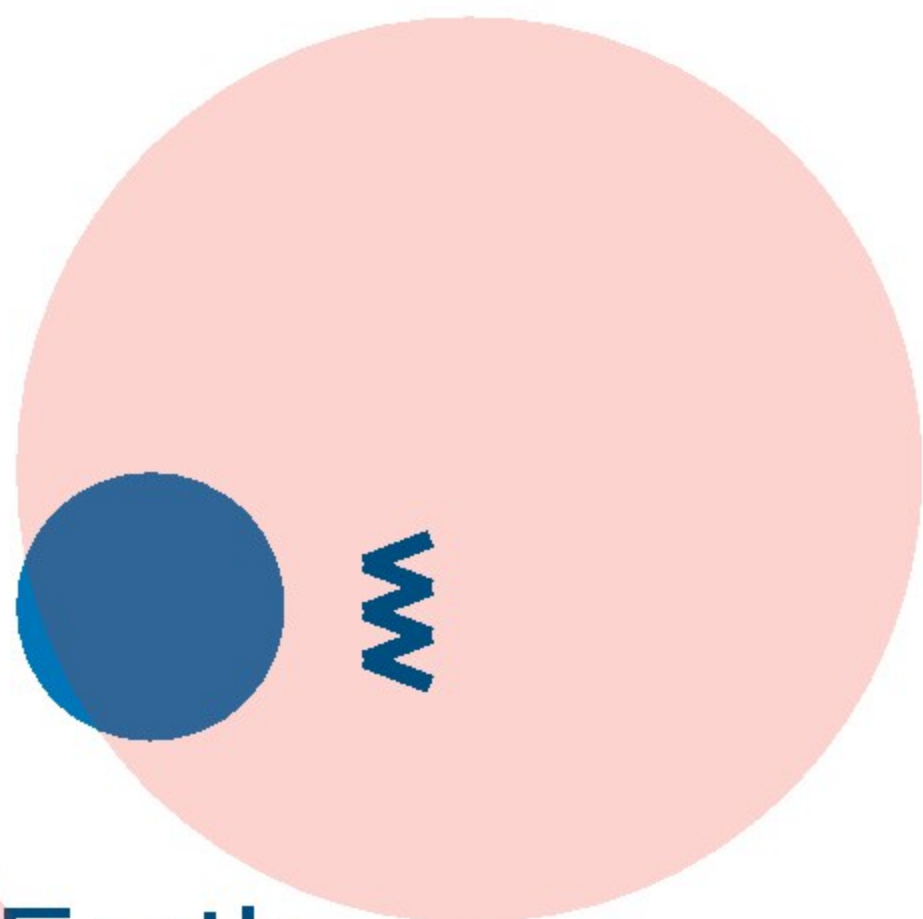
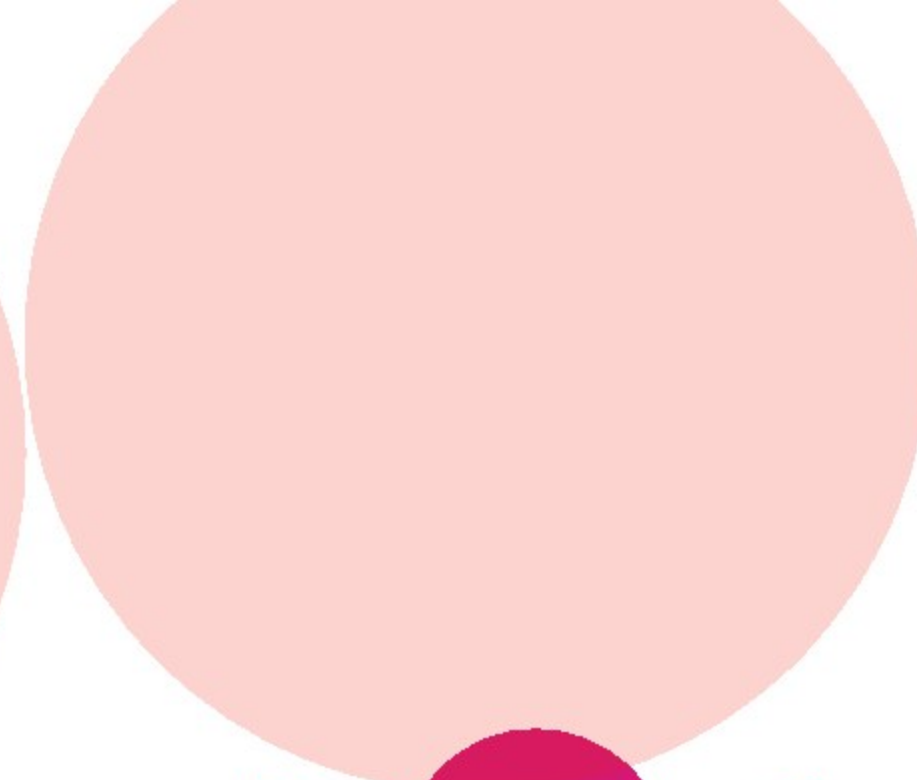
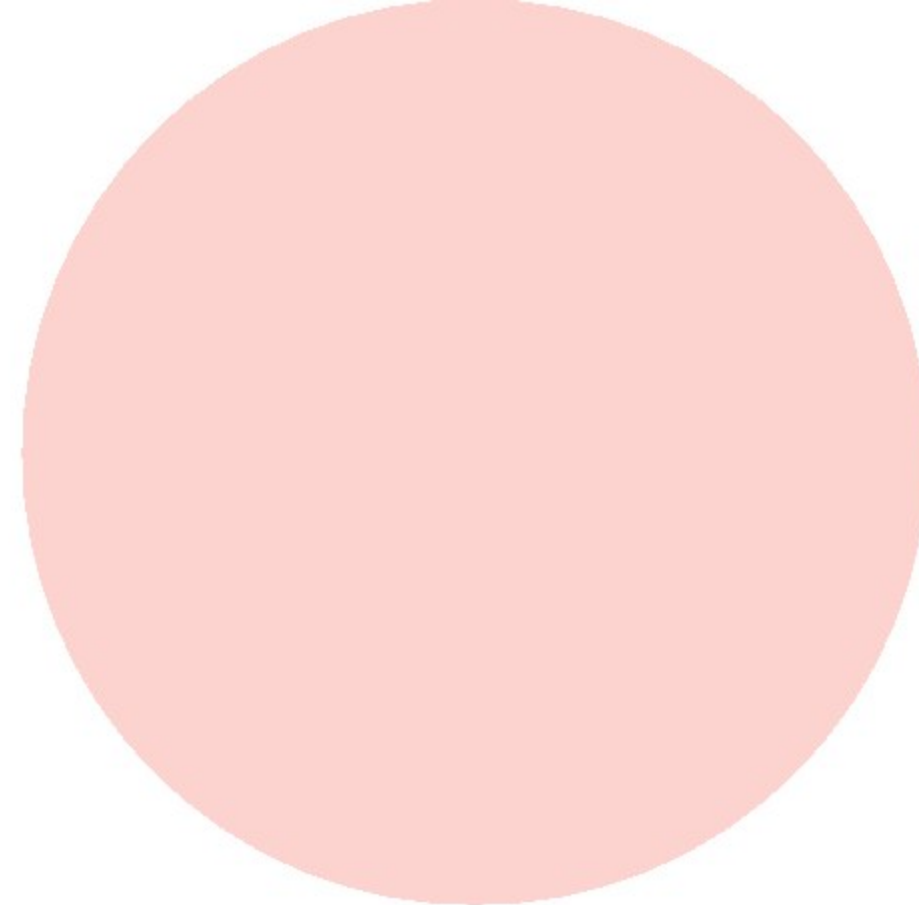
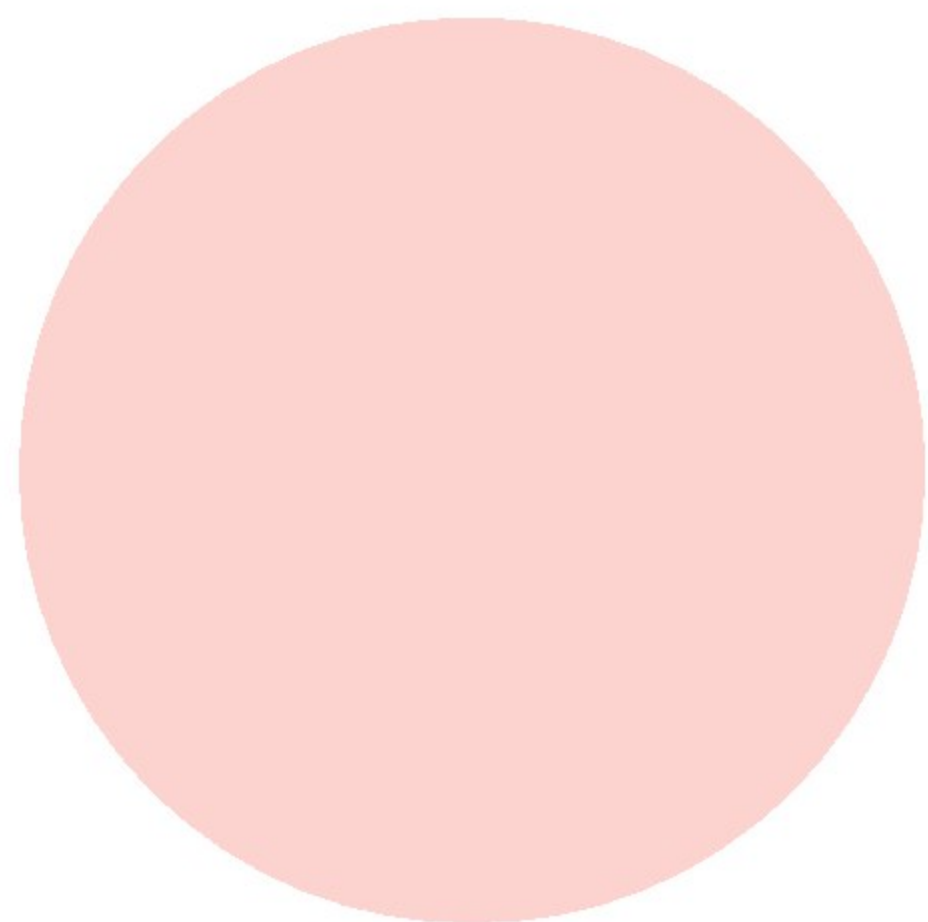
Pulsar



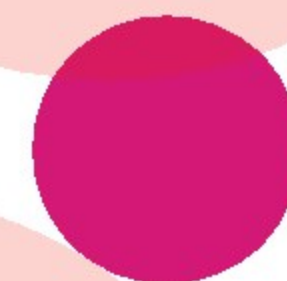
NANOGrav



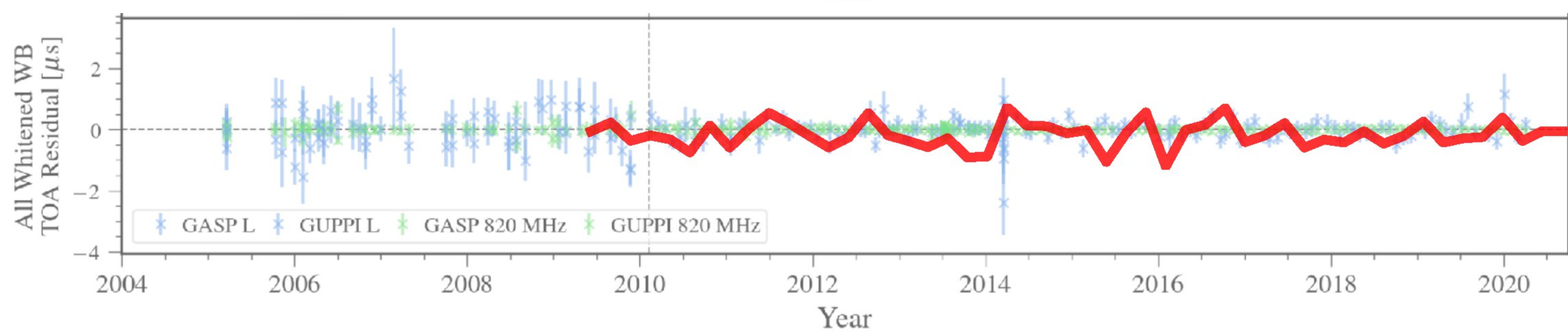
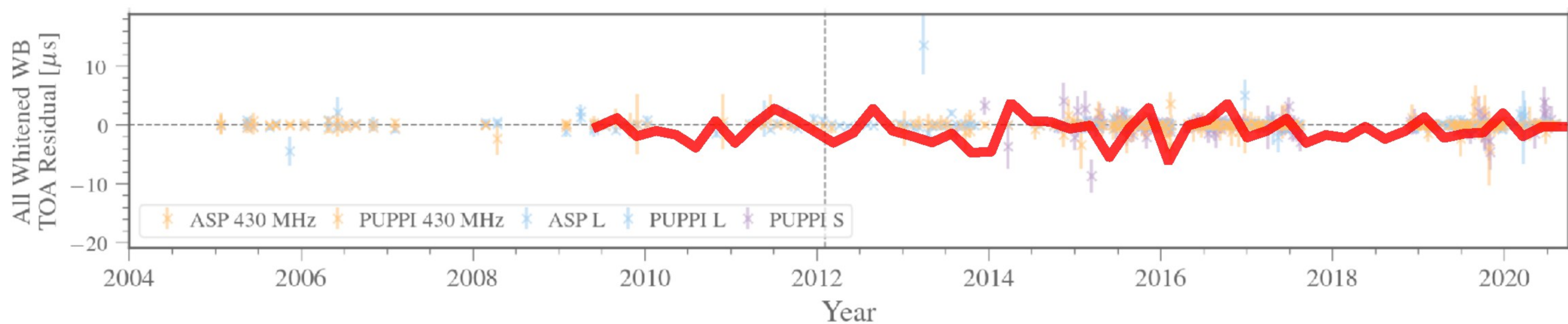
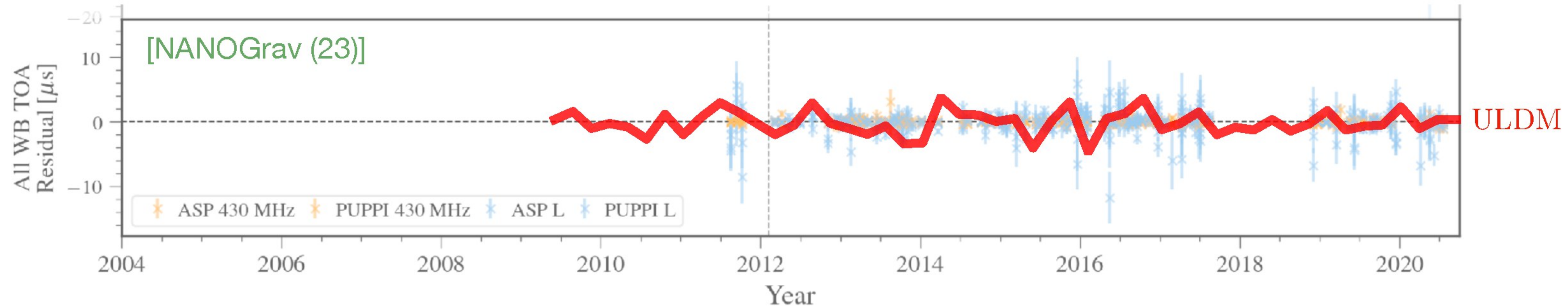




Earth



Pulsar

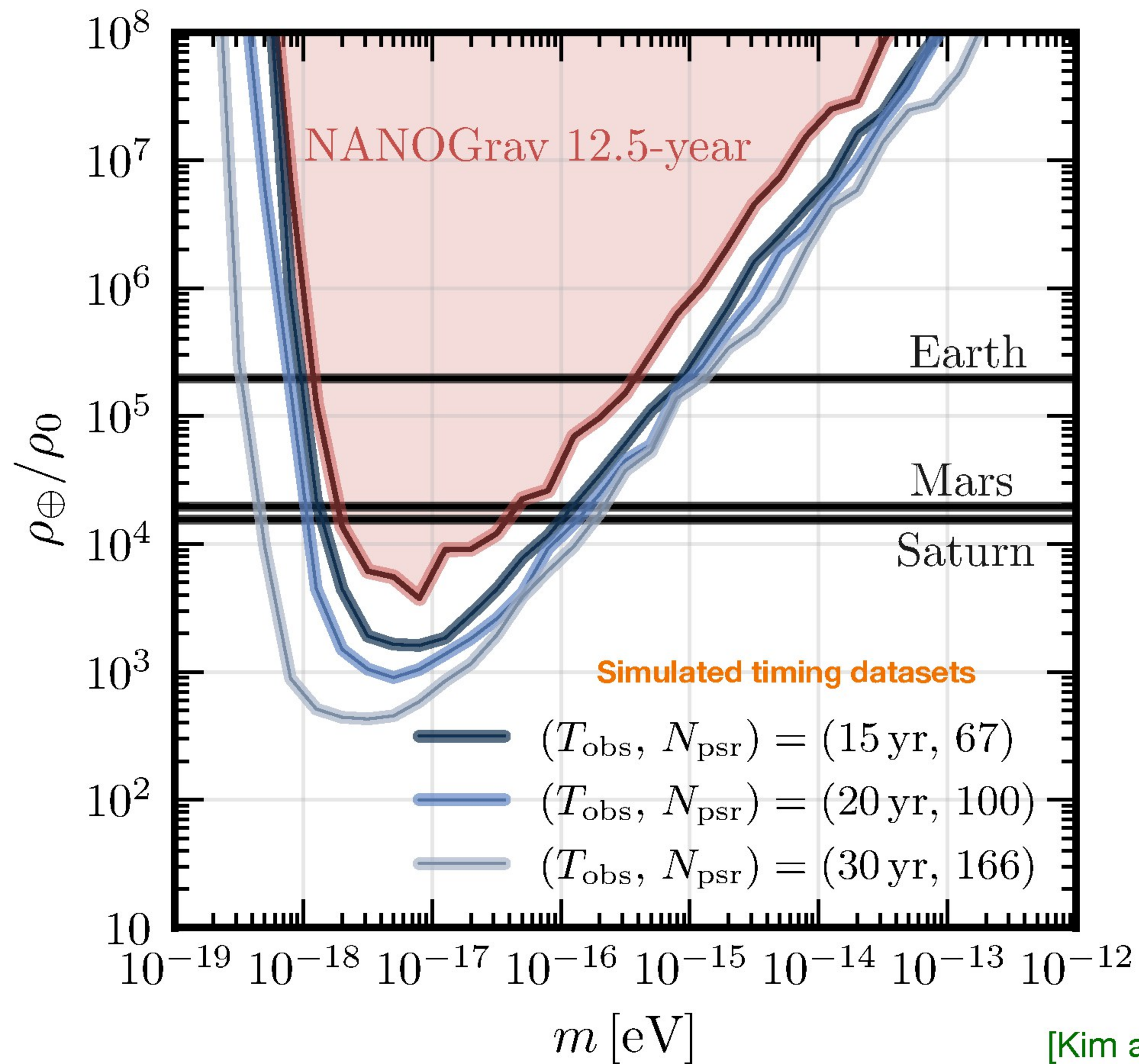


$$\langle \delta t_a \delta t_b \rangle = \int df \, \Gamma_{ab}^{\text{ULDM}} S_{\delta t}^{\text{ULDM}}(f)$$

$$\langle \delta t_a \delta t_b \rangle = \int df \, \Gamma_{ab}^{\text{ULDM}} S_{\delta t}^{\text{ULDM}}(f)$$

$$\langle \delta t_a \delta t_b \rangle = \int df \, \Gamma_{ab}^{\text{ULDM}} S_{\delta t}^{\text{ULDM}}(f)$$

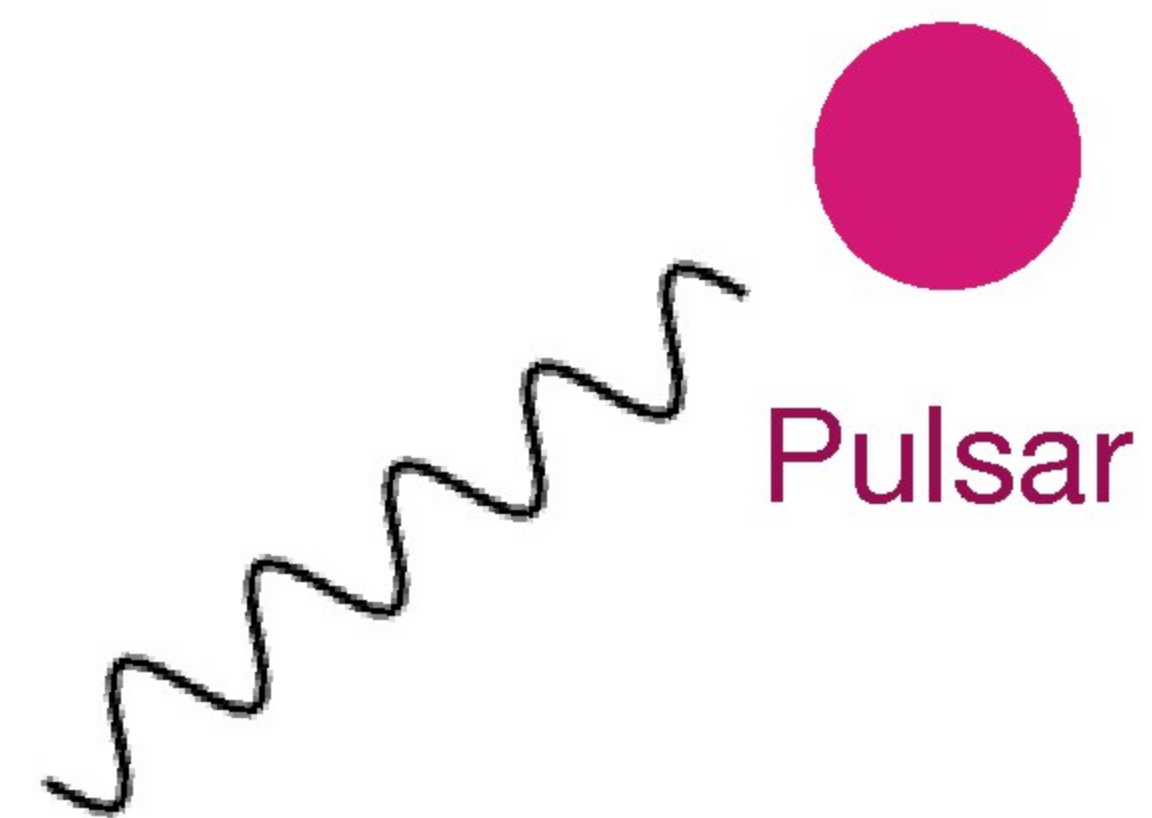
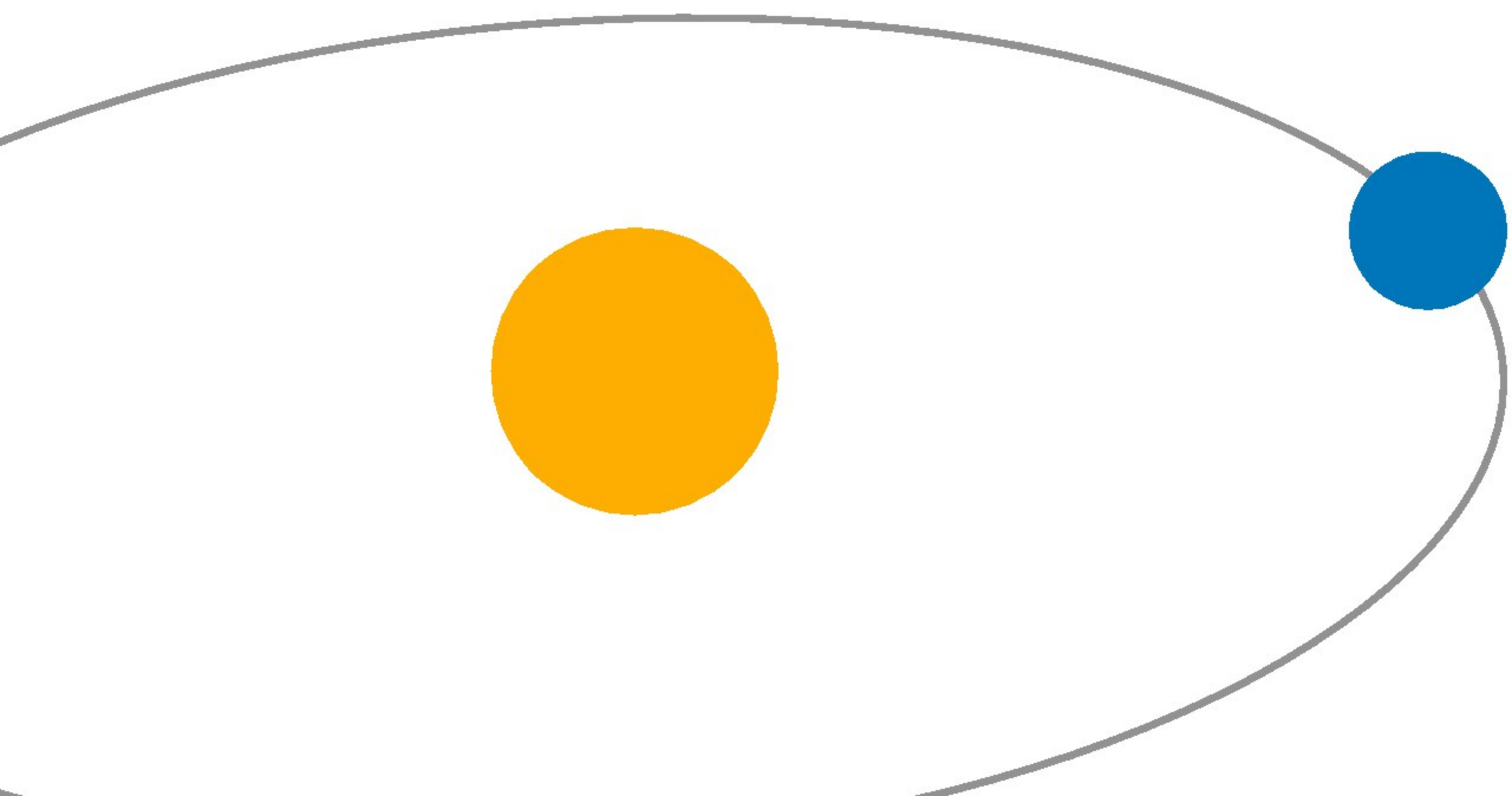
For vector dark matter
[Dror and Wei (25)]



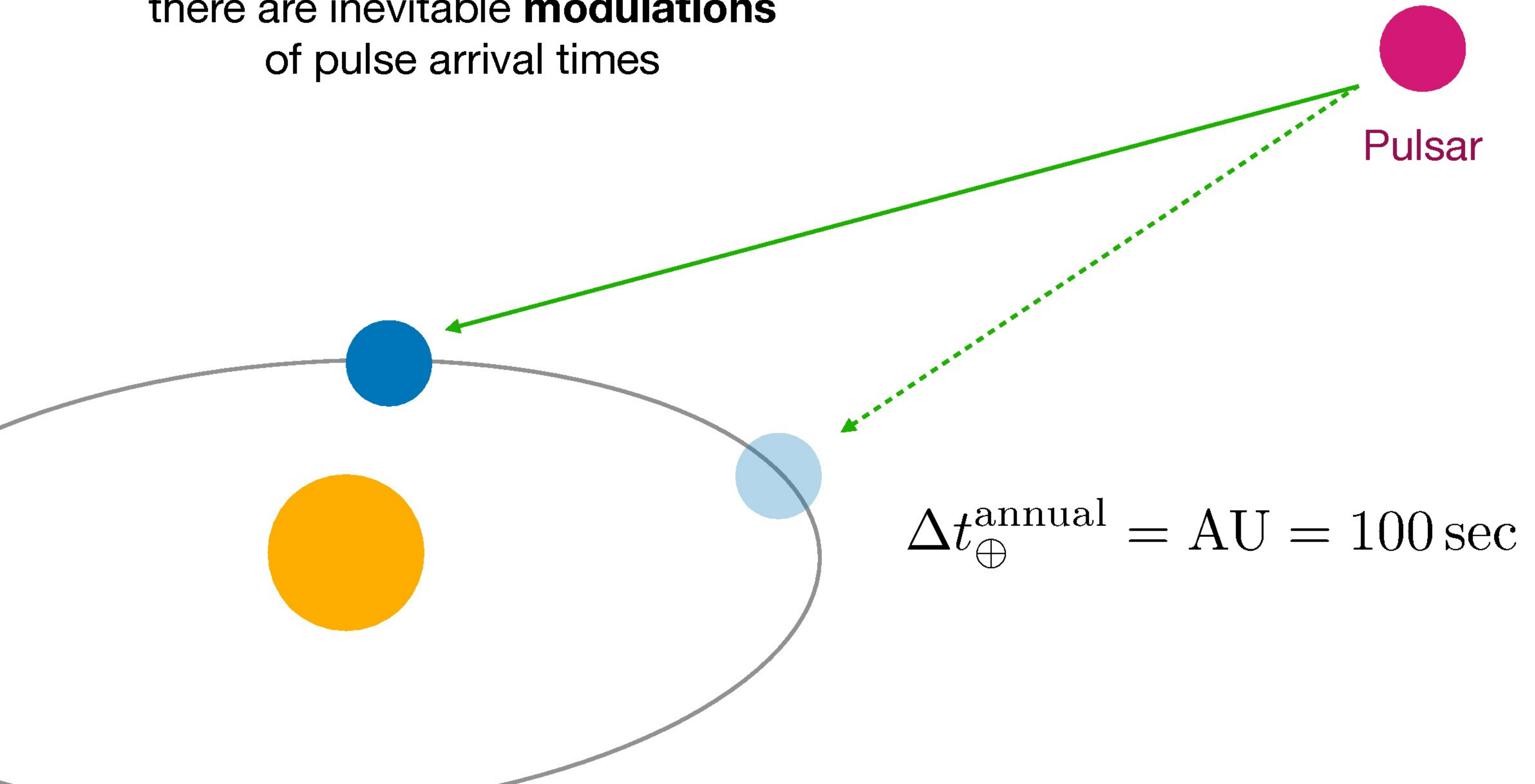
[Kim and Mitridate 2312.12225]

one last example:

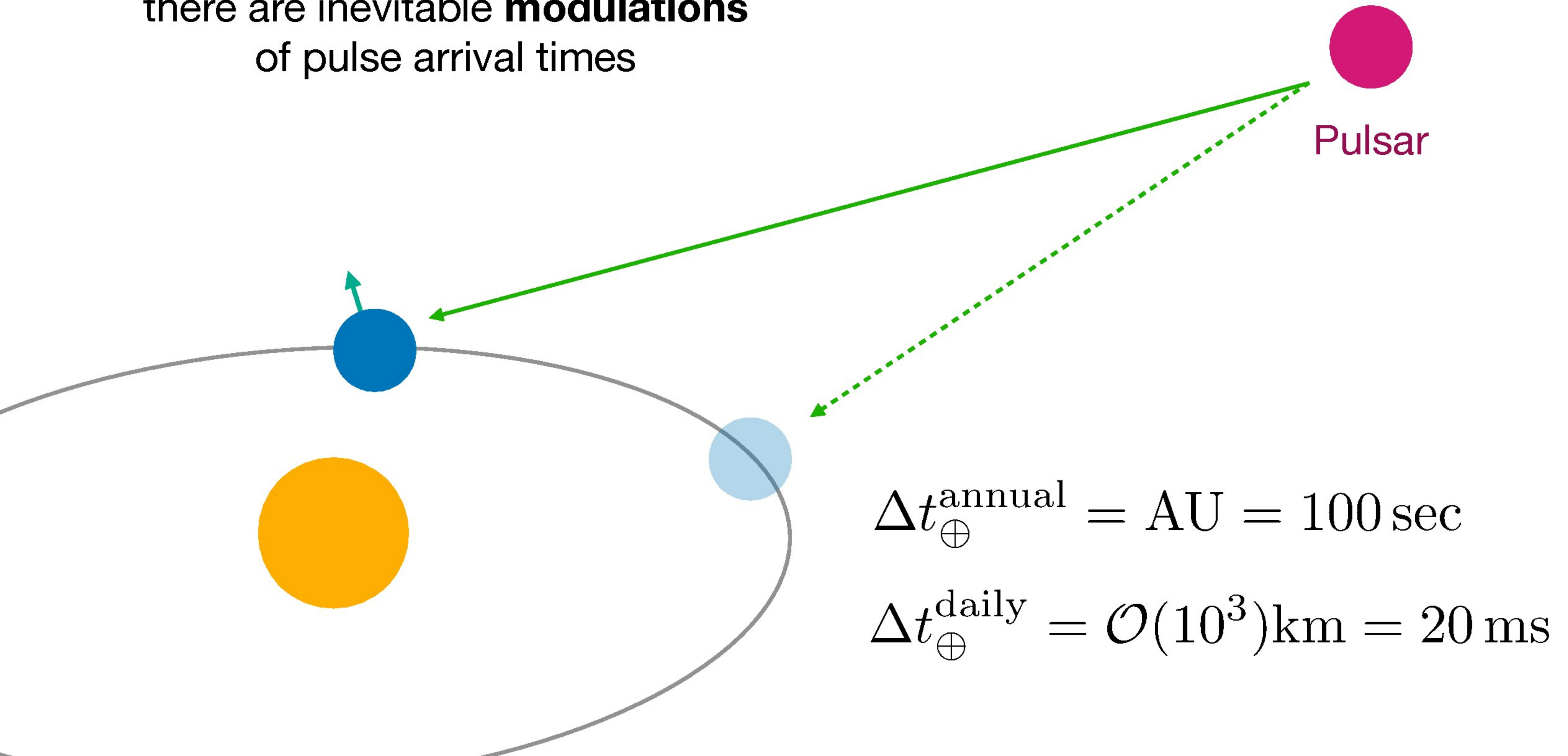
Solar system dynamics



due to the motion of Earth
there are inevitable **modulations**
of pulse arrival times



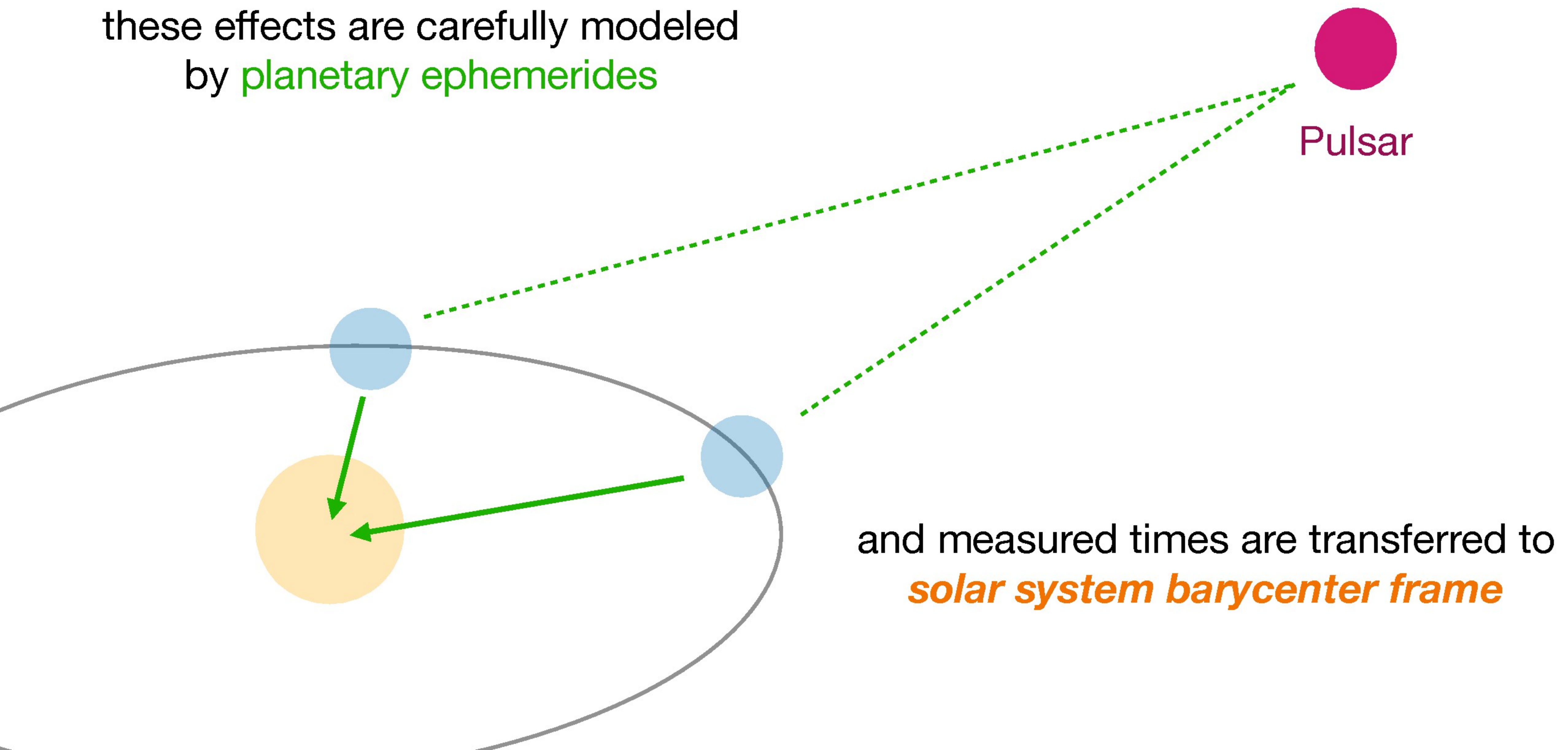
due to the motion of Earth
there are inevitable **modulations**
of pulse arrival times



$$\Delta t_{\oplus}^{\text{annual}} = \text{AU} = 100 \text{ sec}$$

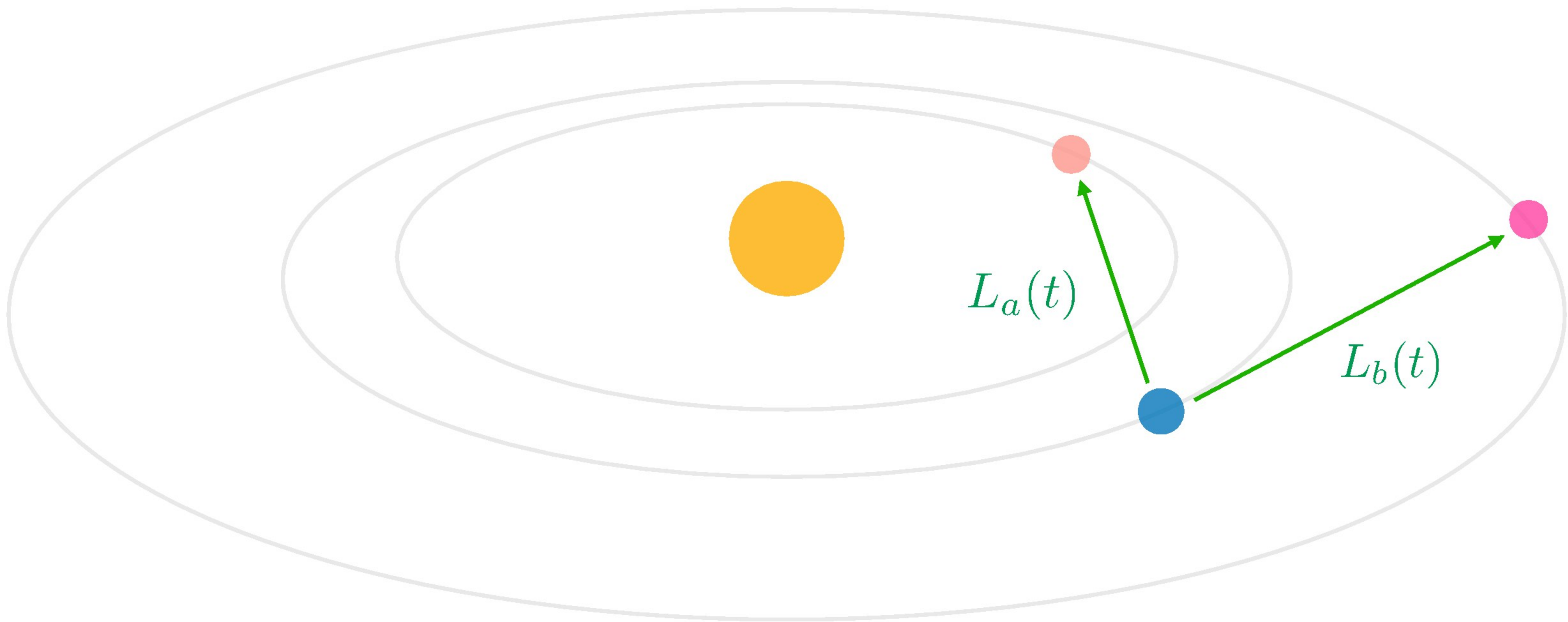
$$\Delta t_{\oplus}^{\text{daily}} = \mathcal{O}(10^3) \text{ km} = 20 \text{ ms}$$

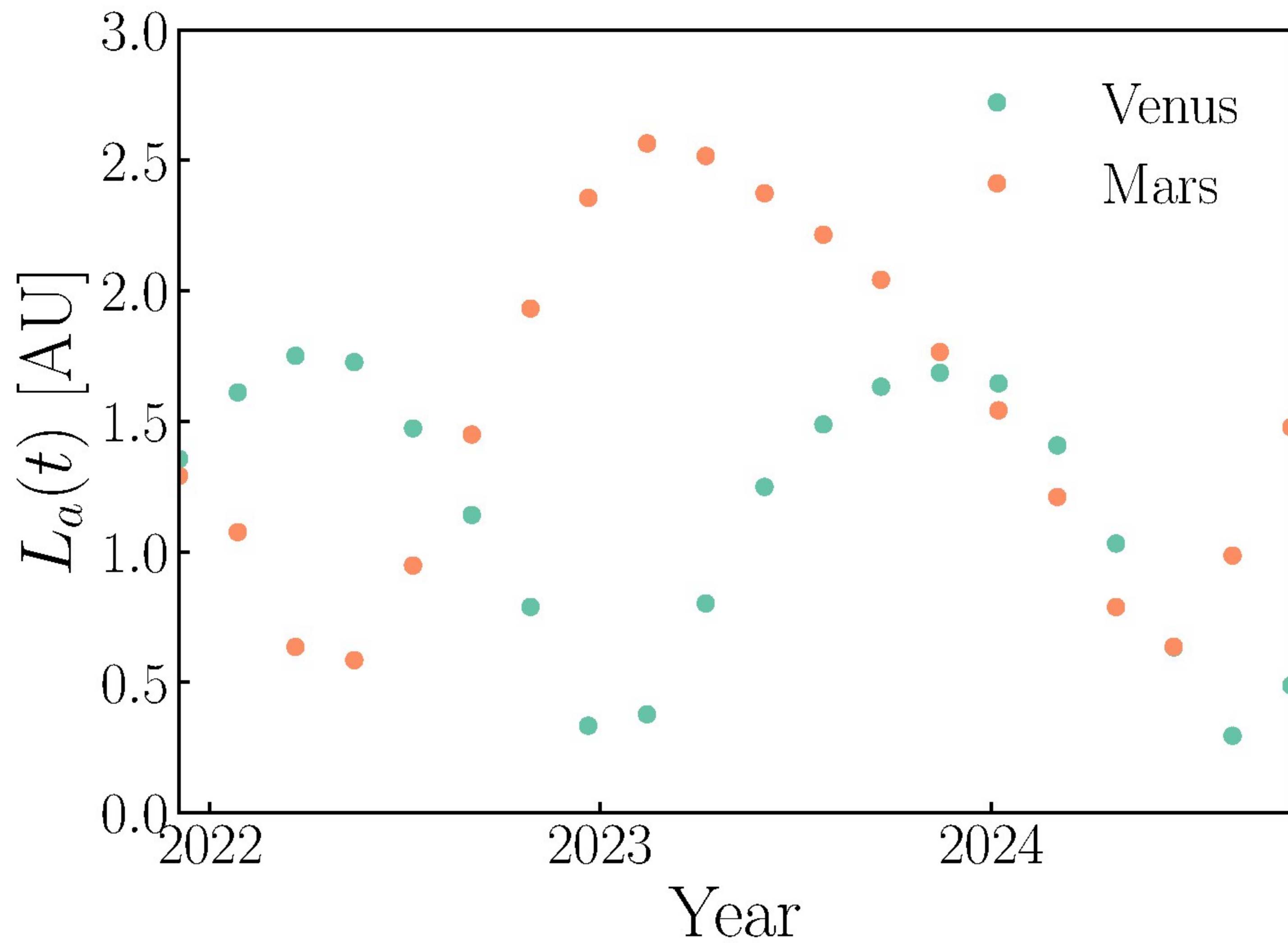
these effects are carefully modeled
by **planetary ephemerides**



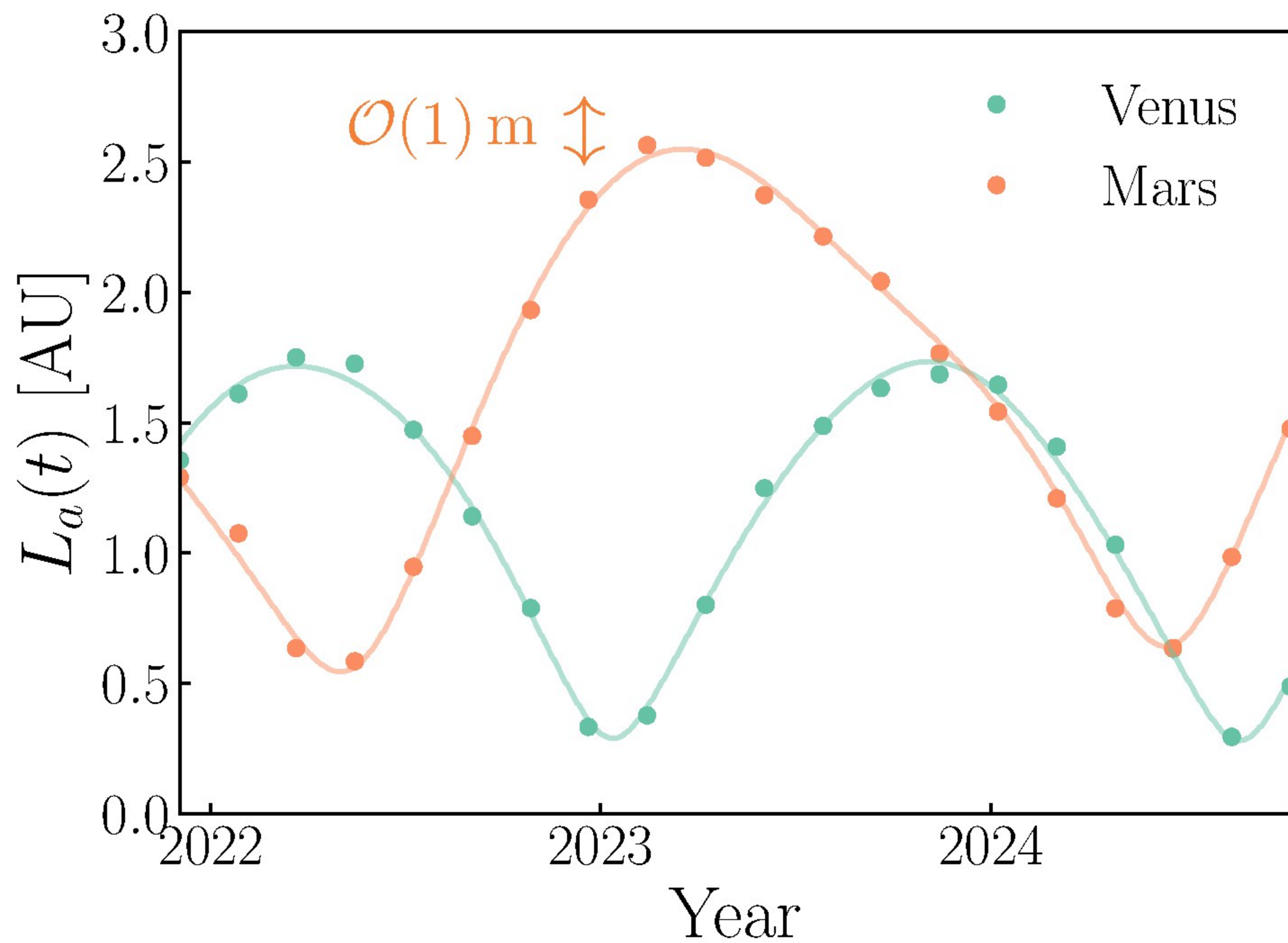
Pulsar

and measured times are transferred to
solar system barycenter frame



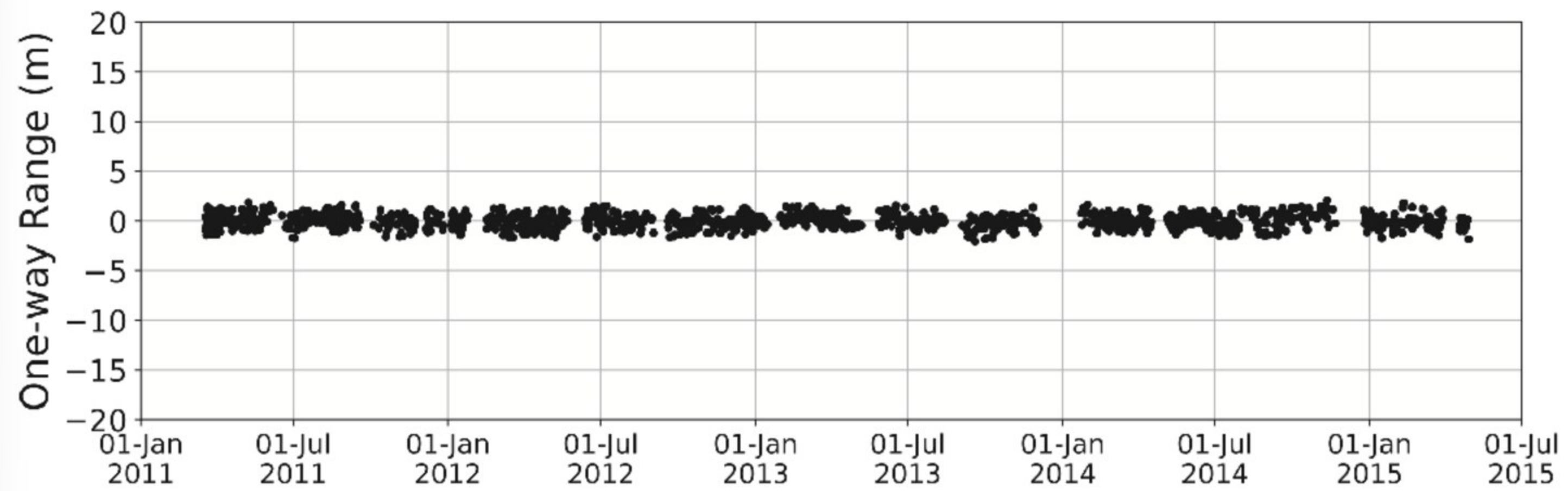


$$L_a(t) = L_a^{\text{model}}(t; \boldsymbol{\theta}_{\text{model}}) + n_a(t)$$

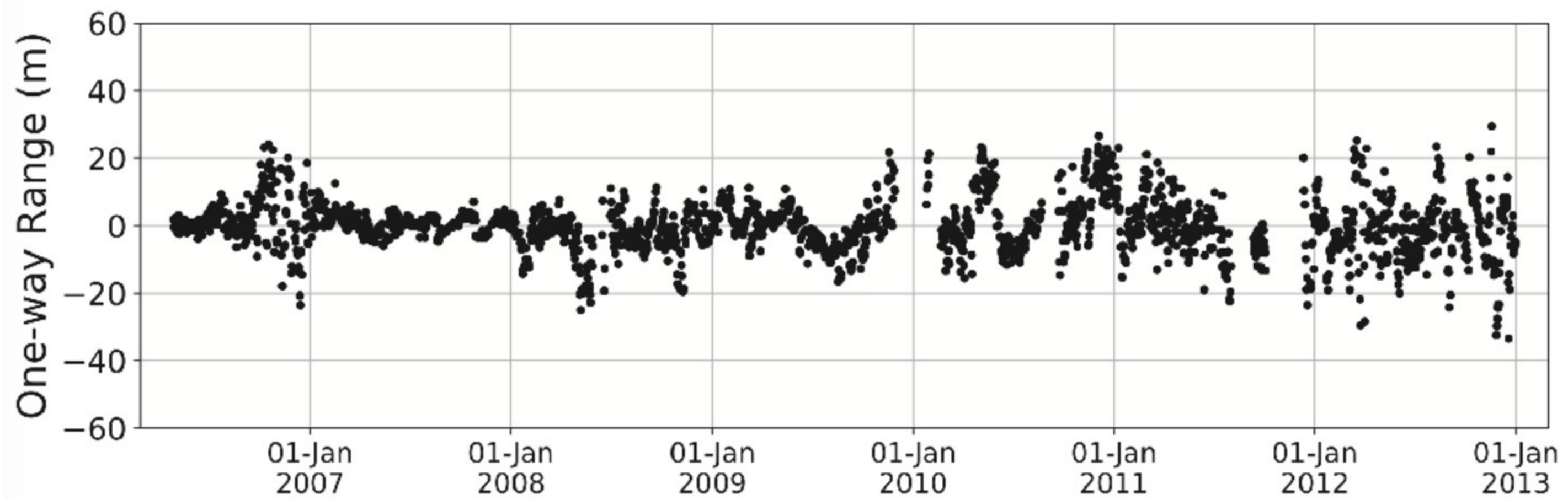


$$\Delta L_a(t) = L_a(t) - L_a^{\text{model}}(t; \boldsymbol{\theta}_{\text{fit}})$$

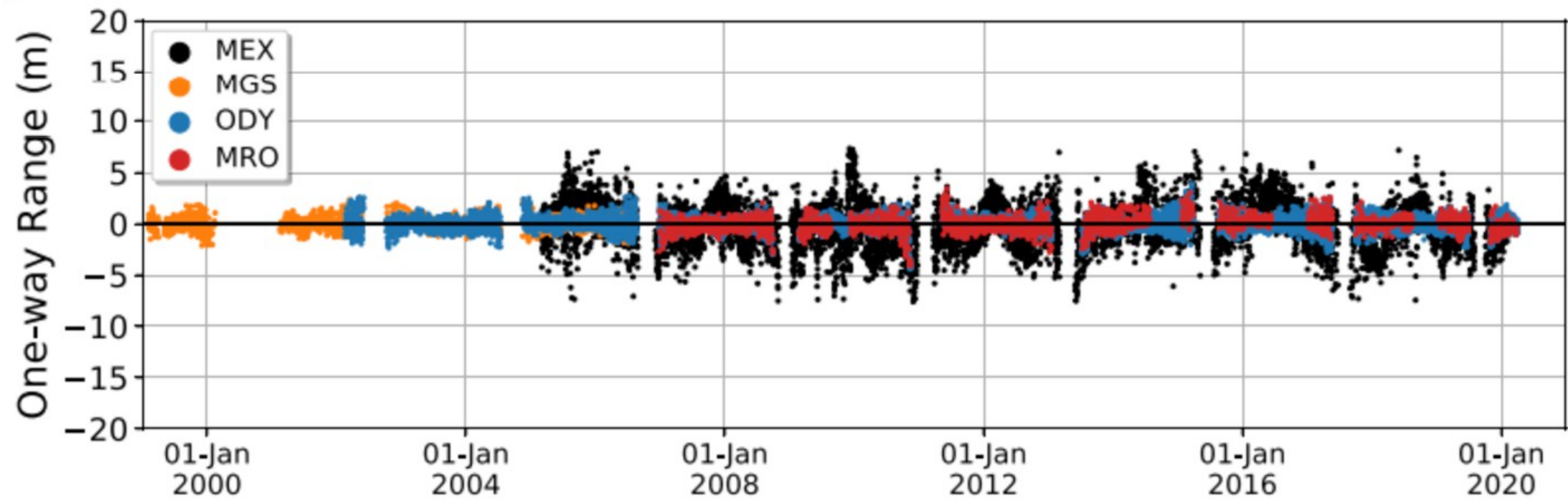
Mercury

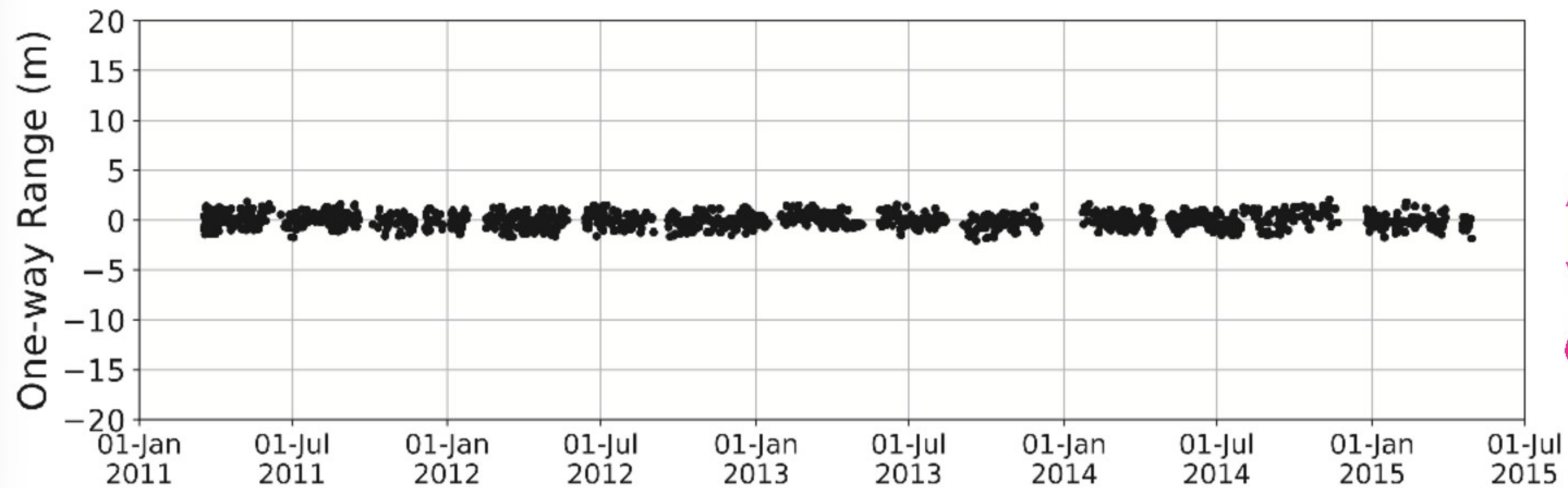


Venus



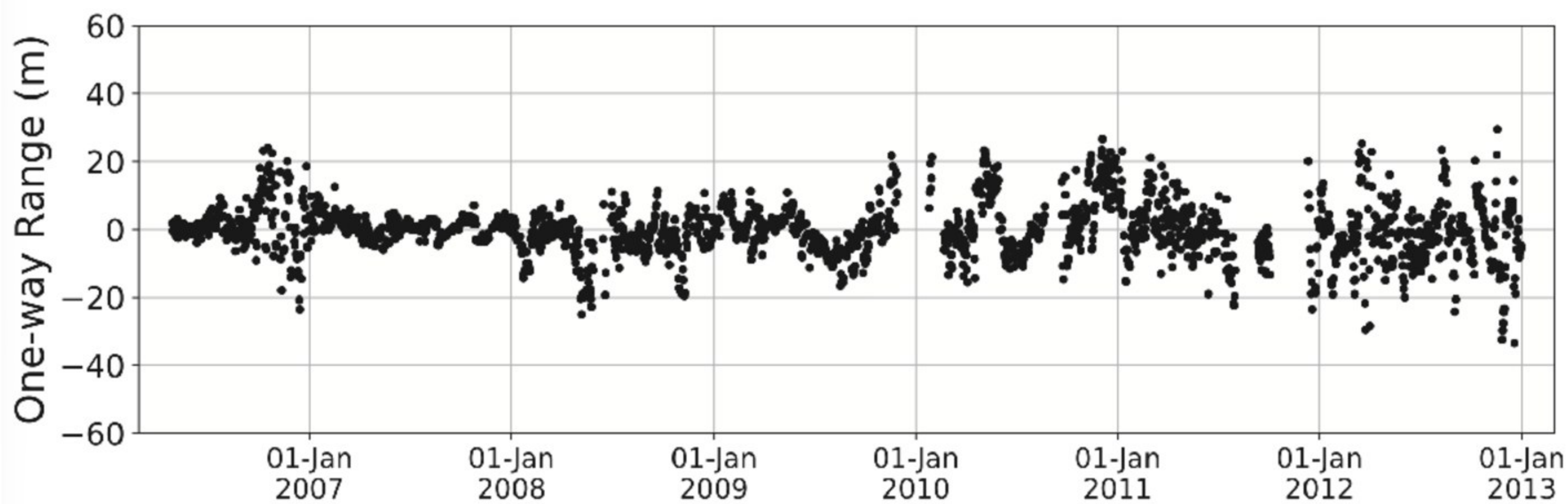
Mars



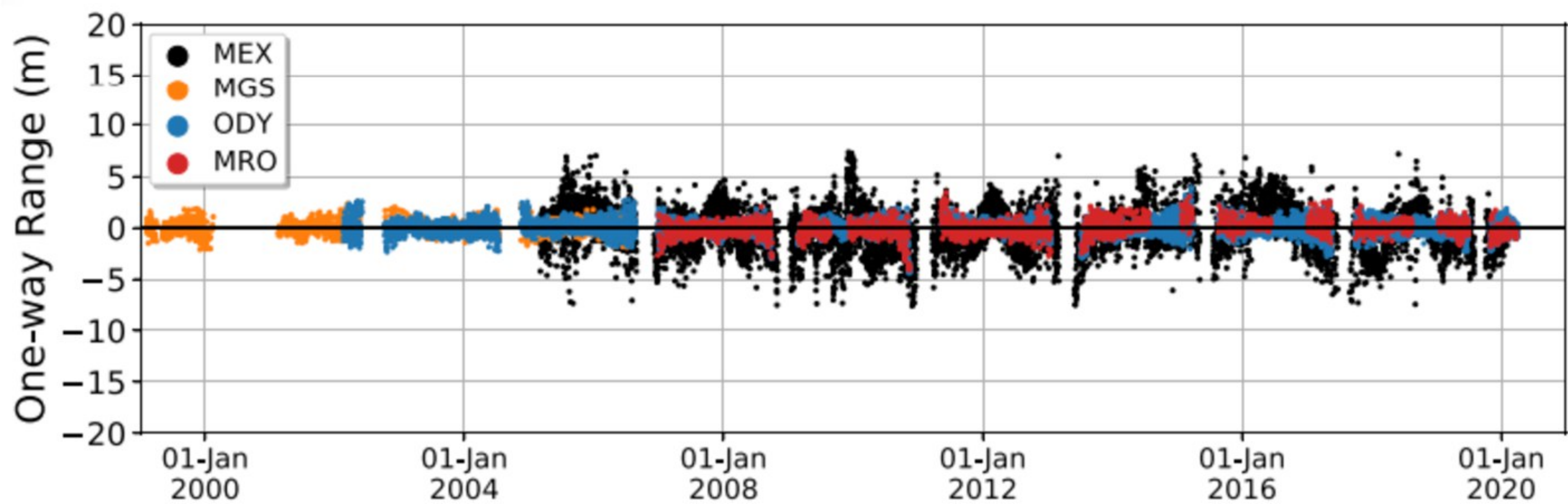


Mercury

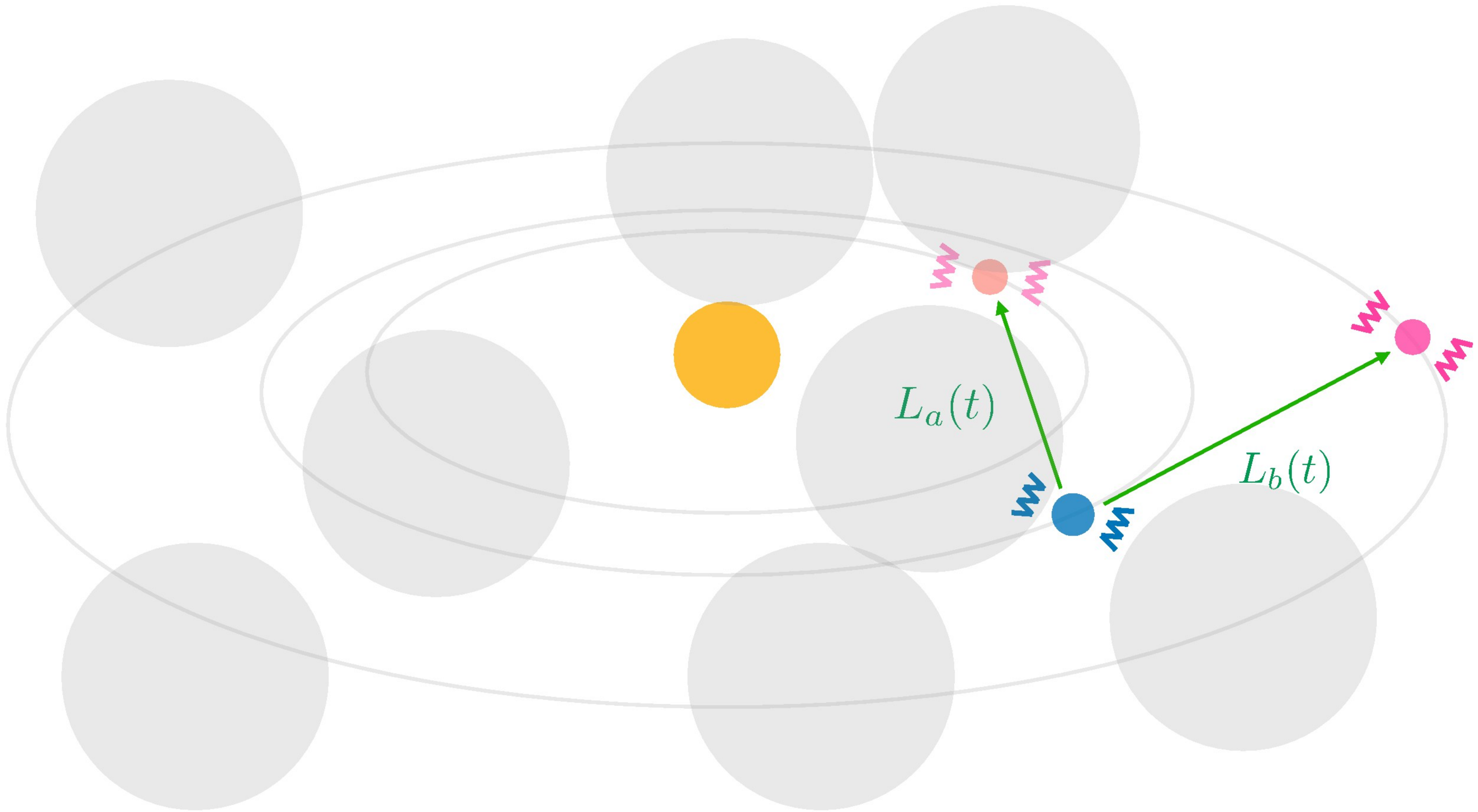
$O(1)$ m



Venus



Mars



signal is similarly characterized by
two-point function

$$\left\langle \widetilde{\Delta L}_a(f) \widetilde{\Delta L}_b(f') \right\rangle = \frac{1}{2} \delta(f - f') S_{aa}(f) \Gamma_{ab}(f, \mathbf{L}_a, \mathbf{L}_b)$$

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$$\left\langle \widetilde{\Delta L}_a(f) \widetilde{\Delta L}_b(f') \right\rangle = \frac{1}{2} \delta(f - f') S_{aa}(f) \Gamma_{ab}(f, \mathbf{L}_a, \mathbf{L}_b)$$

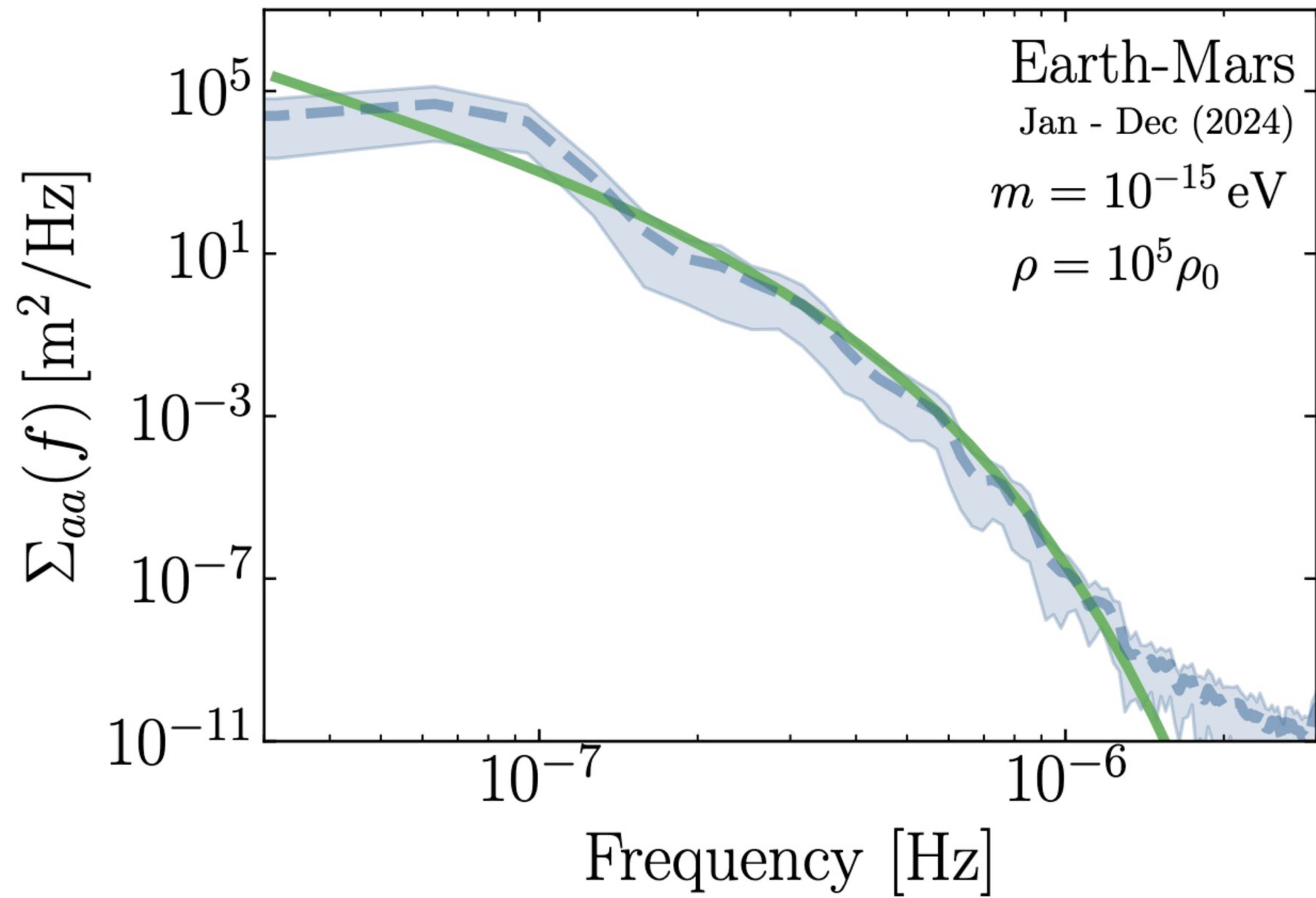
signal is similarly characterized by
two-point function

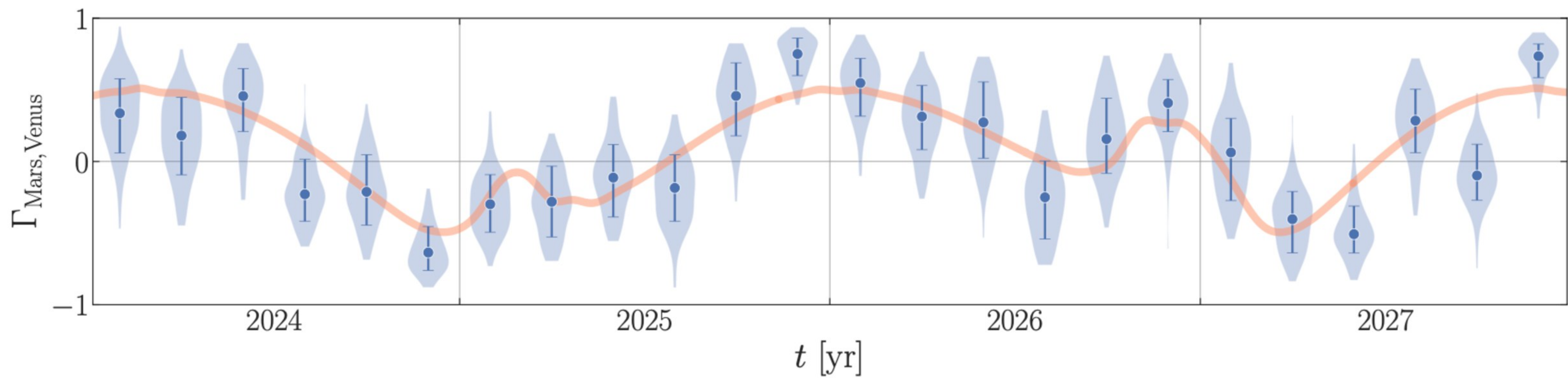
$$\left\langle \widetilde{\Delta L}_a(f) \widetilde{\Delta L}_b(f') \right\rangle = \frac{1}{2} \delta(f - f') S_{aa}(f) \Gamma_{ab}(f, \mathbf{L}_a, \mathbf{L}_b)$$

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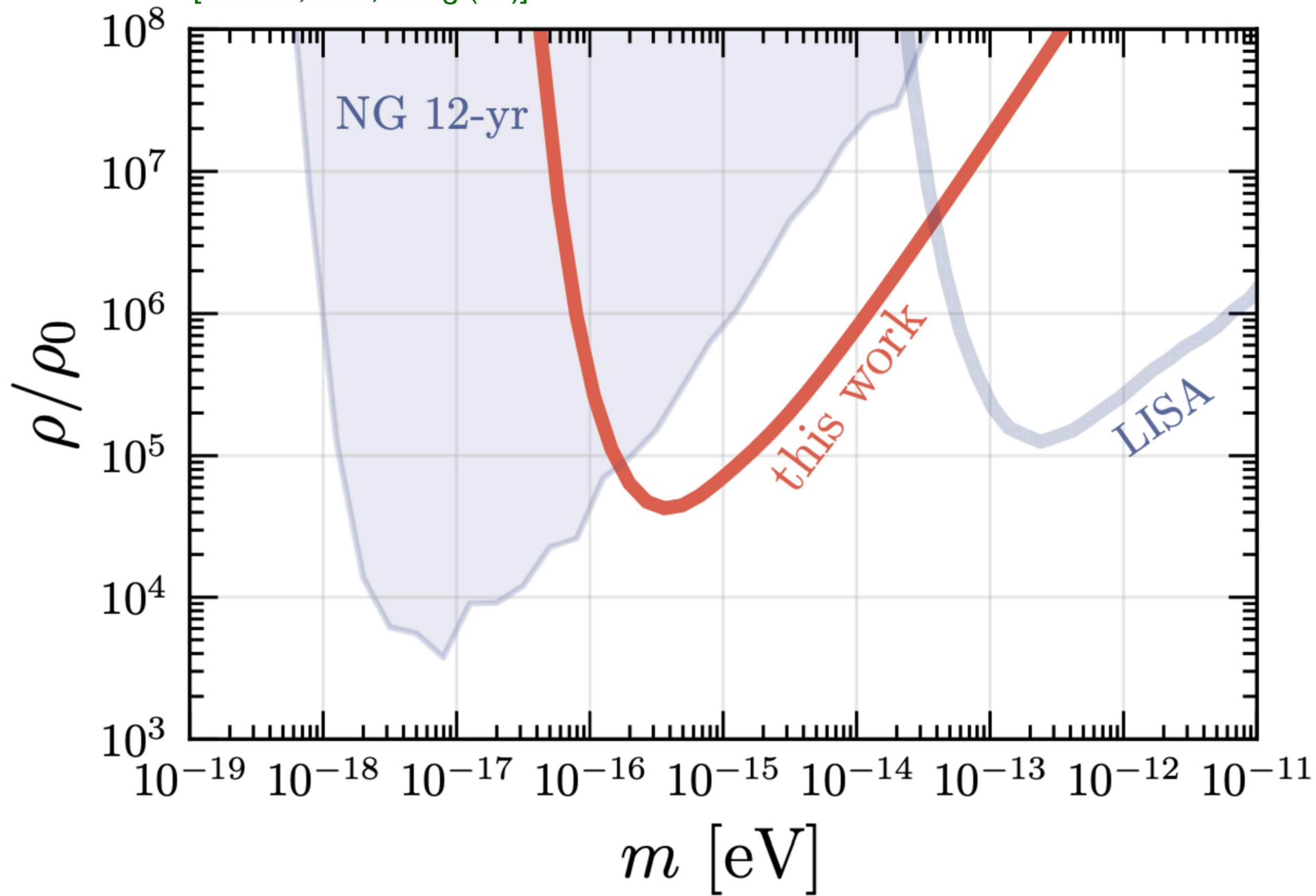
$$\left\langle \widetilde{\Delta L}_a(f) \widetilde{\Delta L}_b(f') \right\rangle = \frac{1}{2} \delta(f - f') S_{aa}(f) \Gamma_{ab}(f, \mathbf{L}_a, \mathbf{L}_b)$$

these quantities are now **time-dependent**





[Frerick, Kim, Kling (26)]



For binary systems
[Foster et al (25)]

A remark

all of the results shown here are sensitive to
ULDM density around/within the solar system

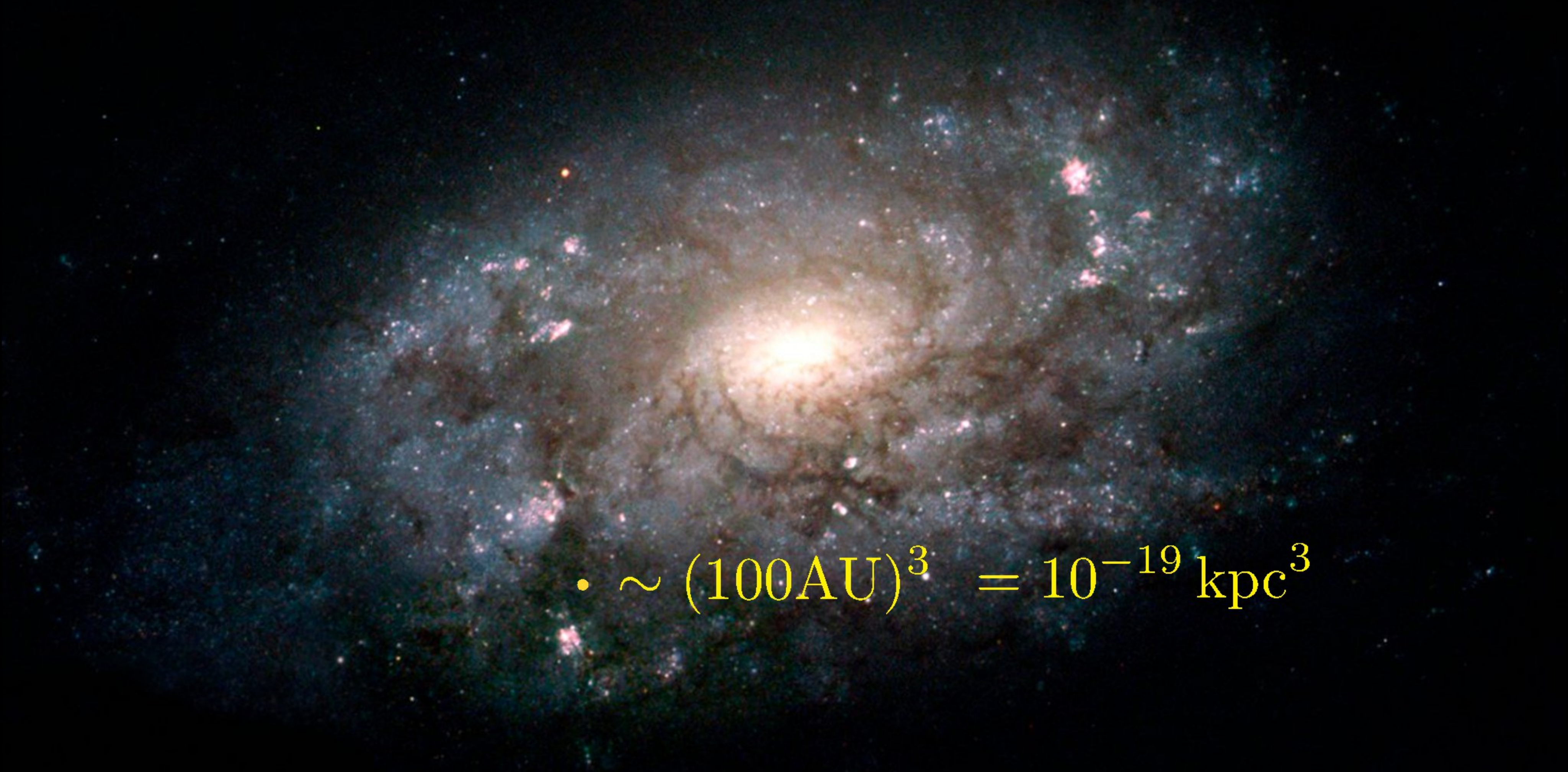
local dark matter density is often derived over kpc scales



$$\rho_0 = 0.4 \text{ GeV}/\text{cm}^3$$

is an ***average density over the volume of kpc***

what we are probing is
(or what matters for all terrestrial DM detector is)



• $\sim (100\text{AU})^3 = 10^{-19} \text{ kpc}^3$

currently no measurement on this scale exists

only constraints exist

only constraints exist

$$\rho/\rho_0 \lesssim 10^{11}$$

From geodetic satellite and LLR
[Adler (08)]

only constraints exist

$$\rho/\rho_0 \lesssim 10^{11}$$

From geodetic satellite and LLR
[Adler (08)]

$$\rho/\rho_0 \lesssim 6 \times 10^6$$

From asteroids in the solar system
[Tsai, Eby et al (22)]

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From asteroids in the solar system
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$$\rho/\rho_0 \lesssim 2 \times 10^4$$

From solar system ephemerides
[Pitjev, Pitjeva (13)]

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From asteroids in the solar system
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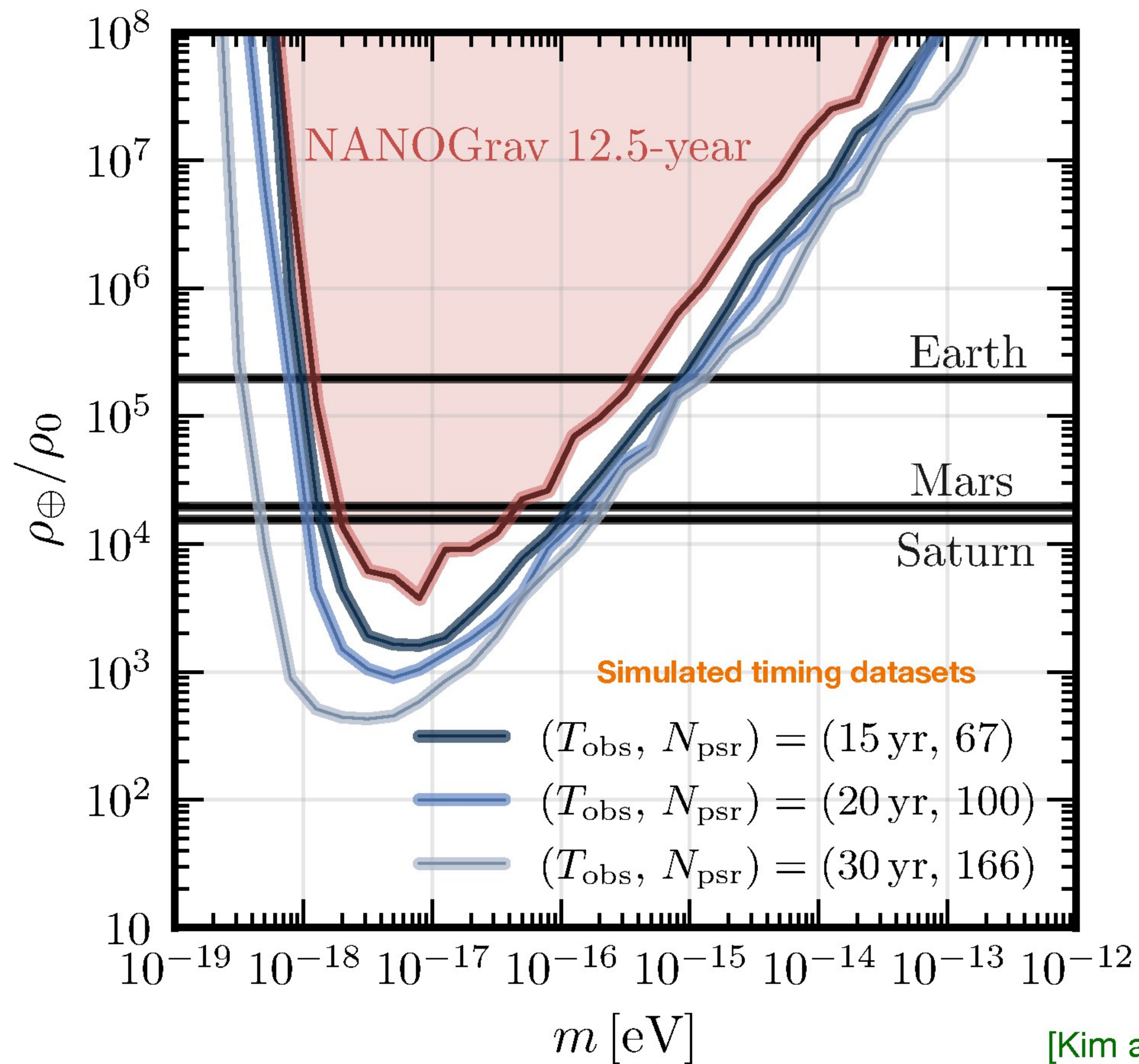
From solar system ephemerides
[Pitjev, Pitjeva (13)]

GW detectors will provide
one of the strongest probes of ULDM density
within/around the solar system

Another remark

prospects of gravitational probes

For vector dark matter
[Dror and Wei (25)]



[Kim and Mitridate 2312.12225]

Next International Pulsar Timing Array (IPTA) data release
is expected to include

$$N_{\text{psr}} \sim 100$$

$$T_{\text{obs}} = 20^{+} \text{ yr}$$

with next-gen radio telescope (e.g. Square Kilometer Array)
an order of magnitude or more improvement might be feasible

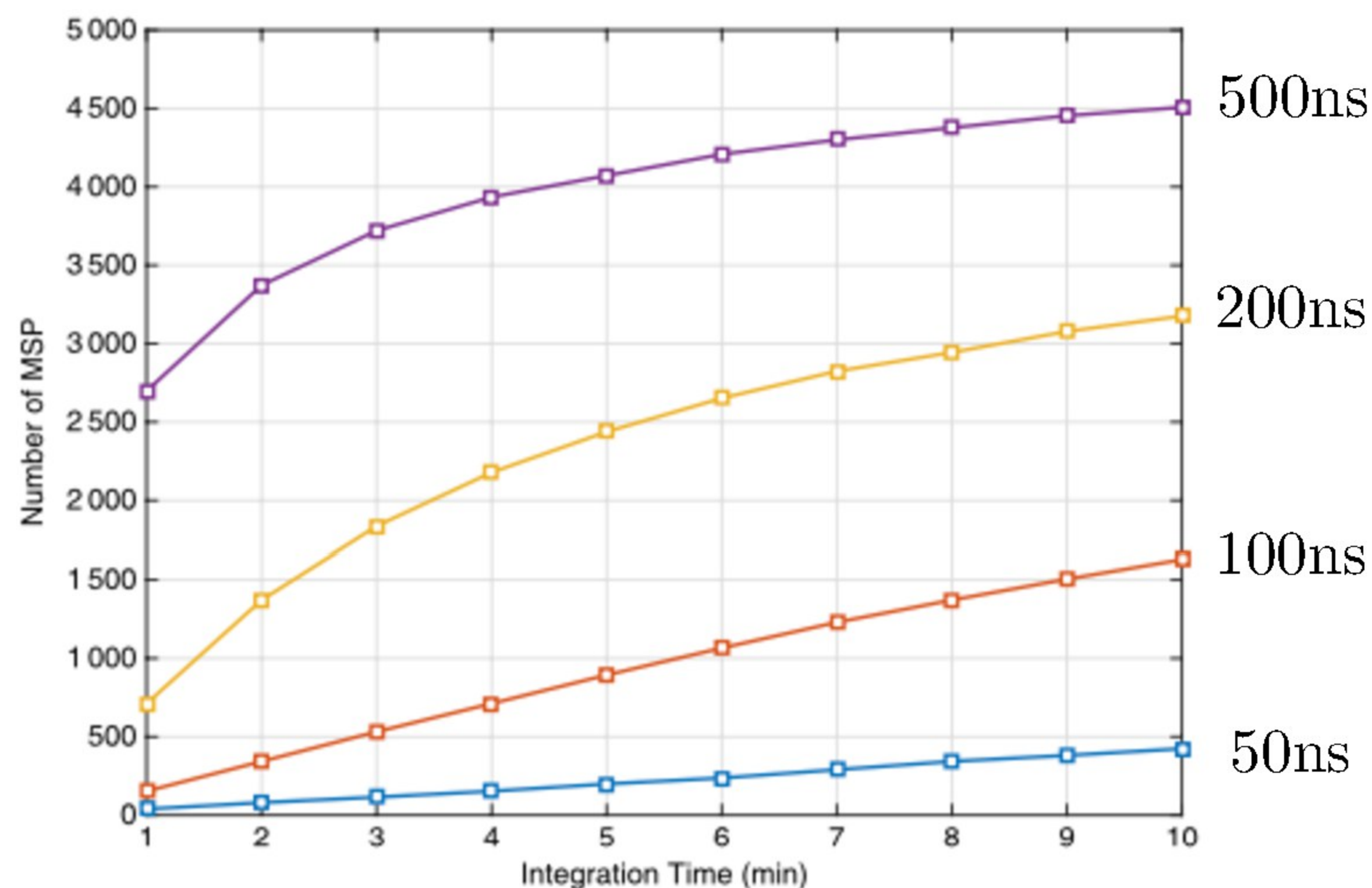


Figure 10. Numbers of MSPs that can archive a certain RMS noise level (or better) with varying integration time. Colour lines indicate different RMS noise levels (from bottom to top): 50 ns (blue), 100 ns (red), 200 ns (yellow), and 500 ns (purple).

BepiColombo: Exploring Mercury

The European Space Agency's mission to Mercury will travel 8,524,000,000 km in seven years to reach the planet closest to the Sun. Upon arrival, the European and Japanese spacecraft will separate and each enter their own orbits around the planet.

The Mercury Transfer Module (MTM), carries the interplanetary propulsion system. It is jettisoned on arrival.

Mercury

MPO Orbit: 480 – 1,500 km

MPO

The Mercury Planetary Orbiter (MPO) will focus on the surface and interior science of Mercury with instruments that will point continuously at the planet.

As prime contractor for ESA, Airbus is responsible for designing and building all of the European spacecraft hardware.

The Heat shield (MOSIF) protects the Japanese orbiter MMO during the long journey. It is jettisoned at Mercury.

The Mercury Magnetospheric Orbiter (MMO) is the only part of the mission supplied by the Japanese space agency JAXA. It will focus on exploring Mercury's magnetic environment.

MMO Orbit: 590 – 11,640 km

MMO

AIRBUS