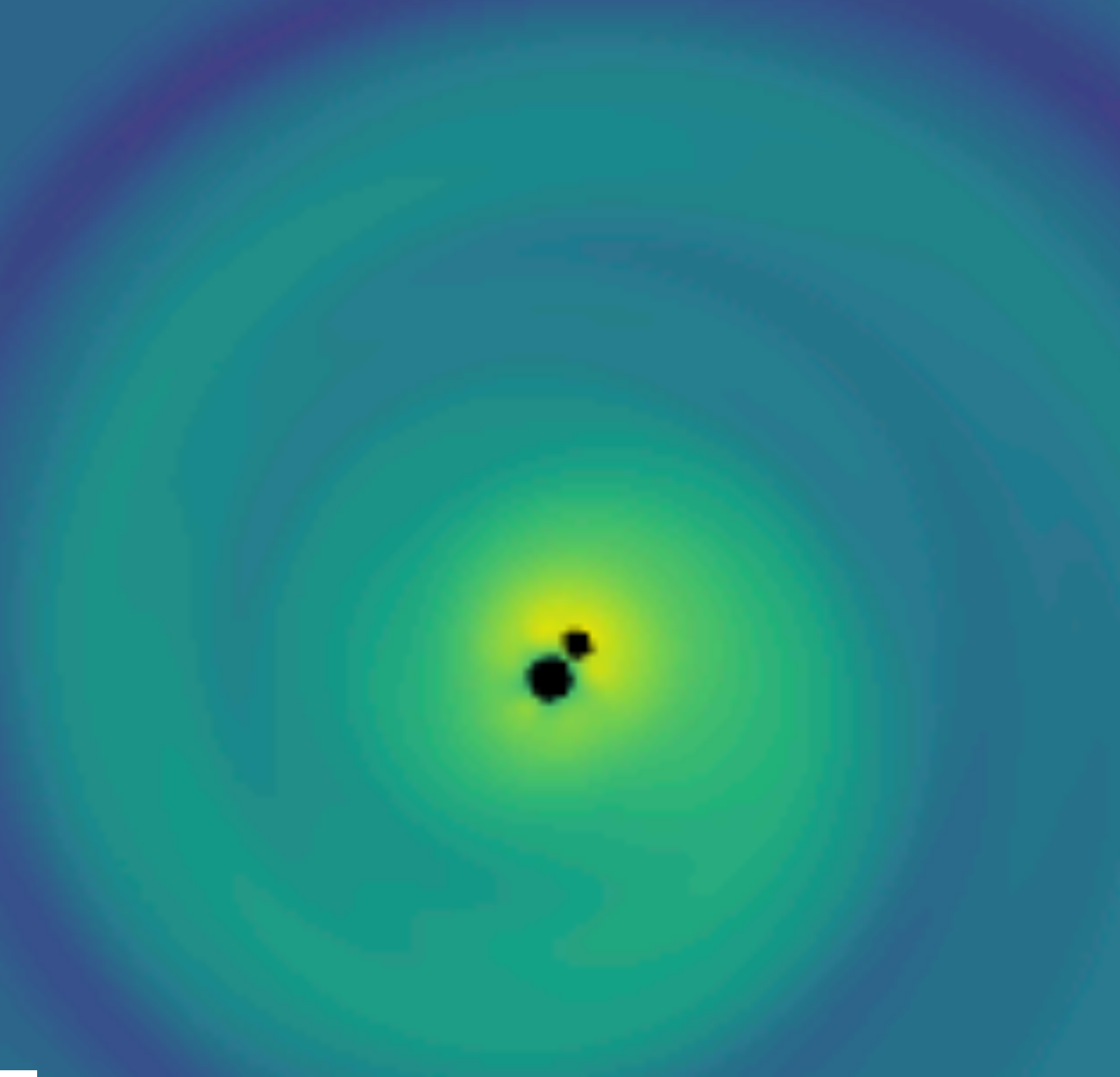


# Extreme-mass-ratio inspirals in scalar cloud environments

Richard Brito, CENTRA, Instituto Superior Técnico, Lisboa



**TÉCNICO**  
UNIVERSIDADE  
DE LISBOA



**centra**

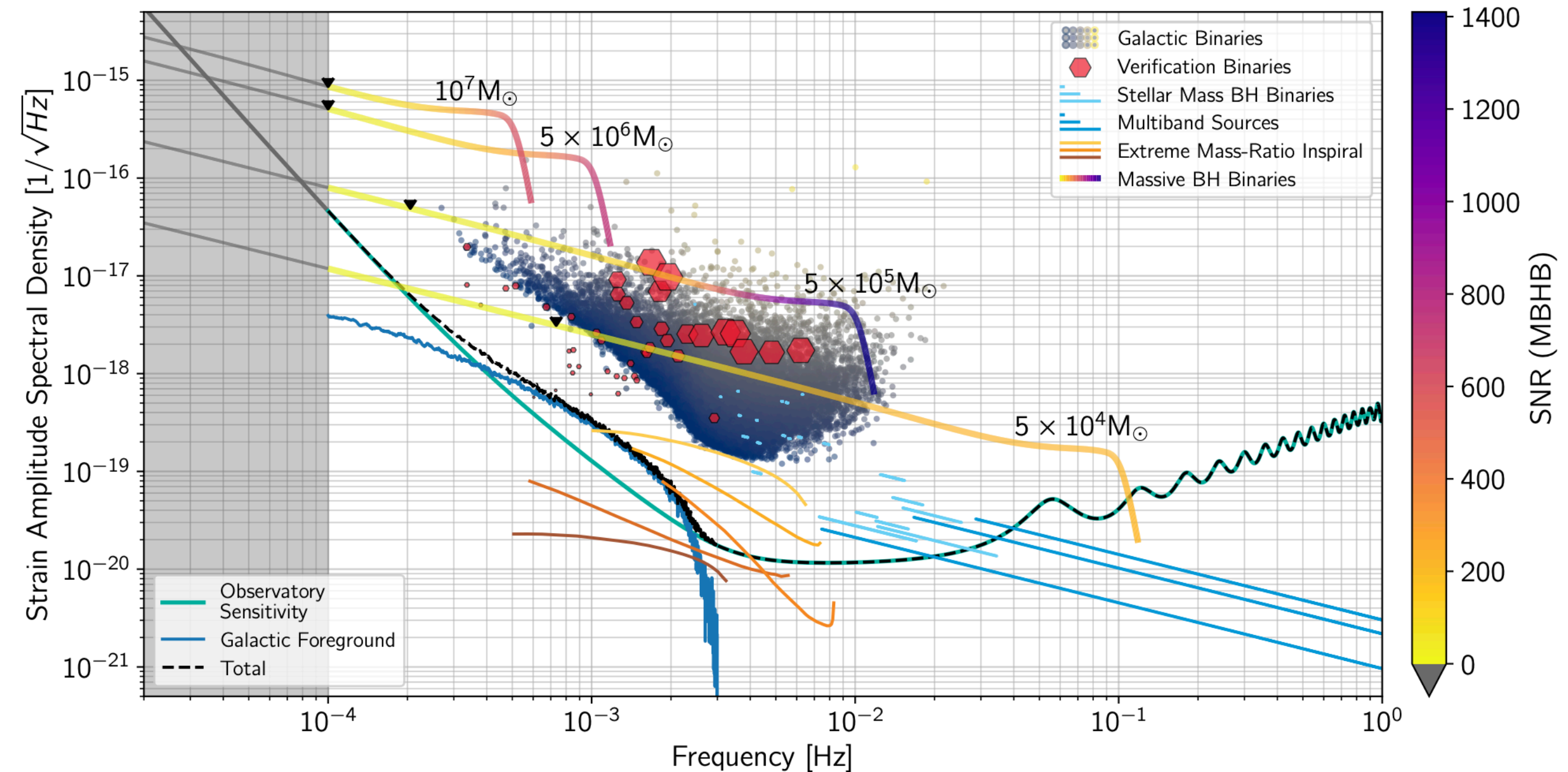
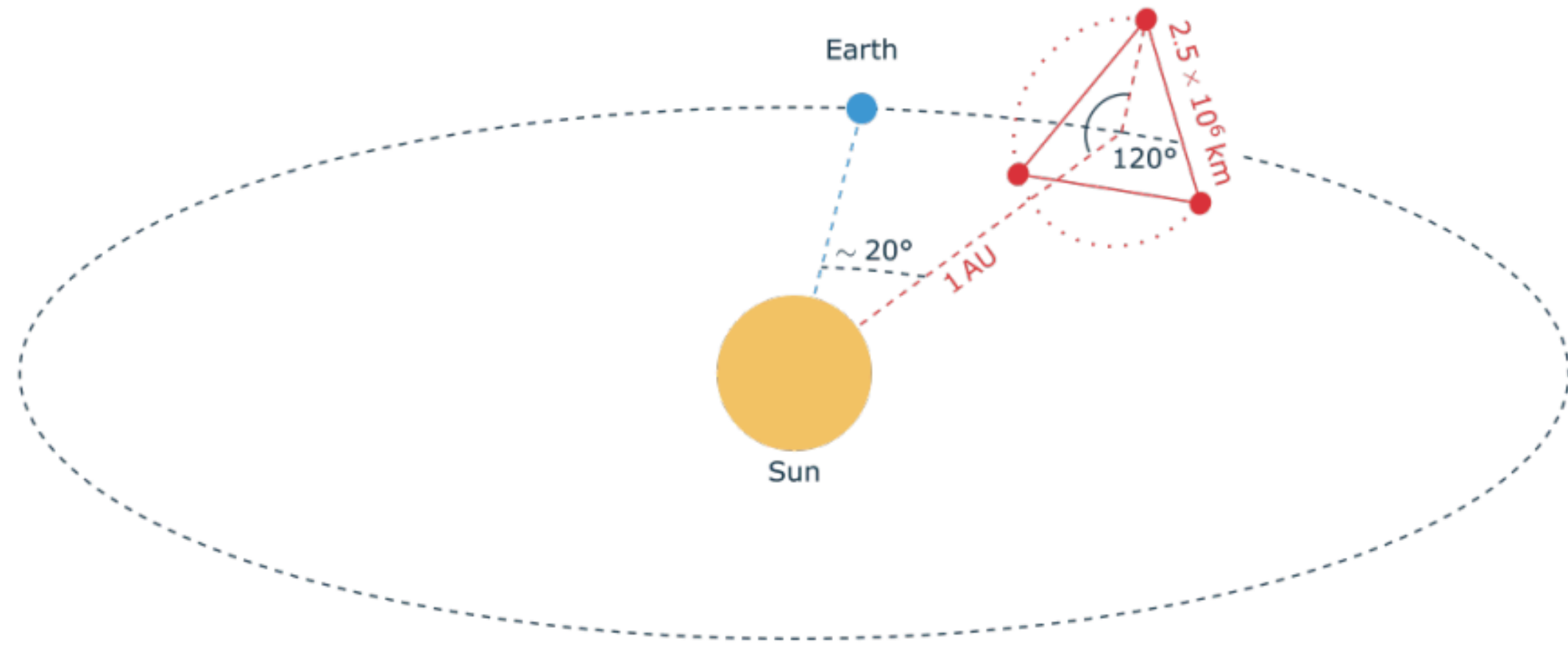
**fct**

Fundação  
para a Ciência  
e a Tecnologia

with C. Dyson, R. Della Monica, K. Destounis, S. Dolan, R. Keijzer,  
T. May, A. Pound, S. Shah, T. Spiexsma, S. Upton, M. van de Meent,  
R. Vicente, C. Yuan, Q.-X. Xu

# Space-based GW detectors & EMRIs

**LISA, Taiji, TianQin:** gravitational-wave (GW) detectors in the mHz band

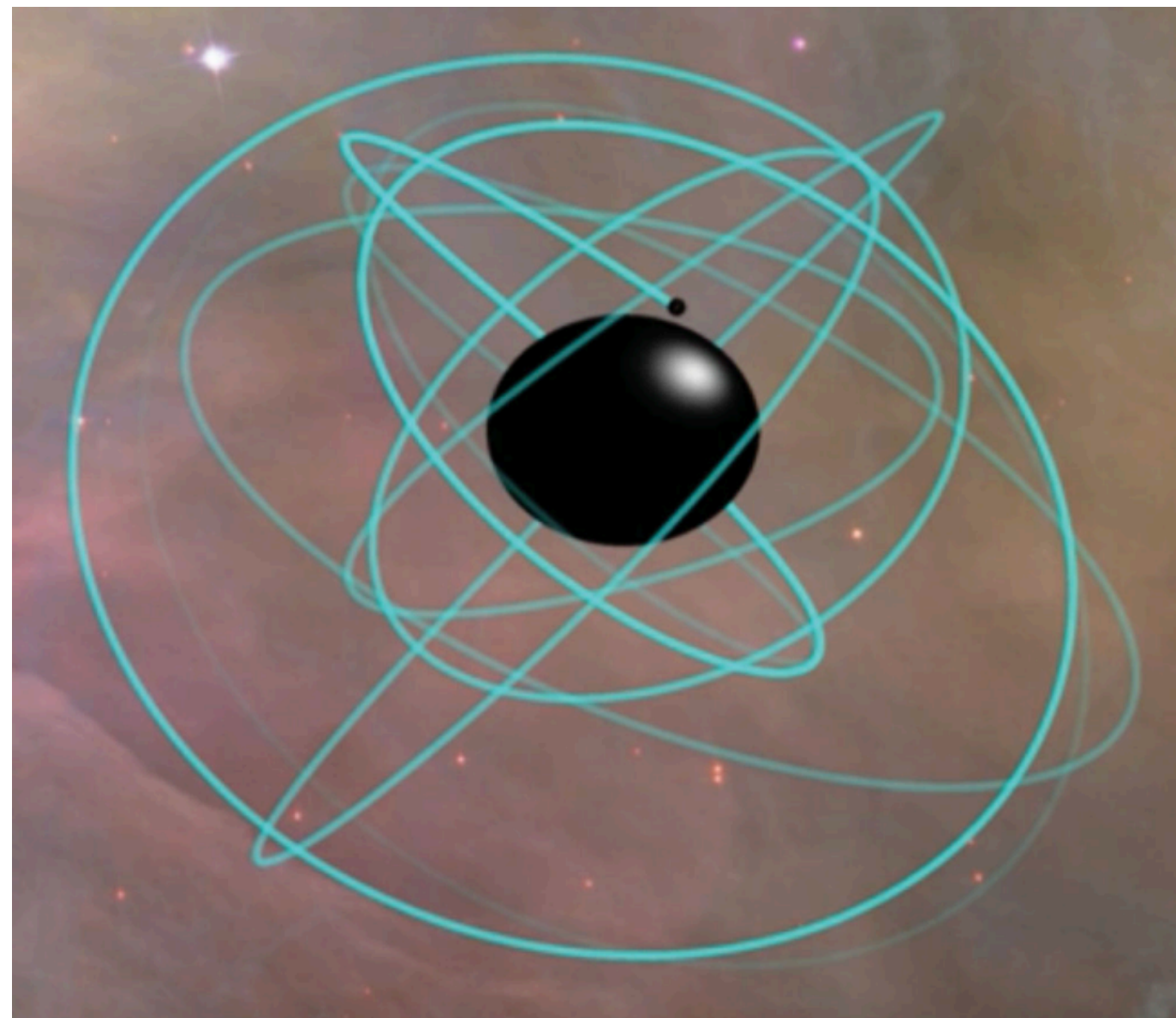


From: LISA Red Book, arXiv: 2402.07571

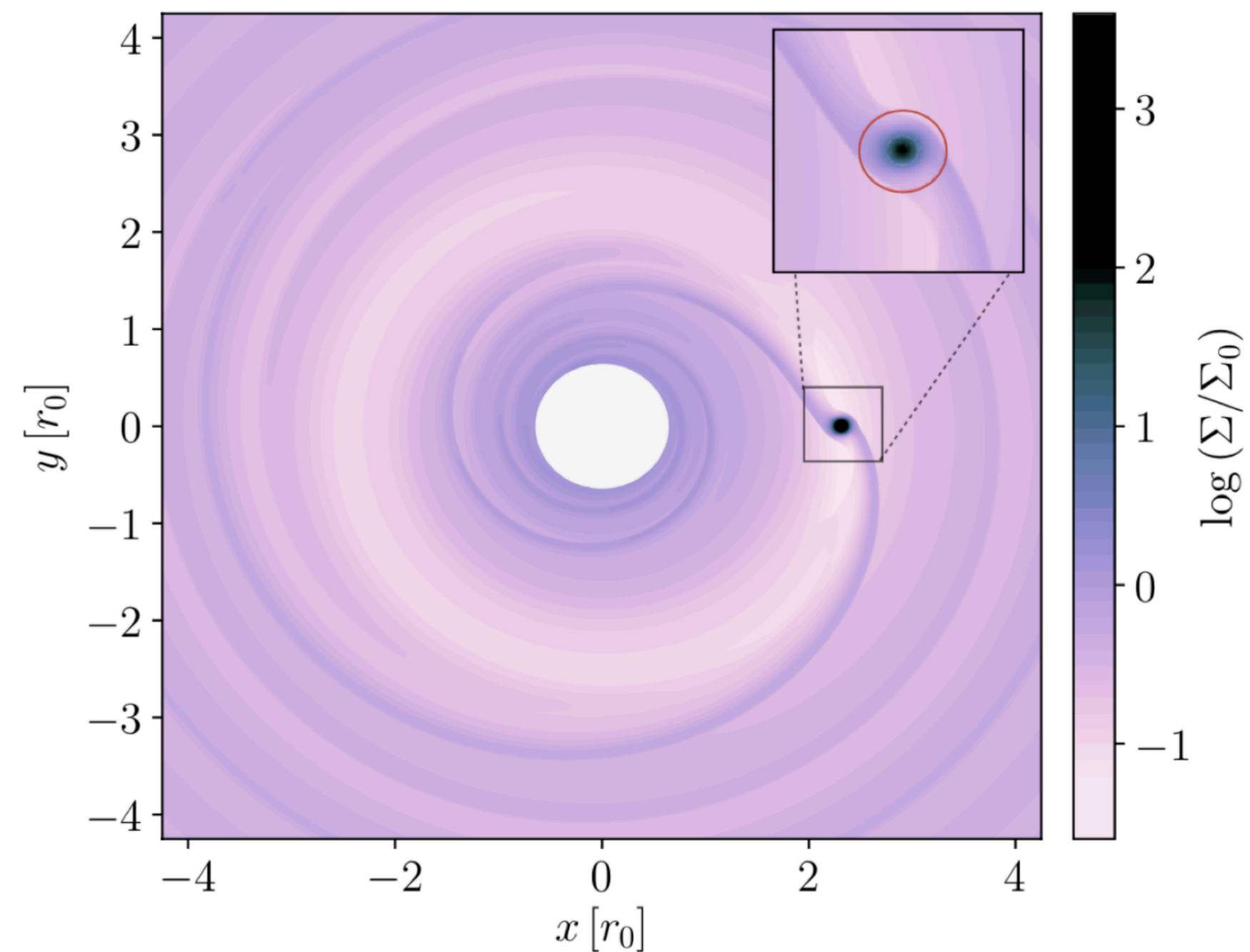
**Extreme mass-ratio inspirals (EMRIs):**  
binaries with very asymmetric masses

$$m_1 \sim 10^6 M_\odot, m_2 \sim 10 M_\odot$$

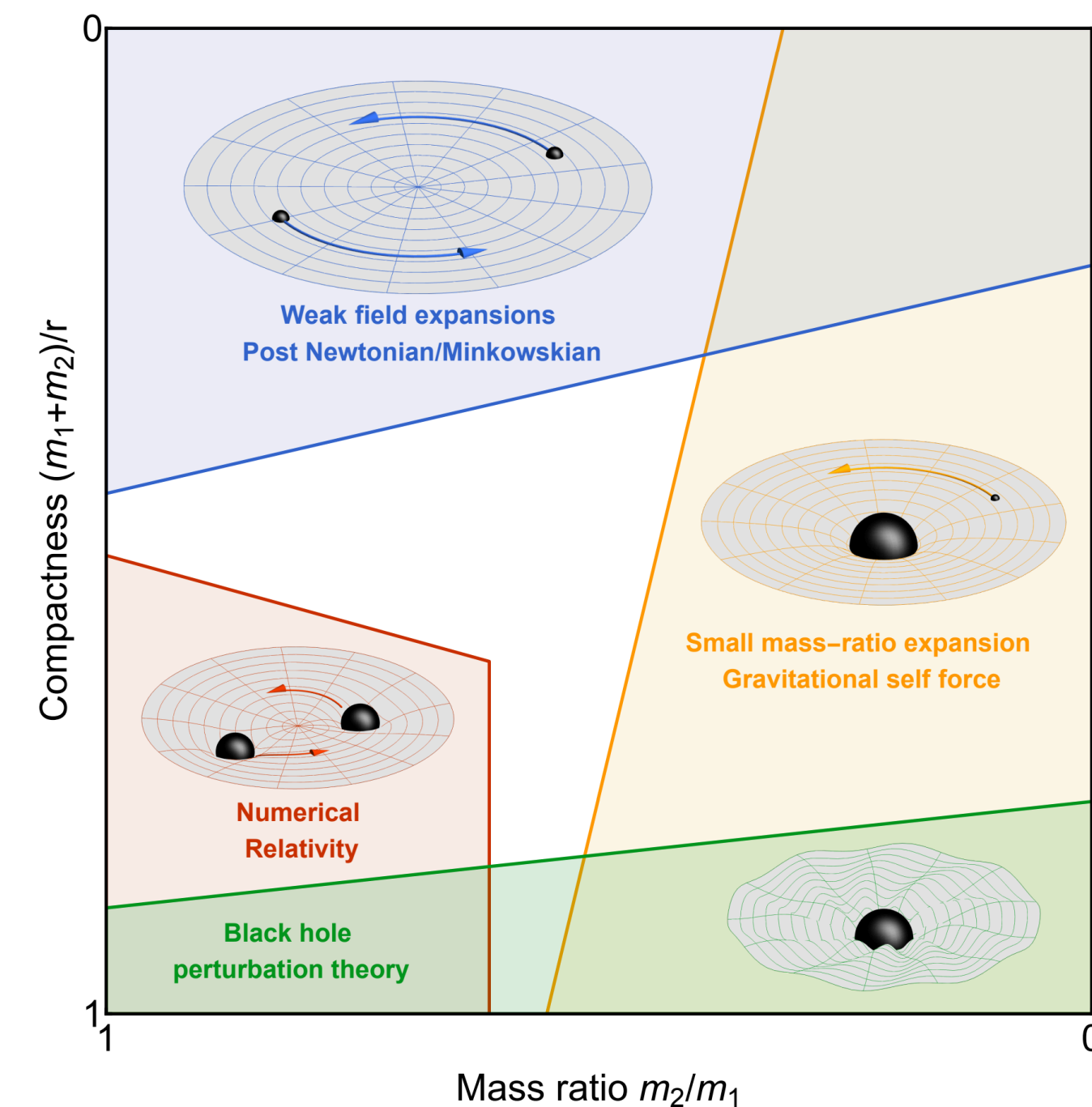
# EMRIs as precision probes



Credit: LISA WavWG,  
2311.01300



Credit: Derdzinski+  
MNRAS501, 3540 '21

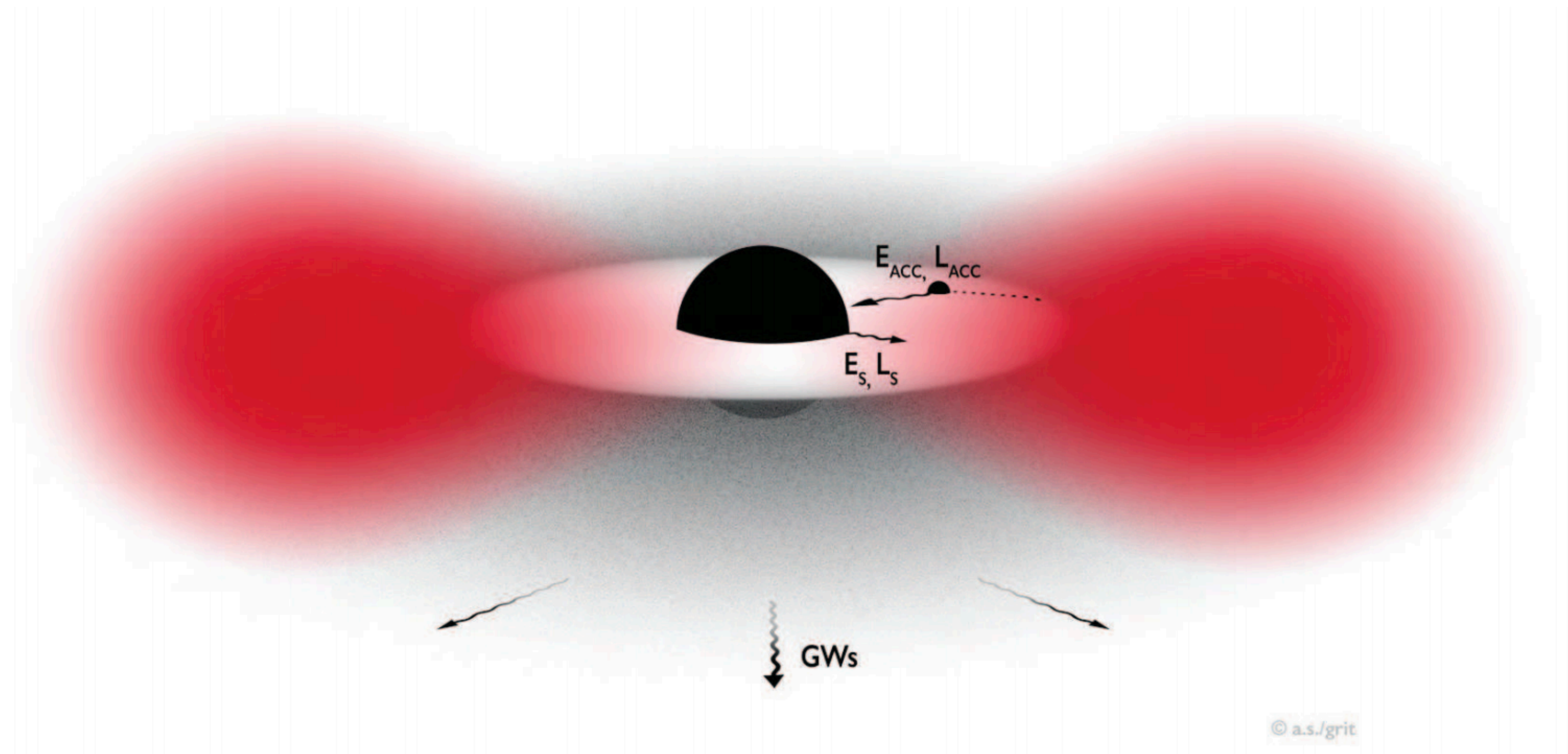


Credit: Steve Drasco

- ❖ GWs from EMRIs can last for **months/years in LISA/Taiji/TianQin band**
- ❖ Unique probes to test gravity but also the **environment** around massive black holes
- ❖ **Challenge:** Accurate modelling requires using relativistic perturbation theory methods.

# Black-hole environment: scalar clouds

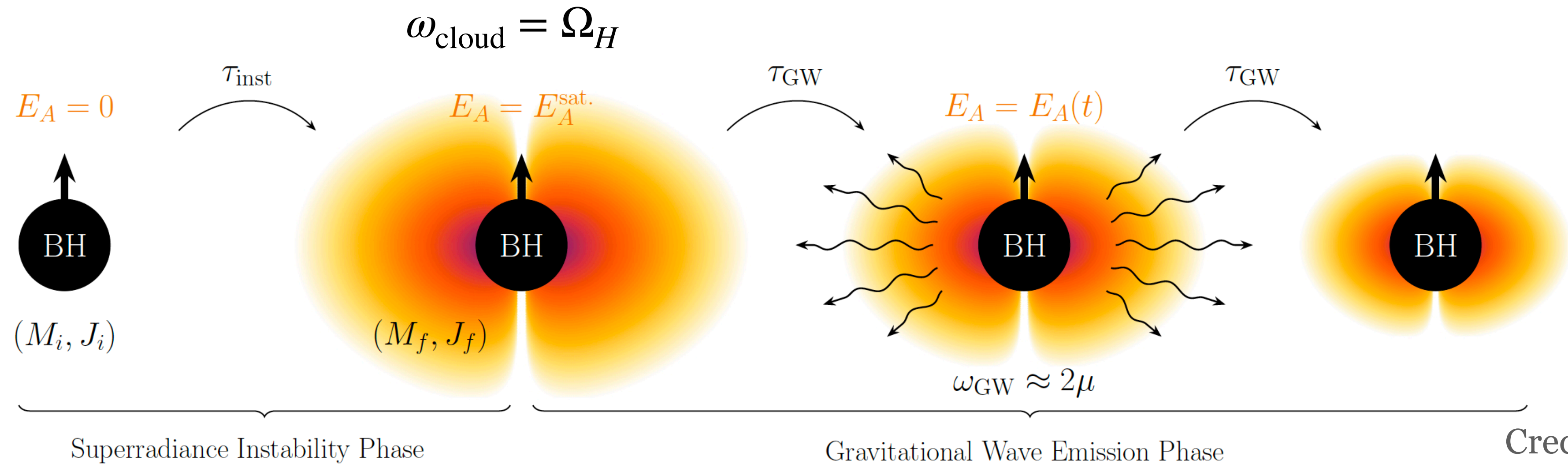
$$\square_g \Phi - \mu^2 \Phi = 0, \quad \mu = m_b c / \hbar = \lambda_c^{-1}$$



From: RB, Cardoso & Pani, CQG32 (2015) 13, 134001

# Scalar clouds

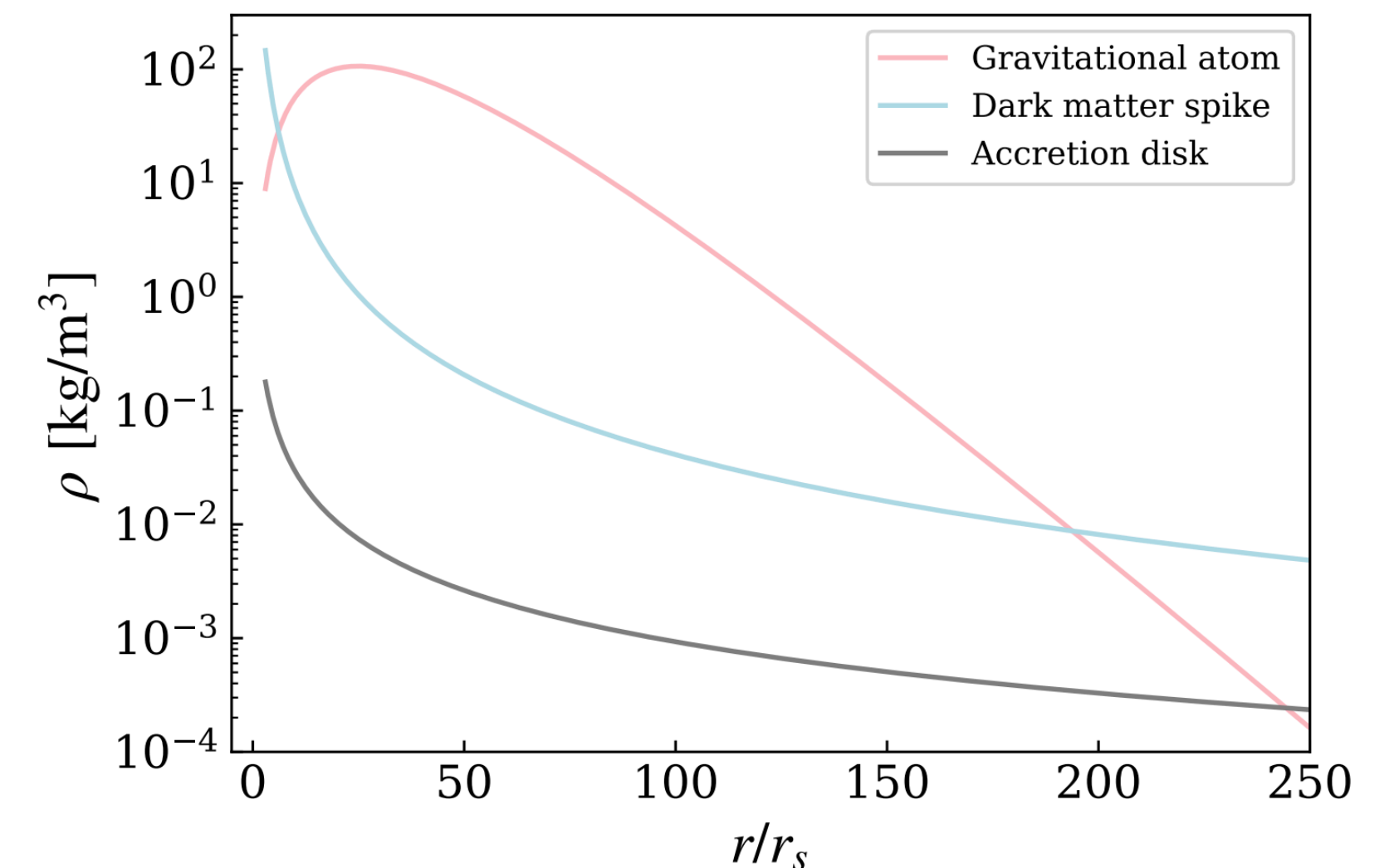
Review: RB, Cardoso & Pani “Superradiance” Lect. Notes Phys. 971 (2020), 2nd ed.



Astrophysically relevant when:

$$M\mu = \frac{R_G}{\lambda_c} \lesssim 0.5 \approx \left( \frac{M}{7 \times 10^5 M_\odot} \right) \left( \frac{m_b c^2}{10^{-16} \text{eV}} \right)$$

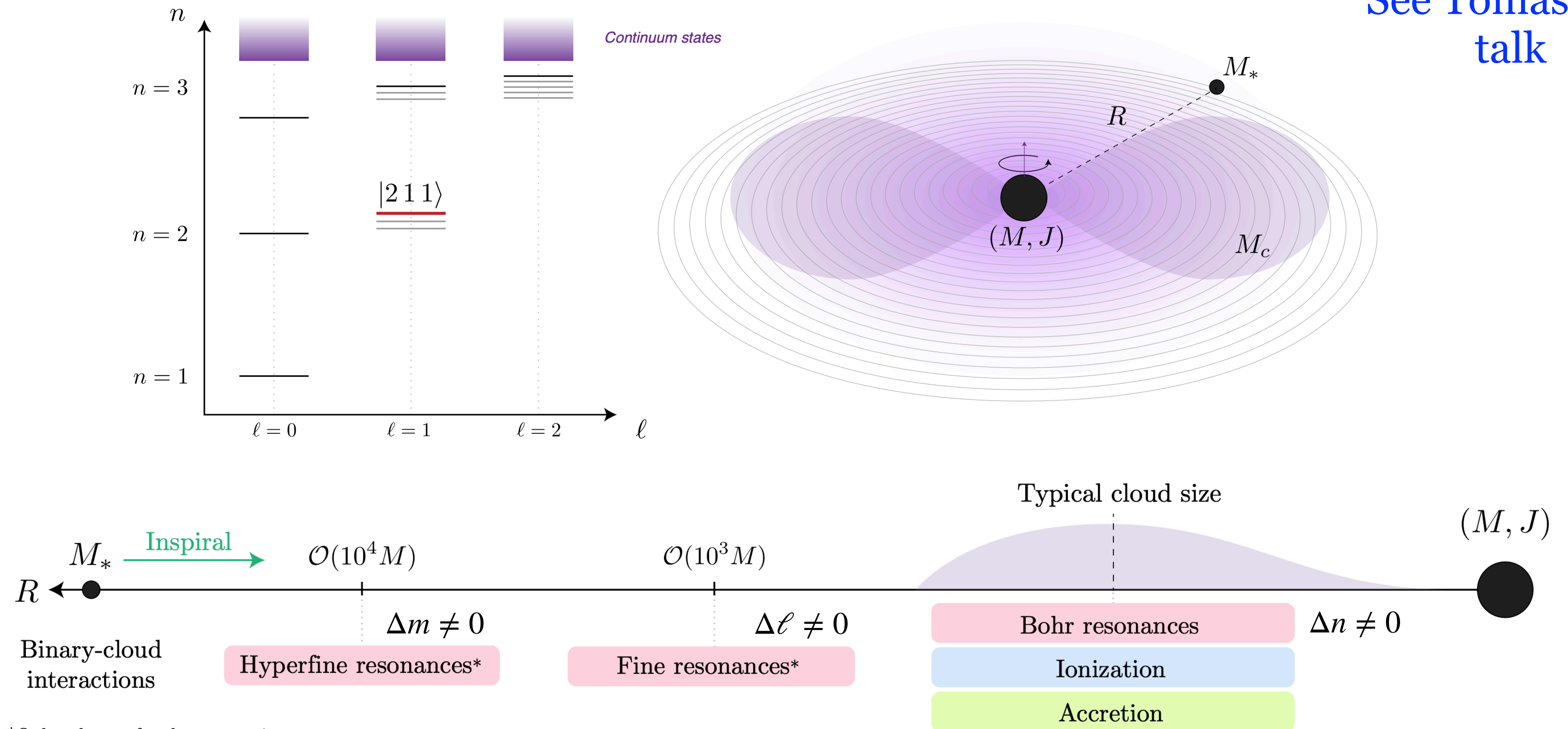
**Ultralight bosons:** axion-like particles;  
dark photons; ultralight dark matter, ...



# Probing scalar clouds with binaries

Baumann+'18-'21; Tomaselli+'23-'25; RB & Shah '23; Dyson+ '25; Li+ '25; Keijzer+ '26; Xu+ '26...

See Tomaselli's talk



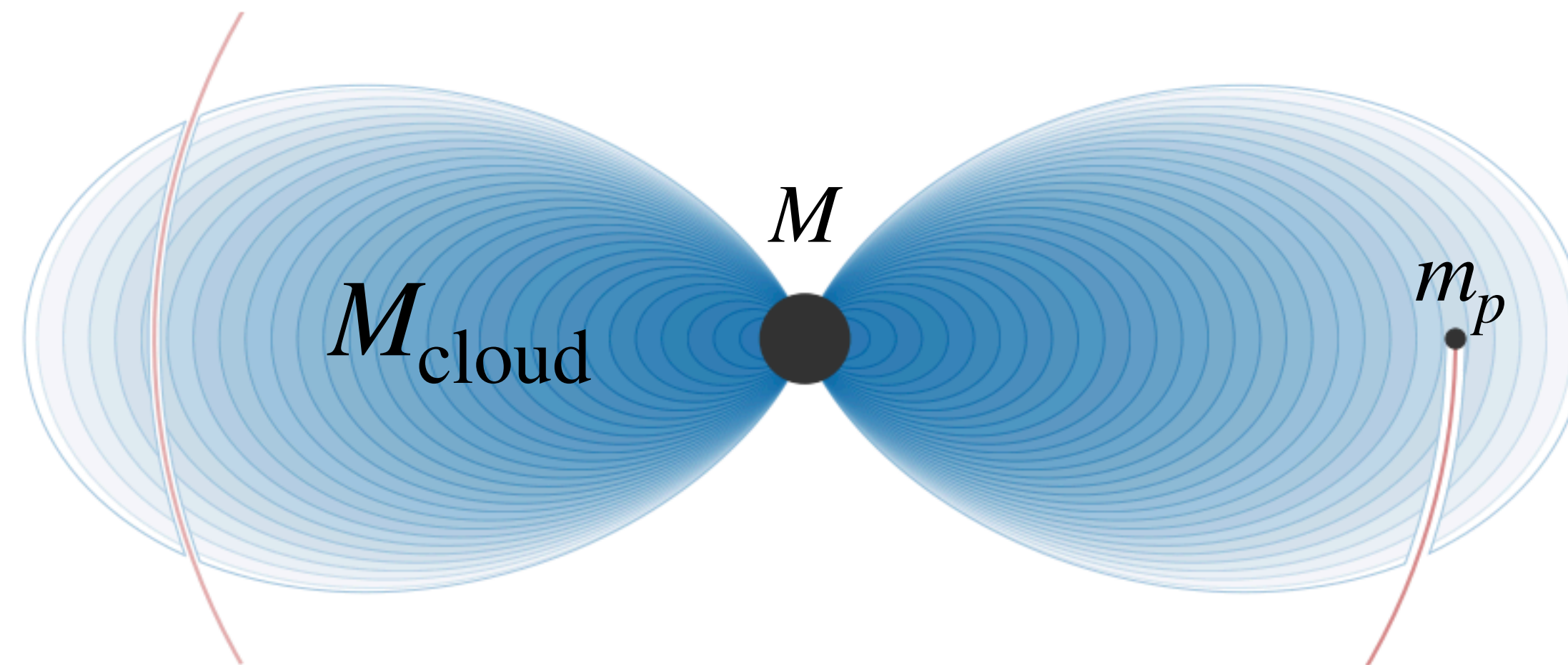
\*Only relevant for the co-rotating case

Credit: Riccardo Della Monica (based on Baumann+ & Tomaselli+)

# Modelling EMRIs in boson clouds

RB & Shah '23; Dyson+ '25; Li+ '25; Keijzer+ '26; Xu+ '26

**Goal:** develop (accurate) EMRI waveform model to probe scalar clouds



From: Baumann+, PRD105, 115036 (2022)

$$S = \int d^4x \sqrt{-\mathbf{g}} \left( \frac{\mathbf{R}}{16\pi} - \partial_\mu \bar{\Phi} \partial^\mu \Phi - \mu^2 |\Phi|^2 \right) - m_p \int_\gamma \sqrt{-\mathbf{g}_{ab} u^a u^b} d\tau$$

Scalar field “environment”

Secondary object

# Modelling EMRIs in boson clouds

---

RB & Shah '23; Dyson+ '25; Li+ '25; Keijzer+ '26; Xu+ '26

**Idea:** use two-parameter perturbative expansion

$$q \equiv m_p/M \ll 1, \quad \epsilon^2 \sim \rho_{\text{cloud}} M \ll 1$$

$$\mathbf{g}_{\mu\nu} = g_{\mu\nu}^{(0)} + q h_{\mu\nu}^{(0,1)} + \epsilon^2 h_{\mu\nu}^{(2,0)} + \epsilon^2 q h_{\mu\nu}^{(2,1)} + \dots$$

$$\Phi = \epsilon \phi^{(1,0)} + q \epsilon \phi^{(1,1)} + \dots$$

# Modelling EMRIs in boson clouds

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BH background

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Scalar  
cloud



$$(\square^{(0)} - \mu^2)\Phi^{(1,0)} = 0$$

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1st order GSF

$$\Phi = \boxed{\epsilon\phi^{(1,0)}} + q\epsilon\phi^{(1,1)} + \dots$$

Scalar  
cloud

$$\delta G_{\alpha\beta}^{(0)}[h^{(1,0)}] = 8\pi T_{\alpha\beta}^{pp(1,0)}$$

$$(\square^{(0)} - \mu^2)\Phi^{(1,0)} = 0$$

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1st order GSF

Cloud's "self-gravity"

$$\Phi = \boxed{\epsilon \phi^{(1,0)}} + q \epsilon \phi^{(1,1)} + \dots$$

Scalar cloud

$$\delta G_{\alpha\beta}^{(0)}[h^{(1,0)}] = 8\pi T_{\alpha\beta}^{pp(1,0)}$$

$$\delta G_{\alpha\beta}^{(0)}[h^{(2,0)}] = 8\pi T_{\alpha\beta}^{c(2,0)}[\phi^{(1,0)}, \bar{\phi}^{(1,0)}]$$

$$(\square^{(0)} - \mu^2)\Phi^{(1,0)} = 0$$

# Modelling EMRIs in boson clouds

RB & Shah '23; Dyson+ '25; Li+ '25; Keijzer+ '26; Xu+ '26

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Scalar cloud

Cloud's response to the secondary

$$(\square^{(0)} - \mu^2)\Phi^{(1,0)} = 0$$

$$(\square^{(0)} - \mu^2)\Phi^{(1,1)} = S[h^{(0,1)}, \Phi^{(1,0)}]$$

# Modelling EMRIs in boson clouds

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1st order GSF      Cloud's "self-gravity"      Interaction between cloud and secondary

$$\delta G_{\alpha\beta}^{(0)}[h^{(1,0)}] = 8\pi T_{\alpha\beta}^{pp(1,0)}$$

$$\delta G_{\alpha\beta}^{(0)}[h^{(2,0)}] = 8\pi T_{\alpha\beta}^{c(2,0)}[\phi^{(1,0)}, \bar{\phi}^{(1,0)}]$$

$$\delta G_{\alpha\beta}^{(0)}[h^{(2,1)}] = S[h^{(0,1)}, h^{(2,0)}, \Phi^{(1,0)}, \Phi^{(1,1)}]$$

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$$(\square^{(0)} - \mu^2)\Phi^{(1,0)} = 0$$

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# Post-adiabatic expansion

---

RB, Dyson, Keijzer, Pound, Spieksma, Upton & Xu, *work in progress*

$$u^\nu \nabla_\nu^{(0)} u^\mu = q f_{(0,1)}^\mu + \epsilon^2 f_{(2,0)}^\mu + \epsilon^2 q f_{(2,1)}^\mu + q^2 f_{(0,2)}^\mu + \dots$$

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On radiation-reaction time, the orbital phases can be expanded as:

$$\varphi_{\text{orb}} = \frac{1}{q} \left[ \varphi_{\text{vac}}^{(0)}(qt) + \epsilon^2 \varphi_{\Phi}^{(0)} \right] + q^0 \varphi_{\text{vac}}^{(1)} + \mathcal{O}(\epsilon^2, q)$$

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$f_{(0,1)}^{\text{diss}}$



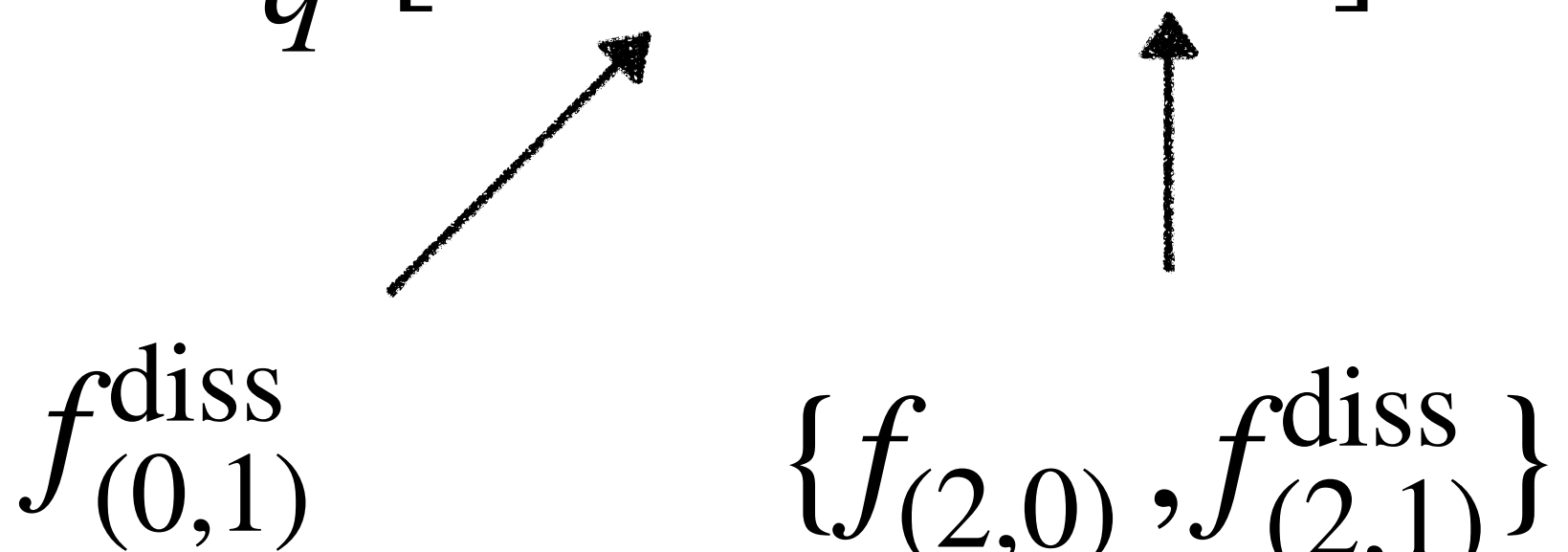
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$f_{(0,1)}^{\text{diss}}$                        $\{f_{(2,0)}, f_{(2,1)}^{\text{diss}}\}$

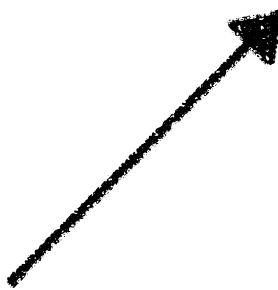
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$f_{(0,1)}^{\text{diss}}$



$\{f_{(2,0)}, f_{(2,1)}^{\text{diss}}\}$



$\{f_{(0,1)}^{\text{cons}}, f_{(0,2)}^{\text{diss}}\}$

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$f_{(0,1)}^{\text{diss}}$ 
 $\{f_{(2,0)}, f_{(2,1)}^{\text{diss}}\}$ 
 $\{f_{(0,1)}^{\text{cons}}, f_{(0,2)}^{\text{diss}}\}$

$\{\phi^{(1,0)}, \phi^{(1,1)}, h^{(0,1)}, h^{(2,0)}, h^{(2,1)}\}$

# Radiative losses

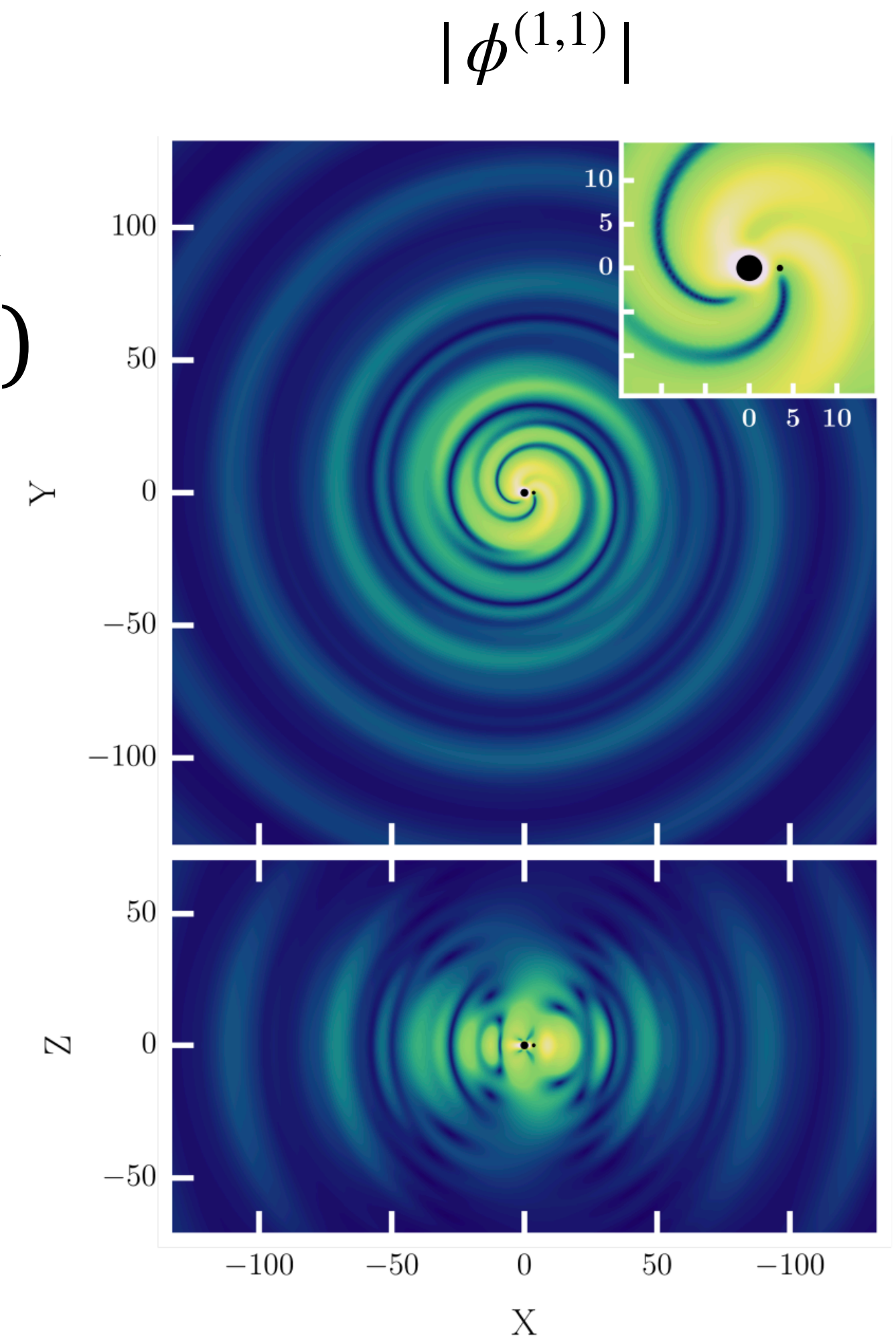
RB & Shah '23; Dyson+ '25; Li+ '25; Xu+ '26

For dipolar clouds, only **scalar losses** due to  $\phi^{(1,1)}$  have been computed so far (but in Kerr, only for circular orbits)

[See also Duque+ '23 and Keijzer+ '26 for spherical clouds]

$$\dot{E}_{\text{orb}} + \dot{M}_{\text{cloud}} = \underbrace{-\dot{E}^{g,\infty} - \dot{E}^{g,H}}_{\text{GW fluxes}} - \underbrace{\dot{E}^{\Phi,\infty} - \dot{E}^{\Phi,H}}_{\text{scalar fluxes}}$$

$$\dot{L}_{\text{orb}} + \dot{L}_{\text{cloud}} = \underbrace{-\dot{L}^{g,\infty} - \dot{L}^{g,H}}_{\text{GW fluxes}} - \underbrace{\dot{L}^{\Phi,\infty} - \dot{L}^{\Phi,H}}_{\text{scalar fluxes}}$$

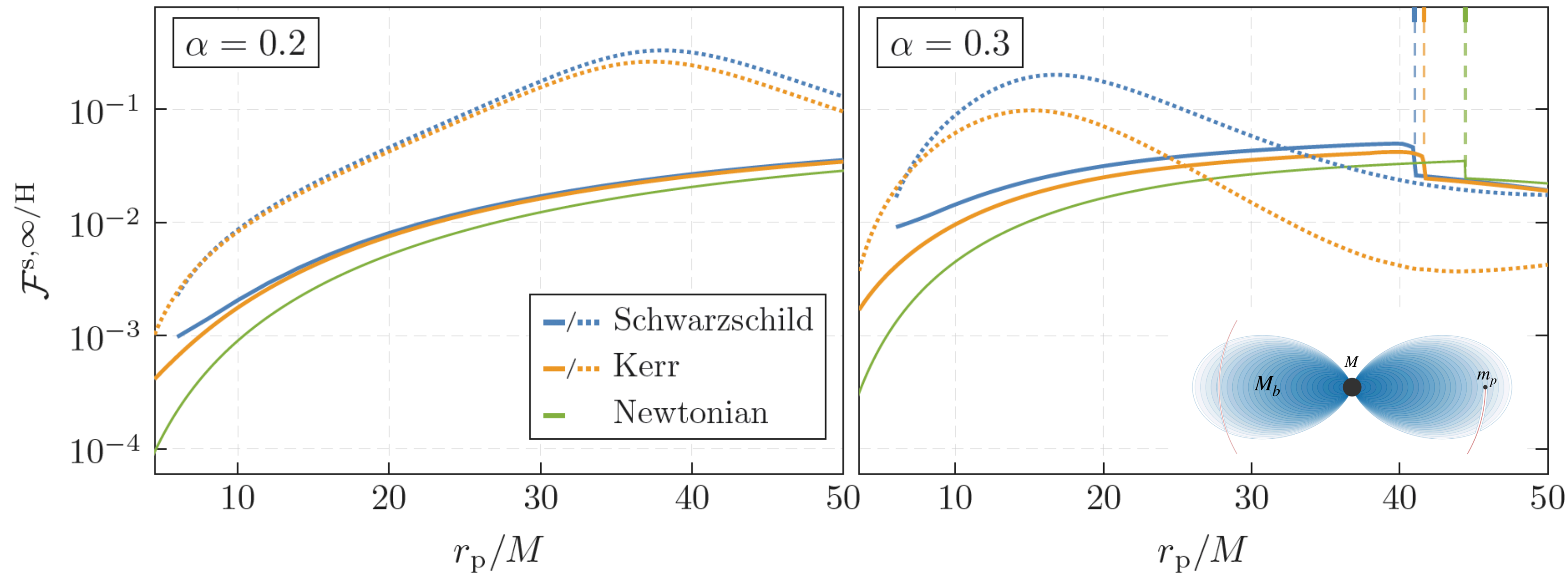


From: Dyson+ '25,  
PRL134 (2025) 21, 211403

# Circular, equatorial orbits (in Kerr)

Dyson, Spieksma, RB, Dolan & van de Meent '25

$$\mathcal{F}^{s,\infty/H} \equiv q^{-2} \epsilon^{-2} \left( \dot{E}^{\Phi,\infty/H} + \dot{M}_{\text{cloud}}^{\infty/H} \right)$$



$r_p$  - orbital radius

$$\alpha := M\mu$$

**Dotted:** horizon fluxes

**Solid:** fluxes at infinity

**Note:** In these plots, prograde orbits with

$$\mathcal{F}^{s,H} \propto (\Omega_{\text{orb}} - \omega_{\text{cloud}}) < 0$$

**Interpretation:**

for  $\Omega_{\text{orb}} < \omega_{\text{cloud}} \rightarrow$  net transfer energy/angular momentum from cloud to orbit (*tidal acceleration*)

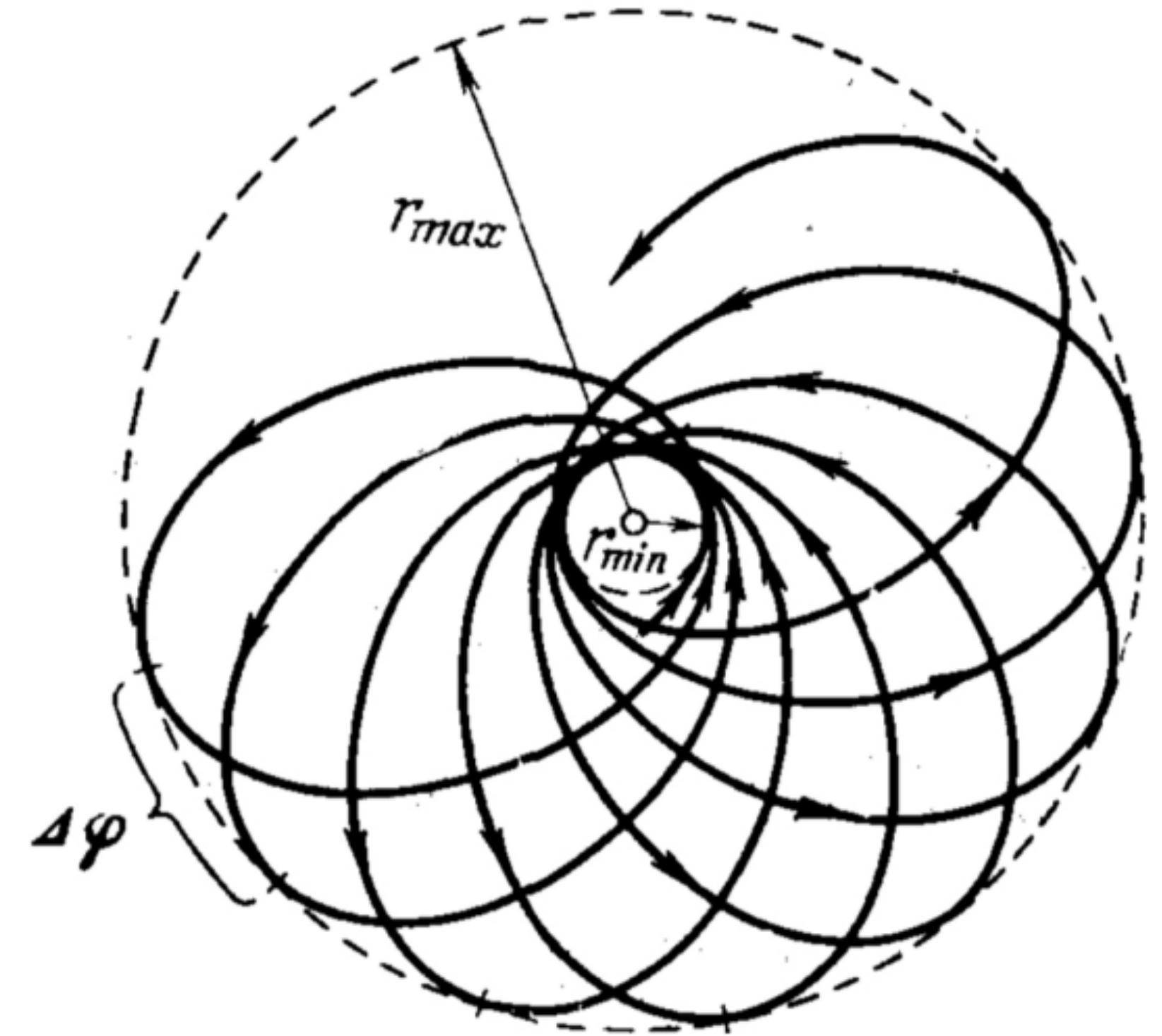
# Eccentric orbits (in Schwarzschild)

$$r_{\max} = \frac{pM}{1-e}, \quad r_{\min} = \frac{pM}{1+e}$$

$\Omega_r \rightarrow$  radial motion's frequency

$$\varphi_p(t) = \Omega_\varphi t + \sum b_k e^{i\Omega_r t}$$

$$h_{\mu\nu} \sim \sum_{m,n} e^{-i(m\Omega_\varphi + n\Omega_r)t}$$



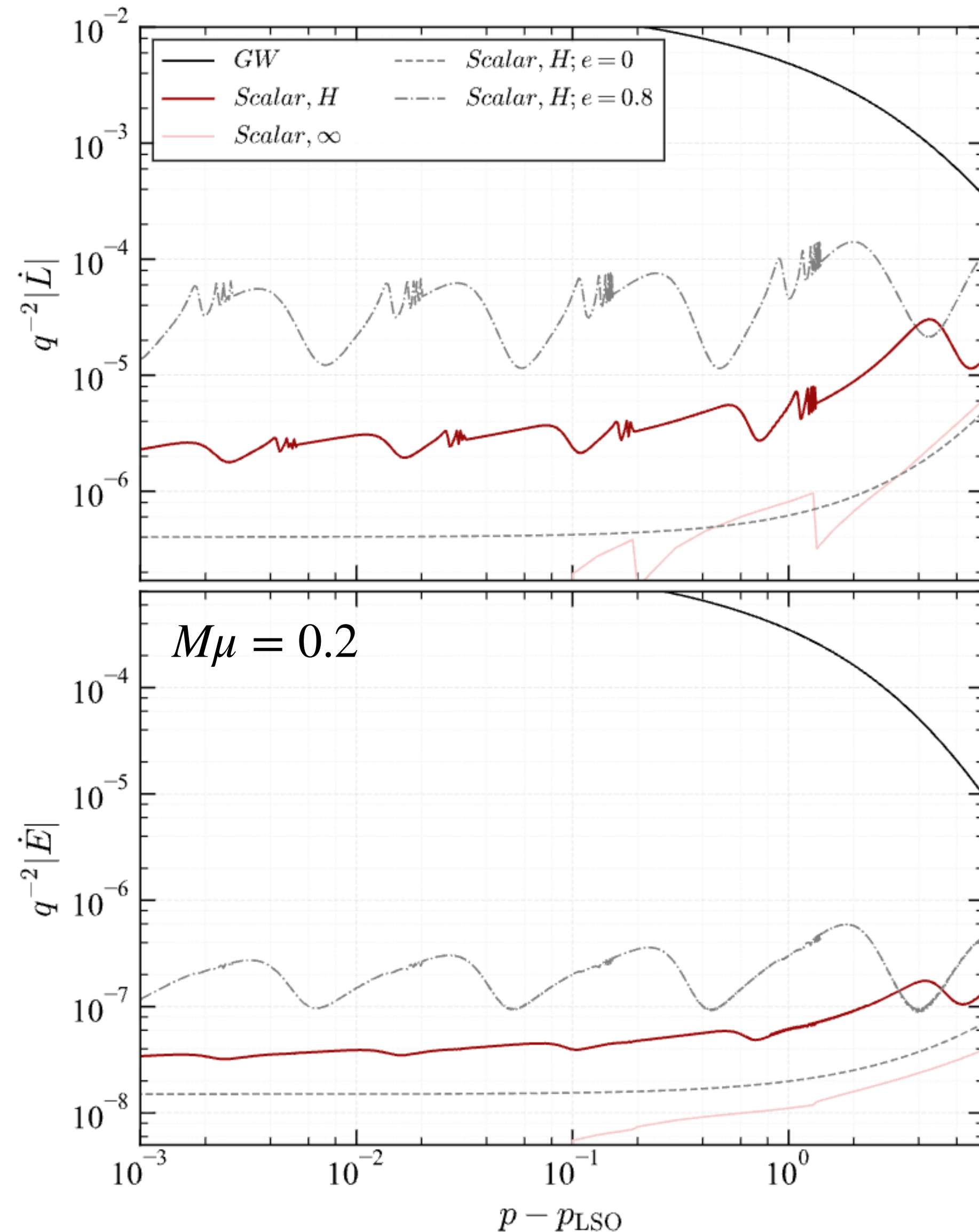
From: Landau & Lifshitz, Vol 1 "Mechanics"

$$\frac{\Delta\varphi}{2\pi} = \frac{\Omega_\varphi}{\Omega_r} - 1$$

# Eccentric, equatorial orbits (in Schwarzschild)

Xu, RB, Della Monica, Vicente, Yuan '26

See Xu's talk  
on Thursday



$$\Phi^{(1,1)} \sim \sum_{m,n} e^{-i(\omega_{\text{cloud}} + \sigma_{mn})t}$$

$$\sigma_{mn} = m\Omega_{\varphi} + n\Omega_r$$

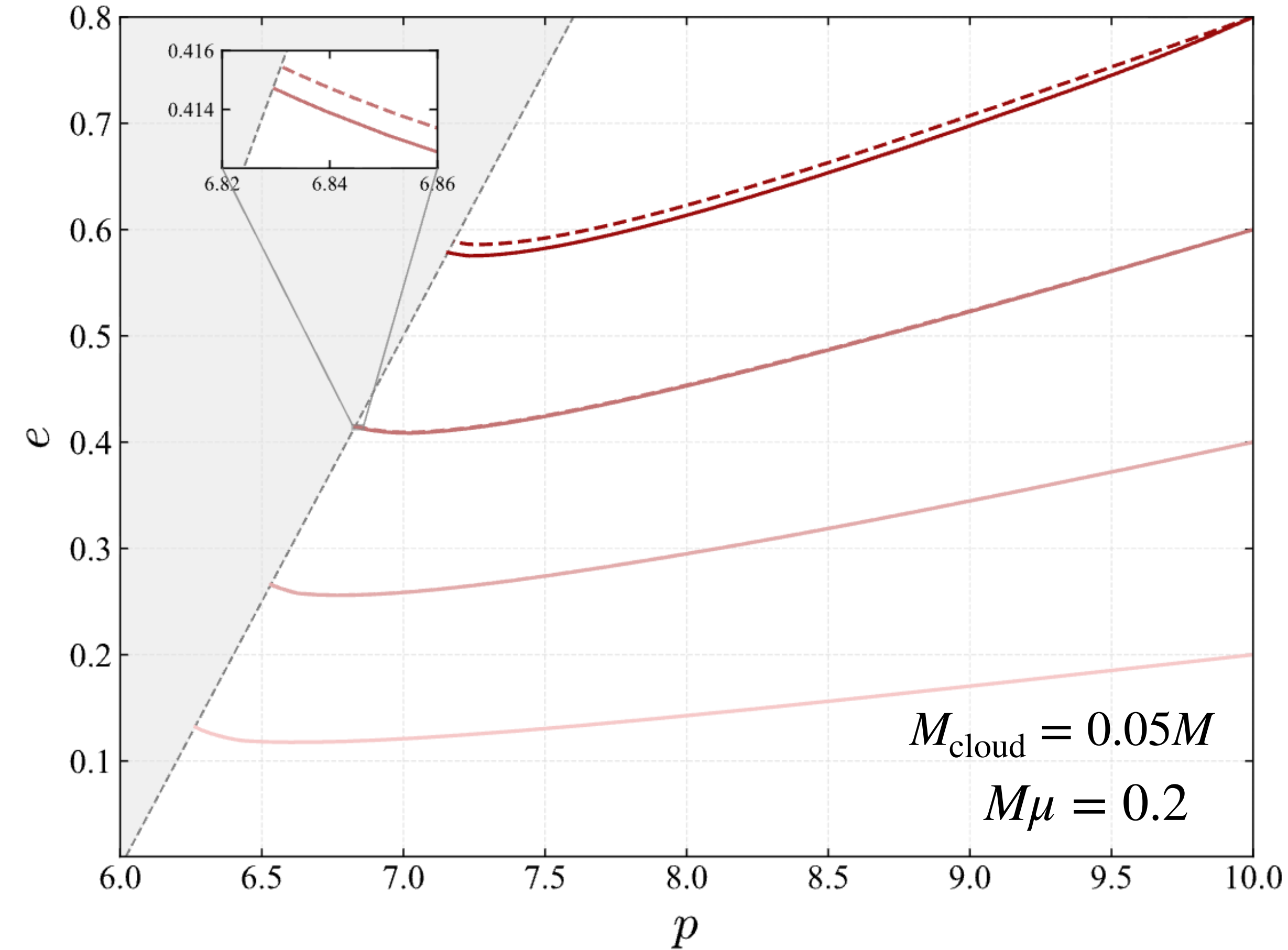
**Resonances when:**

$$\omega_{\text{cloud}} + \sigma_{mn}^{n_f} = \text{Re}(\omega_{n_f \ell_f m_f})$$

Strong-field resonances  
intrinsically tied to **relativistic  
precession** ( $\Omega_r \neq \Omega_{\varphi}$ )

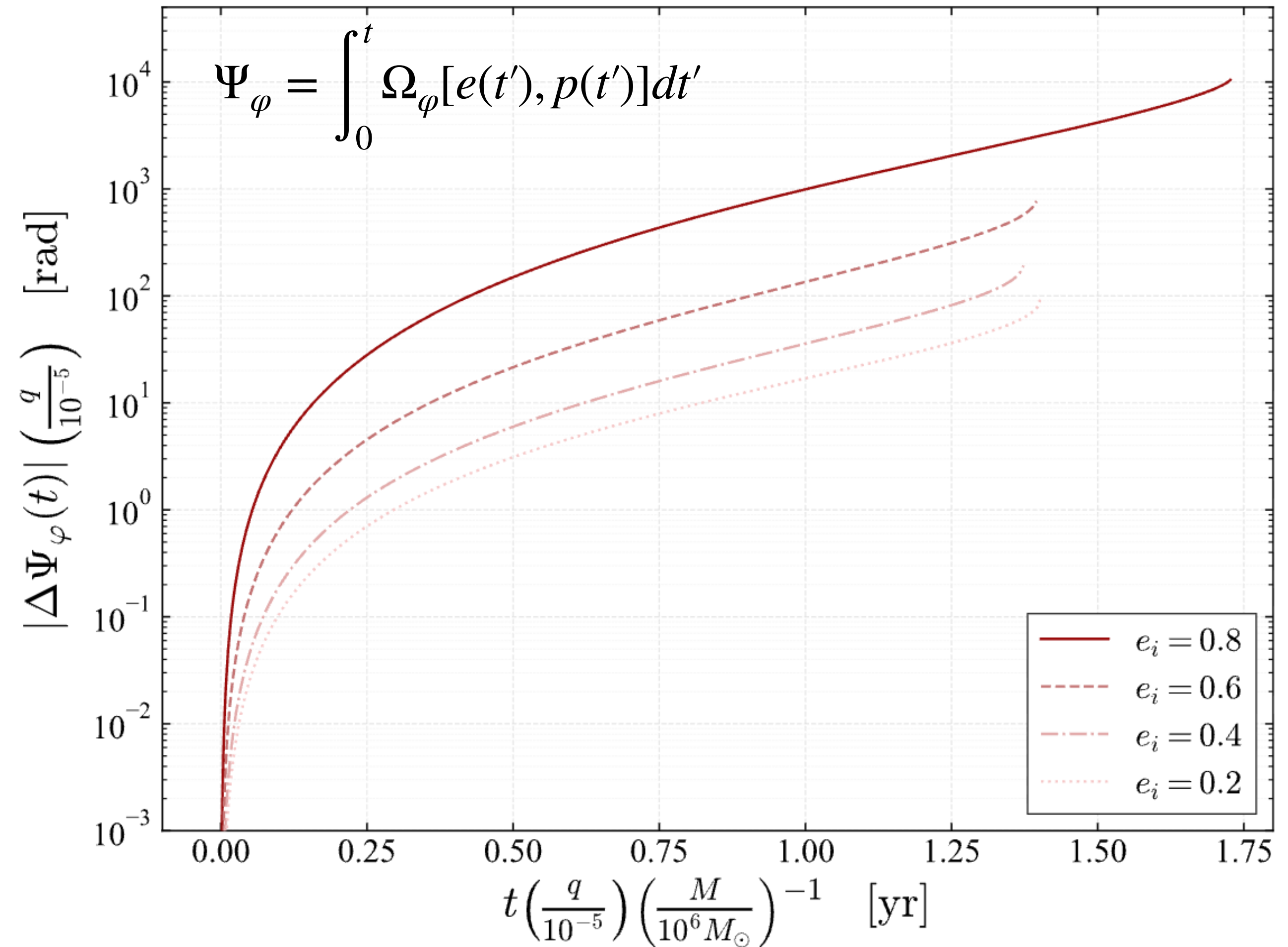
# Orbital evolution

Xu, RB, Della Monica, Vicente, Yuan '26



**Dotted:** vacuum (only driven by GWs)

**Solid:** with scalar cloud



$\Delta\Psi_{GW} \sim \mathcal{O}(10^4)$  rad in  $T_{\text{obs}} \sim 2$  years

# Parameter estimation: proof-of-principle

Khalvati+ (including RB), '25

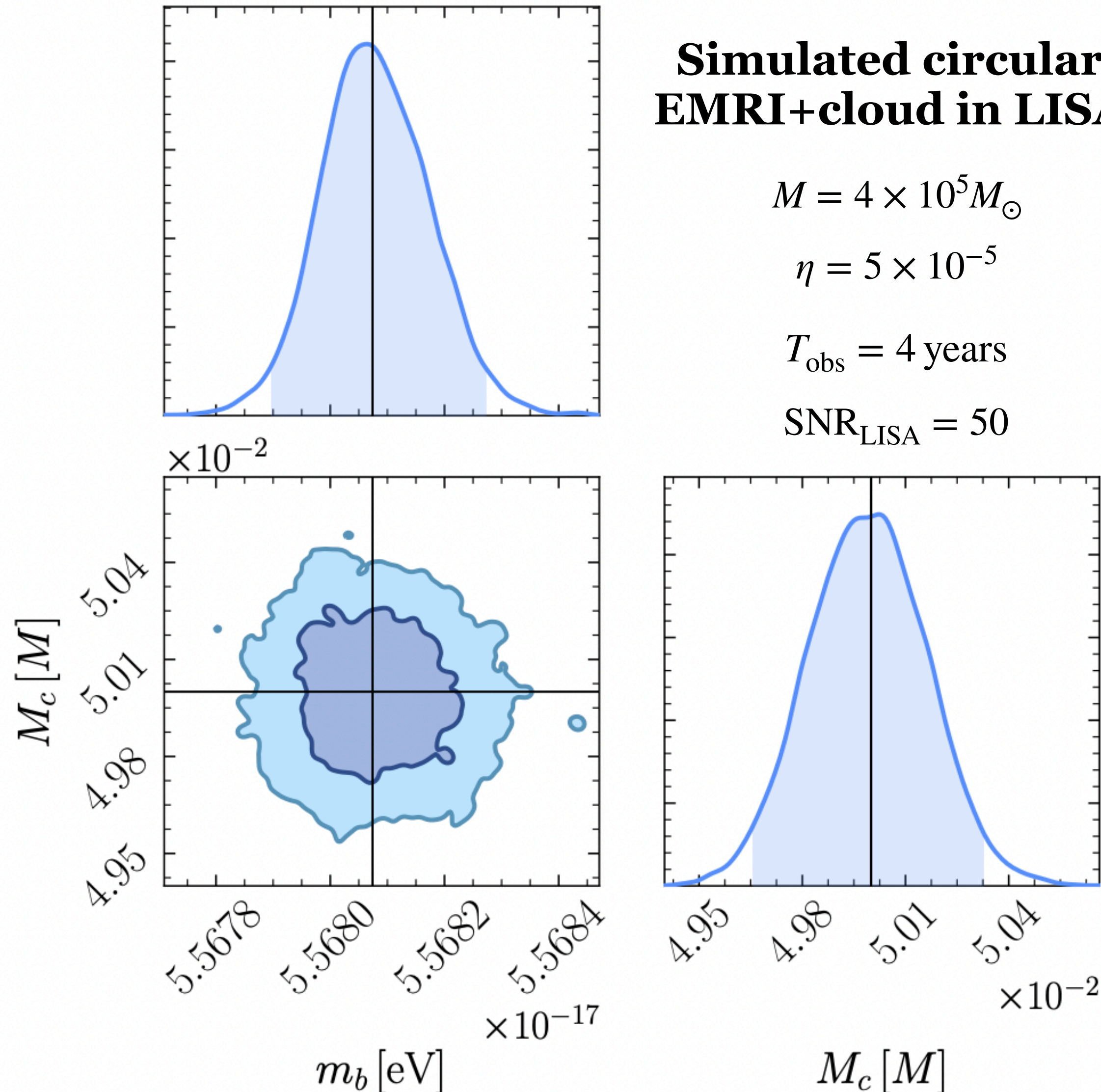
## Simulated circular EMRI+cloud in LISA

$$M = 4 \times 10^5 M_\odot$$

$$\eta = 5 \times 10^{-5}$$

$$T_{\text{obs}} = 4 \text{ years}$$

$$\text{SNR}_{\text{LISA}} = 50$$



*Proof-of-principle results:*  
parameters of boson cloud  
can be recovered with  
**exquisite precision!**

**Distinguishable** from other  
environments (e.g. accretion  
disks, dark matter spikes)?  
*Probably* yes, but a lot of work  
remains to be done to be certain.

See P.S. Cole+,  
Nature Astron. 7 (2023) 8, 943-950

# Conclusions

---

- ❖ **Fully relativistic, perturbative framework** to model EMRIs in the presence of scalar clouds from first-principles
- ❖ **Scalar radiative losses** already indicate rich physics: tidal acceleration; rich structure of resonances
- ❖ ***Work in progress:***
  - Waveform model for quasi-circular EMRIs in scalar clouds (in Kerr) including all relevant effects [with Dyson, Keijzer, Pound, Spieksma, Upton & Xu]
  - Generic orbits in Kerr: prolonged orbital resonances and chaotic motion [with Xu, Destounis, Guo & May]
- ❖ ***Future (long-term) goal:*** waveform model for generic orbits in Kerr

# Thank you!

This work is supported by national funds through FCT – Fundação para a Ciência e a Tecnologia, I.P., in the scope of the ERC-Portugal program, Project “GravNewFields”.

