

Virtual absorption modes of Schwarzschild-de Sitter spacetimes in semi-open systems

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- 1 Introduction and motivation
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Why look beyond quasinormal modes

- Black hole spectroscopy: after a perturbation, the spacetime ringdowns at a discrete set of complex frequencies. Those frequencies are called the quasinormal modes (QNMs) with boundary condition

$$\Psi \sim e^{\pm i\omega x}, \quad x \rightarrow \pm\infty.$$

We can estimate the parameters of the remnant by **receiving** the ringdown signal obtained from the GW source.

- However, scattering contains more information than resonances alone. We can ask a complementary question: If we **emit** a wave towards a black hole, at which frequencies does reflection disappear? That question leads from QNMs to total transmission modes (TTMs).

Total transmission modes

- **Total transmission modes (TTMs)** are defined by a vanishing reflection amplitude. An incident wave crosses the effective potential barrier without producing a reflected component. Depending on the propagation direction, one distinguishes right and left TTMs. The right TTM (TTM_R) and left TTM (TTM_L) are defined as:

$$\text{TTM}_R: \quad \psi \sim e^{-i\omega x}, \quad \text{as } x \rightarrow \pm\infty;$$

$$\text{TTM}_L: \quad \psi \sim e^{+i\omega x}, \quad \text{as } x \rightarrow \pm\infty.$$

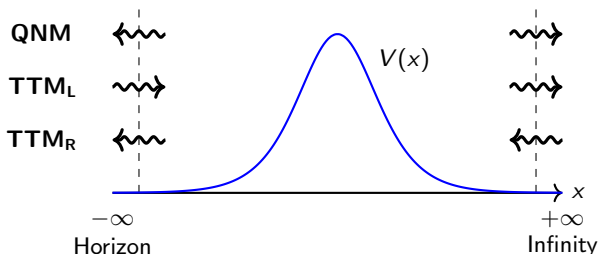


Figure 1: The definitions of QNMs and TTMs.

Virtual absorption modes

- In scattering language (Green's functions), QNMs appear as poles of the reflection amplitude, whereas TTMs appear as its zeros.
- A TTM has a positive imaginary part. For a QNM, a positive imaginary part normally signals an instability. For a TTM, the interpretation is different.
- Such a TTM can be selectively stimulated by an incoming waveform designed to grow exponentially in time, the rate of which is precisely set by the mode's imaginary part. If we prepare that time dependence with such TTM spectrum, destructive interference suppresses the reflected wave. Therefore, TTMs are also called the **virtual absorption modes (VAMs)**.

From black holes to semi-open systems

- Considering that black holes are not completely “black”. Instead of assuming a perfectly absorbing event horizon, we place a partially reflective wall close to it. The boundary conditions around the event horizon will be modified into

$$\psi \sim e^{-i\omega(x-x_0)} + \mathcal{K}(\omega)e^{i\omega(x-x_0)}.$$

- We consider $\mathcal{K}(\omega)$ to be constant. This is a simple effective model for an exotic compact object (ECO). Because one boundary is reflective while the outer boundary remains outgoing, we call this a **semi-open system**.

Questions addressed in this work

- First, how do the reflectivity and the wall position move the VAM spectrum in the complex-frequency plane?
- Second, if we tune an incident waveform to one of these frequencies, can we see the absorption process directly in time-domain evolution meaning **no reflected wave**?
- Schwarzschild-de Sitter spacetime is especially useful because the relevant homogeneous solutions can be expressed through **Heun's functions**.

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Master perturbation equations

- The SdS spacetime metric function is

$$f(r) = 1 - \frac{2M}{r} - \frac{\Lambda r^2}{3} = -\Lambda \frac{(r - r_c)(r - r_e)(r - r_n)}{3r}.$$

- The perturbation equation can be reduced to the Schrödinger like equations

$$\left[\frac{d^2}{dx^2} + \omega^2 - V_s(x) \right] \Psi(x) = 0, \quad dx = \frac{dr}{f(r)}.$$

Here, we consider three kinds of potentials

$$V_s(r) = f(r) \left[\frac{\ell(\ell+1)}{r^2} + (1-s) \frac{f'(r)}{r} + \delta_{s,2} f''(r) + \frac{2\Lambda}{3} \delta_{s,0} \right],$$

where $s = 0, 1, 2$ indicate scalar, electromagnetic and gravitational axial perturbations.

Semi-open boundary and spectral conditions

- The “in” and “up” solutions are

$$\Psi_{\text{in}}(x) = \begin{cases} e^{-i\omega x}, & x \rightarrow -\infty \\ A_{\text{in}}(\omega)e^{-i\omega x} + A_{\text{out}}(\omega)e^{i\omega x}, & x \rightarrow +\infty \end{cases},$$

and

$$\Psi_{\text{up}}(x) = \begin{cases} B_{\text{in}}(\omega)e^{-i\omega x} + B_{\text{out}}(\omega)e^{i\omega x}, & x \rightarrow -\infty \\ e^{i\omega x}, & x \rightarrow +\infty \end{cases}.$$

- The solution obeys the following reflective boundary condition

$$\Psi_s(x) = \begin{cases} e^{-i\omega x} + \mathcal{K}(\omega)e^{-2i\omega x_w}e^{i\omega x}, & x \rightarrow x_w \\ S_{\text{in}}(\omega)e^{-i\omega x} + S_{\text{out}}(\omega)e^{i\omega x}, & x \rightarrow +\infty \end{cases}.$$

- TTM spectra are derived by the condition $S_{\text{out}}(\omega) = 0$, where $S_{\text{out}}(\omega)$ reads

$$S_{\text{out}}(\omega) = A_{\text{out}}(\omega) + \frac{\mathcal{K}(\omega)e^{-2i\omega x_w}}{A_{\text{in}}(\omega)} \left[1 - A_{\text{out}}(\omega)B_{\text{in}}(\omega) \right], \quad \omega \in \mathbb{C}.$$

- A VAM satisfies $S_{\text{out}} = 0$.

VAM Spectra

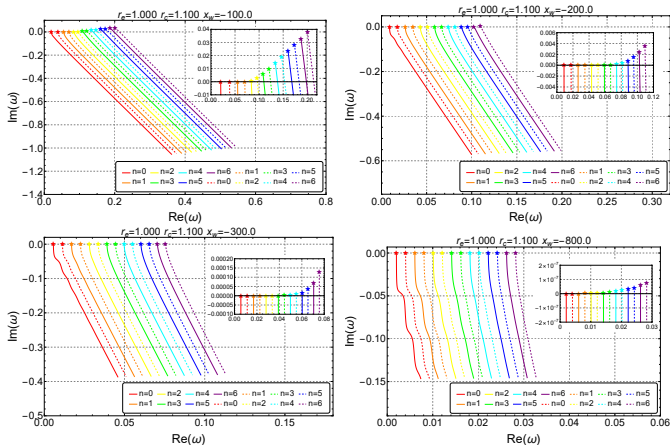


Figure 2: The migrations of the VAM spectra with $\mathcal{K}$ varying with $r_c = 1.1$.

- First, we can see that as $|\mathcal{K}|$ decreases, the modes consistently migrate towards regions of the complex plane characterized by more negative imaginary parts, namely, $-\text{Im}(\omega)$ becomes larger. Their trajectories will cross the real axis.
- Second, for different modes, there always exists a critical $\mathcal{K}$ that causes the imaginary part of the VAM spectrum to vanish, namely $\text{Im}(\omega_{\text{VAM}}) = 0$.

Energy accounting

- We study virtual absorption phenomenon in the semi-open Schwarzschild-de Sitter spacetime.
- The corresponding energy density $\rho(t, x)$ is

$$\rho(t, x) = \frac{1}{2} \left[|\partial_t \Psi|^2 + |\partial_x \Psi|^2 + V_s(x) |\Psi|^2 \right] = \frac{1}{2} \left[|\Pi|^2 + |\partial_x \Psi|^2 + V_s(x) |\Psi|^2 \right],$$

in which we have defined $\Pi(t, x) \equiv \partial_t \Psi(t, x)$.

- At a fixed time t , the total energy on the considered spatial interval $[x_w, x_{\max}]$ is the integration over this interval

$$E_{\text{total}}(t) = \int_{x_w}^{x_{\max}} \rho(t, x) dx = E_{\text{inside}}(t) + E_{\text{outside}}(t),$$

where

$$E_{\text{inside}}(t) = \int_{x_w}^{x_{\text{cut}}} \rho(t, x) dx, \quad E_{\text{outside}}(t) = \int_{x_{\text{cut}}}^{x_{\max}} \rho(t, x) dx.$$

The energy between the wall and x_{cut} is called E_{inside} . It measures energy stored in the wall-barrier cavity.

Tailored initial waveform

- Excite the system with the frequency Ω_0 . We consider the following family of initial data, which consists of an exponentially growing sinusoidal wave truncated by a Gaussian envelope at a truncated radius x_t ,

$$\begin{aligned}\Psi(0, x) &= \mathcal{A} \left[e^{\omega_I(x-x_t)} \Theta(x_t - x) + e^{-(x-x_t)^2/(2\sigma^2)} \Theta(x - x_t) \right] \cos[\omega_R(x - x_t)], \\ \Pi(0, x) &= \frac{d\Psi(0, x)}{dx},\end{aligned}$$

where $\Omega_0 = \omega_R + i\omega_I$ with $\omega_I > 0$, Θ is the Heaviside step function.

- The selection of the initial condition for Π is to ensure that the initial waveform propagates towards the effective potential to simulate **wave injection**.

Time evolution: Exact VAM and non-VAM inputs

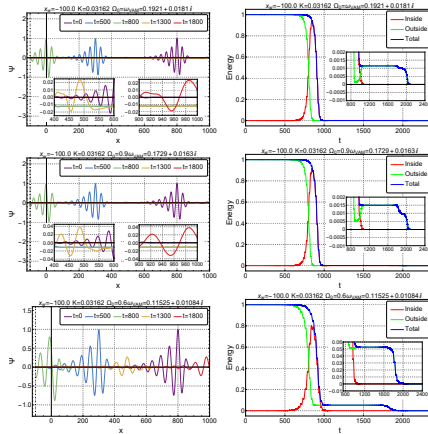


Figure 3: The time evolution of $\Psi(t, x)$ and energy are shown. In the top panels, the overtone $n = 6$ mode $\omega_{\text{VAM}} = 0.1921 + 0.0181i$ is excited, while in the middle and bottom panels, non-VAM spectrum with $\Omega_0 = 0.9\omega_{\text{VAM}} = 0.1729 + 0.0163i$ and non-VAM spectrum with $\Omega_0 = 0.6\omega_{\text{VAM}} = 0.11525 + 0.01084i$ are excited for comparison.

Time evolution: Exact VAM and non-VAM inputs

- The top panels show the dynamic system excited precisely at the VAM spectrum, and the middle and bottom panels show the dynamic system excited at the non-VAM spectra ($\Omega_0 = 0.9\omega_{\text{VAM}}$ and $\Omega_0 = 0.6\omega_{\text{VAM}}$).
- It is found that when Ω_0 is the VAM spectrum, the local minimum of outside energy is minimized. When Ω_0 is not the VAM spectrum, the local minimum of outside energy is relatively larger.
- Describe the magnitude of the reflected wave, values of η are approximately 1.564×10^{-4} , 5.254×10^{-4} , and 5.158×10^{-2} , respectively (The energy has been normalized.). The further away from such modes, the less obvious **virtual absorption phenomenon** becomes.

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Conclusions and outlook

- We present a study of virtual absorption modes (VAMs) in Schwarzschild-de Sitter (SdS) spacetime with semi-open boundary conditions.
- For the frequency-domain aspect, as the reflectivity $|\mathcal{K}|$ decreases, VAM spectra migrate towards regions of the complex plane characterized by more negative imaginary parts with each overtone exhibiting a critical reflectivity at which $\text{Im}(\omega_{\text{VAM}}) = 0$.
- For the time-domain aspect, our results establish VAMs as the spectral signatures of virtual absorption processes for exotic compact objects (ECOs).

Thank you!