

Parameter Estimation of Subsolar-Mass Objects with Mini Extreme Mass-Ratio Inspirals

A Fisher-matrix forecast for ground-based gravitational-wave detectors

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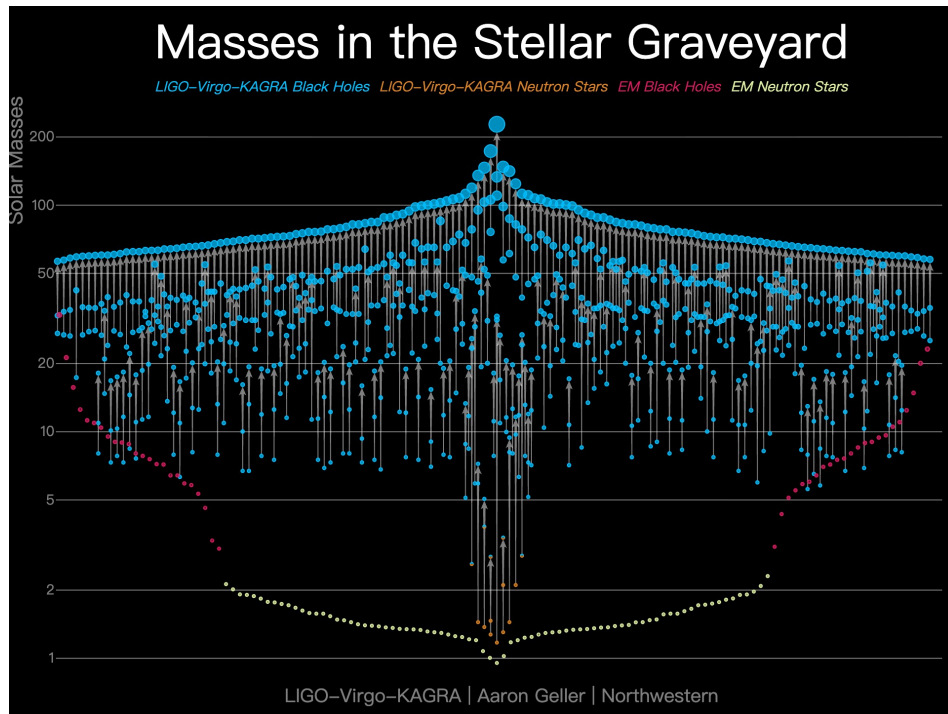
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Why subsolar-mass ultracompact objects?

- Standard astrophysical stellar evolution cannot form ultracompact objects below $1 M_{\odot}$ (Chandrasekhar limit).
- **A single confirmed subsolar-mass ultracompact object would therefore point to new physics.**



≈ 400 compact binary coalescences

0 subsolar-mass compact objects

Primordial black hole? Boson star? Quark star?

Previous searches — no subsolar detection yet

Search	m_1 [$M_\odot$]	m_2 [$M_\odot$]	Mass ratio	Outcome
LVK O1 – O2 (Abbott et al., PRL [1808.04771, 1904.08976])	0.2 – 1	0.2 – 1	comparable	null
LVK O3 – O4a (Abbott et al., PRL [2109.12197]; MNRAS [2212.01477]; Abac et al., arXiv [2605.05444])	0.2 – 10	0.2 – 1	$q \equiv m_2/m_1 \geq 0.1$	null
Nitz et al. (O1–O2) (Nitz & Wang (O1–O2), Nitz & Wang, PRL [2007.03583])	20 – 100	0.01 – 1	down to 10^{-4}	null
Nitz et al. (O3, broad) (Nitz & Wang, PRD [2202.11024])	0.1 – 100	0.01 – 1	down to 10^{-4}	null

- All of these are matched-filtering searches on LVK strain data.
- Independent analyses have pushed to extreme mass ratios ($q \sim 10^{-4}$) with subsolar companions — still no candidate.
- **Much more extreme mass ratios remain unexplored — exactly where the mini-EMRI lives.**

Mini-EMRI: a scaled-down EMRI in the LIGO band

Standard EMRI (Extreme mass ratio inspiral)

- $10^4 - 10^7 M_\odot$ supermassive black hole + stellar-mass companion
- Millihertz band $\rightarrow$ space-borne detectors
- Key source for Taiji, LISA, TianQin



Amaro-Seoane et al., CQG [astro-ph/0703495]
Amaro-Seoane, LRR [1205.5240]

Mini-EMRI

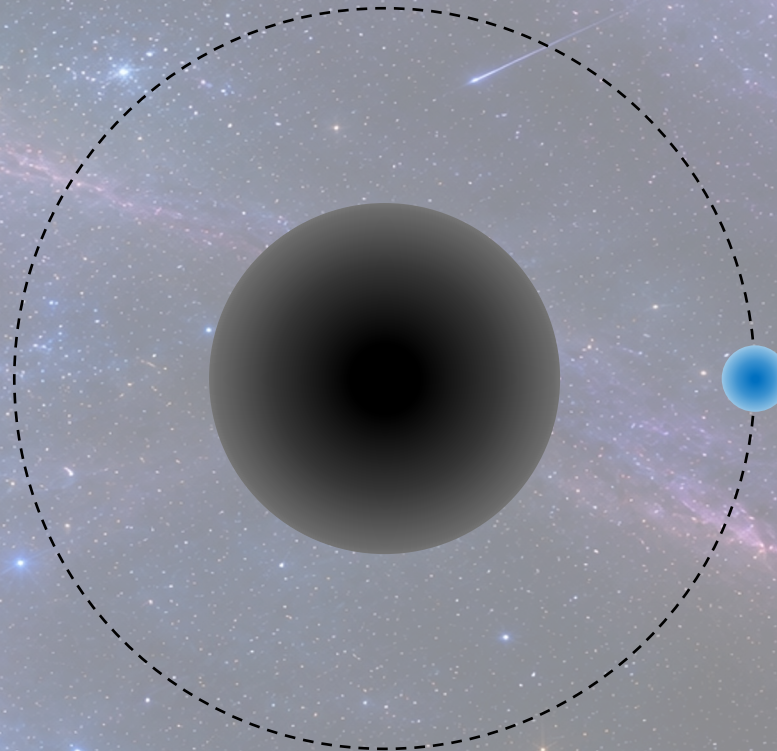
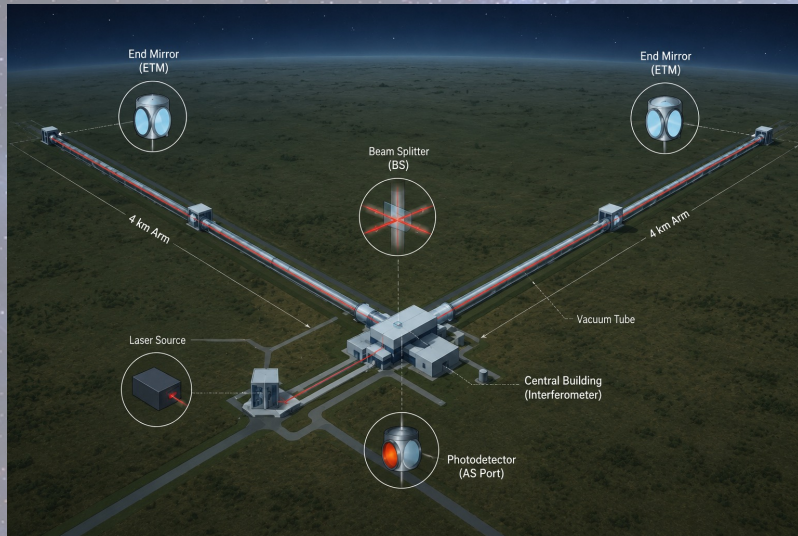
- Stellar black hole + subsolar companion
- $\mathcal{O}(10) - \mathcal{O}(10^3)$ Hz $\rightarrow$ ground-based detectors (LVK)
- Same physics, rescaled masses and frequencies



Guo & Miller [2205.10359]

To what precision can current ground-based gravitational-wave detectors measure the parameters of a mini-EMRI system — in particular, the mass of the subsolar-mass companion?

Fisher Information Matrix



- circular orbits
- non-spinning components,
- single detector (LIGO Hanford)
- directed search (known sky position)

$$\begin{aligned} m_1 &= 20 M_\odot \\ m_2 &= 10^{-4} M_\odot \\ e &= 0 \\ a &= 0 \end{aligned}$$

Waveform models and detector's response

Newtonian quadrupole approximation

- Leading quadrupole approximation — weak field, slow motion; good for the early inspiral.

$$h_+(t) = \frac{4}{r} \left(\frac{GM_c}{c^2} \right)^{5/3} \left(\frac{\pi f}{c} \right)^{2/3} \left(\frac{1+\cos^2 \theta}{2} \right) \cos \Phi$$

$$h_\times(t) = \frac{4}{r} \left(\frac{GM_c}{c^2} \right)^{5/3} \left(\frac{\pi f}{c} \right)^{2/3} \cos \theta \sin \Phi$$

$$f(t) = \frac{1}{\pi} \left(\frac{5}{256} \frac{1}{t_c - t} \right)^{3/8} \left(\frac{GM_c}{c^3} \right)^{-5/8}$$

$$\Phi(t) = -2 \left(\frac{5GM_c}{c^3} \right)^{-5/8} (t_c - t)^{5/8} + \Phi_0$$

Relativistically corrected (RC) Finn & Thorne, PRD [gr-qc/0007074]

- Strong-field physics from the Teukolsky–Sasaki–Nakamura formalism.
- Amplitude factor $H_{o,2}$ and frequency-evolution factor N encode the corrections.

$$h_{2+} = \frac{4}{r} \left(\frac{Gm_1^{2/5} m_2^{3/5}}{c^2} \right)^{5/3} \left(\frac{\pi f}{c} \right)^{2/3} \left(\frac{1+\cos^2 \theta}{2} \right) \cos \Phi \mathcal{H}_{o,2}$$

$$h_{2+} = \frac{4}{r} \left(\frac{Gm_1^{2/5} m_2^{3/5}}{c^2} \right)^{5/3} \left(\frac{\pi f}{c} \right)^{2/3} \cos \theta \sin \Phi \mathcal{H}_{o,2}$$

Waveform models and detector's response

FastEMRIWaveforms (FEW)

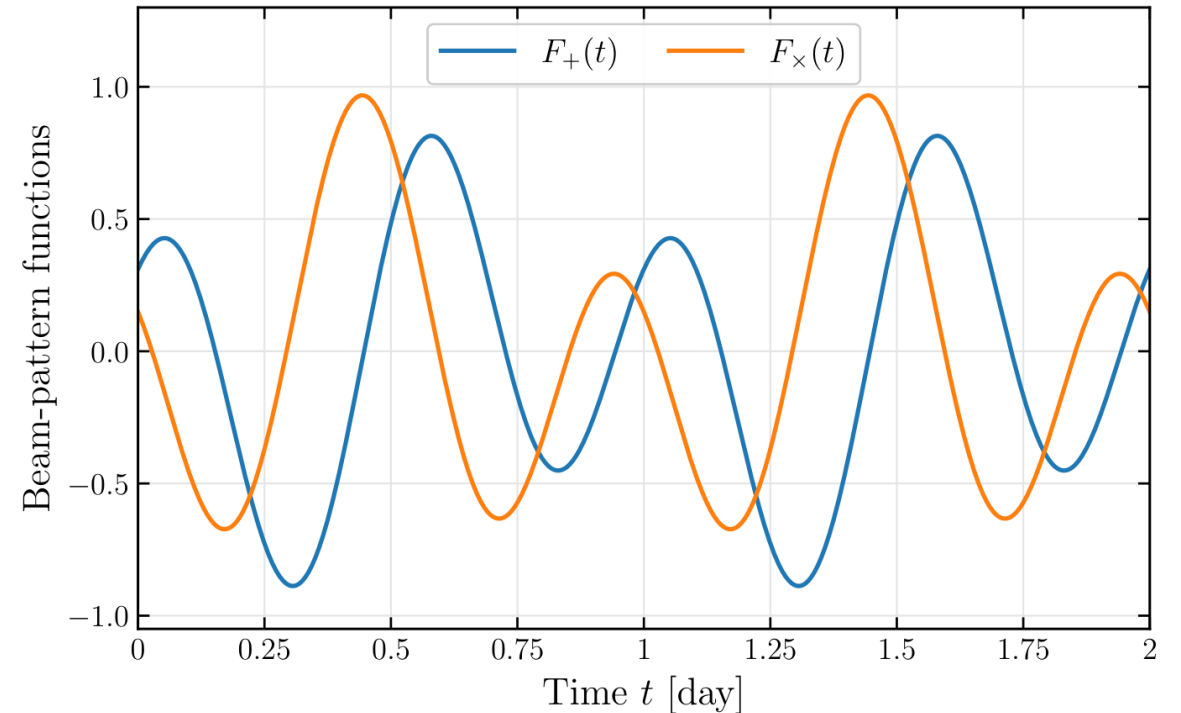
- Black-hole perturbation theory, mode-summed:
- Model: FastSchwarzschildEccentricFlux ($a = 0$)

$$h(t) = h_+ - i h_\times$$

$$h(t) = \frac{G\mu}{c^2 r} \sum_{lmn} A_{lmn}(t) {}_{-2}Y_{lm}(\theta, \phi) \exp[-i \Phi_{mn}(t)]$$

$$\Phi_{mn}(t) = m\Phi_\varphi(t) + n\Phi_r(t) \quad (n = 0 \text{ for circular orbit})$$

- Detector strain: $h(t) = F_+(t)h_+(t) + F_\times(t)h_\times(t)$
- The beam patterns F_+ , $F_\times$ are modulated by the Earth's diurnal rotation.
- **Essential for mini-EMRI signals lasting minutes to days.**

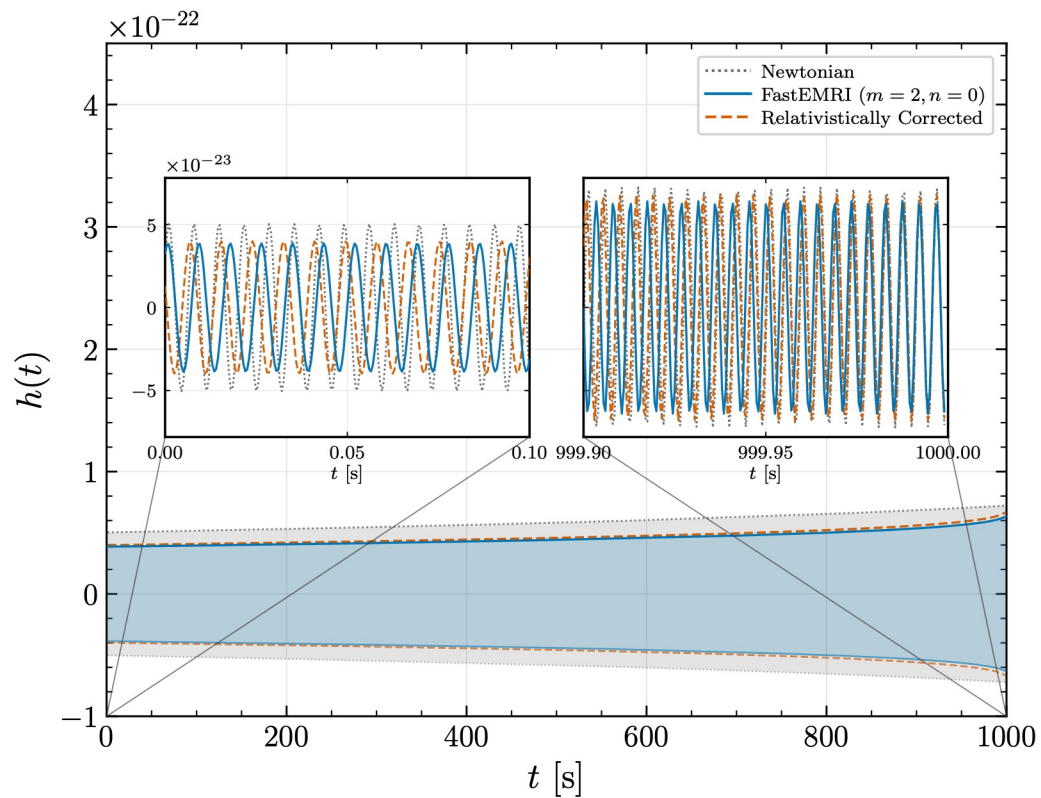


beam-pattern functions of LIGO Hanford over two sidereal days — diurnal antenna-pattern modulation.

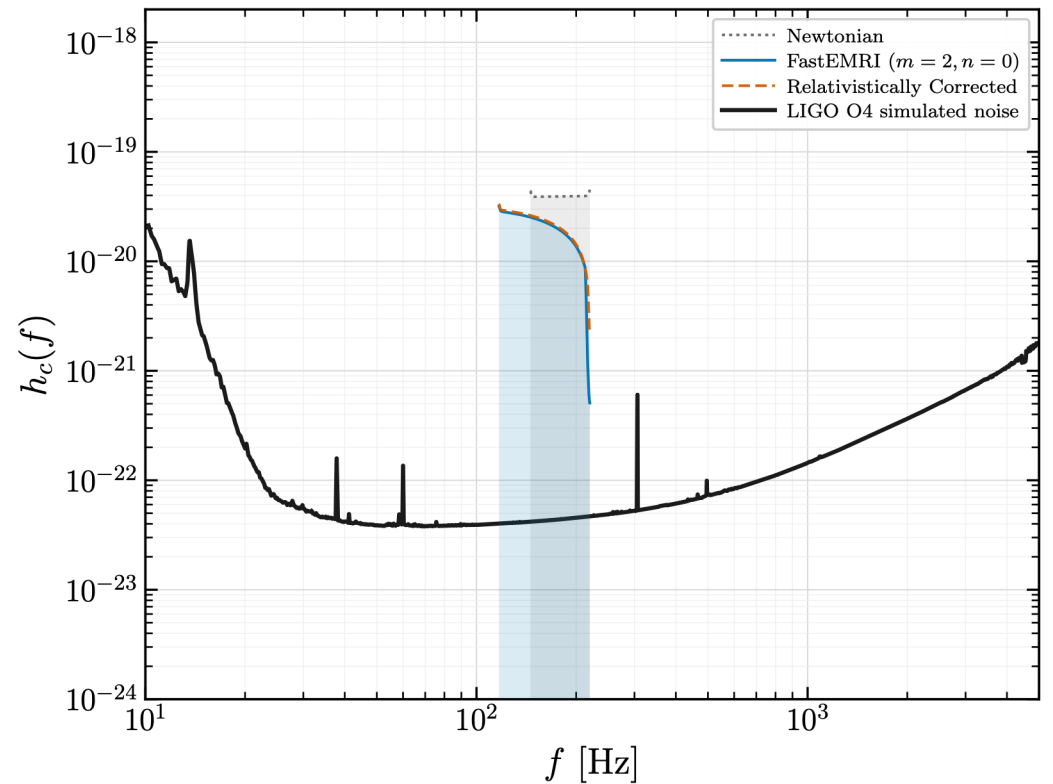
Model comparison and selection

Same system for all models: $m_1 = 20 M_\odot$, $m_2 = 10^{-4} M_\odot$, final 1000 s before the Innermost Stable Circular Orbit (ISCO), aligned at the ISCO.

RC tracks FEW closely; the Newtonian phase drifts away from both. **RC: $\iota - r$ degeneracy** The following results are obtained using **FastEMRI**



(a) Time-domain strains — full segment with zoom insets.



(b) Characteristic strains vs the Advanced LIGO O4 noise.

Parameter Estimation: the Fisher Information Matrix

- signal-to-noise ratio:

$$\text{SNR}^2 = 4 \int_0^\infty df \frac{|\tilde{h}(f)|^2}{S_n(f)}$$

- Inner product:

$$(A|B) = 4 \int_0^\infty df \frac{\tilde{A}^*(f) \tilde{B}(f)}{S_n(f)}$$

- Fisher matrix:

$$\Gamma_{ij} = \left(\frac{\partial h}{\partial \theta_i} \middle| \frac{\partial h}{\partial \theta_j} \right)$$

- Covariance matrix:

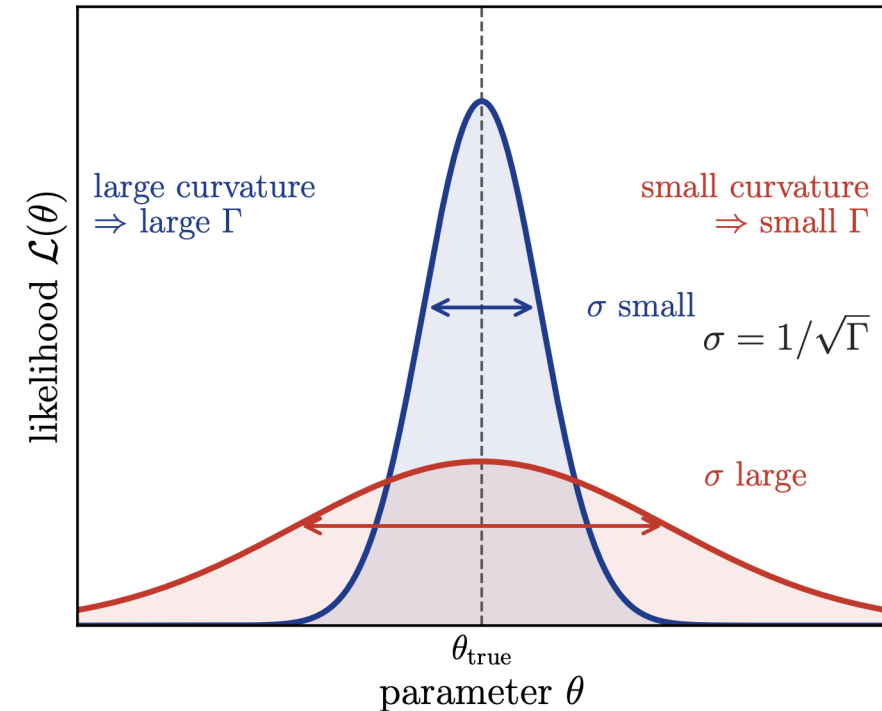
$$\Sigma_{ij} = (\Gamma^{-1})_{ij}$$

$$\Delta m_2 = (\Gamma^{-1})_{m_2 m_2}$$

- Parameters:

primary mass m_1 , secondary mass m_2 , distance r , initial phase Φ_0 , inclination angle ι , polarization angle ψ

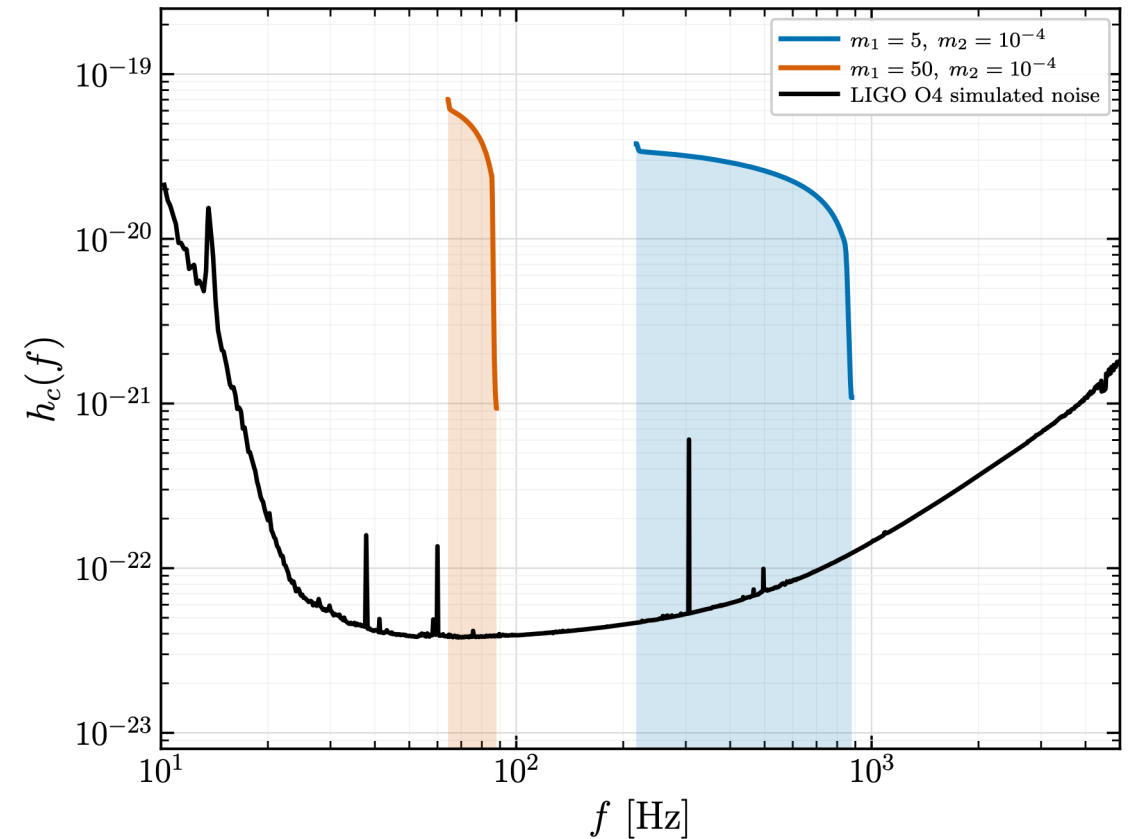
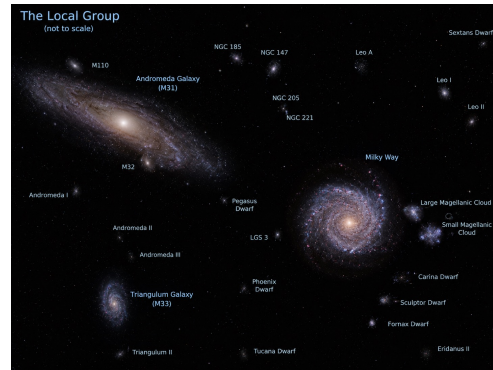
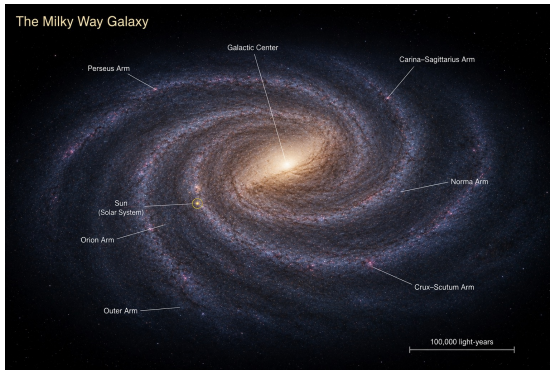
Fisher information = curvature of $\ln \mathcal{L}$ at the peak: $\Gamma_{ij} = -\langle \partial_i \partial_j \ln \mathcal{L} \rangle$



- Assumes high SNR, local linearity, broad priors.

Parameter space: masses, distance

- $m_1 \in [5, 50] M_\odot \rightarrow$ signals at $\approx 64 - 879$ Hz, inside LIGO's most sensitive band.
- $m_2 \in [10^{-5}, 10^{-3}] M_\odot$, subsolar mass
- Distances: from Galaxy up to Local Group ($\sim 10^6$ pc).
- Noise: public Advanced LIGO O4 sensitivity curve (LIGO DCC).



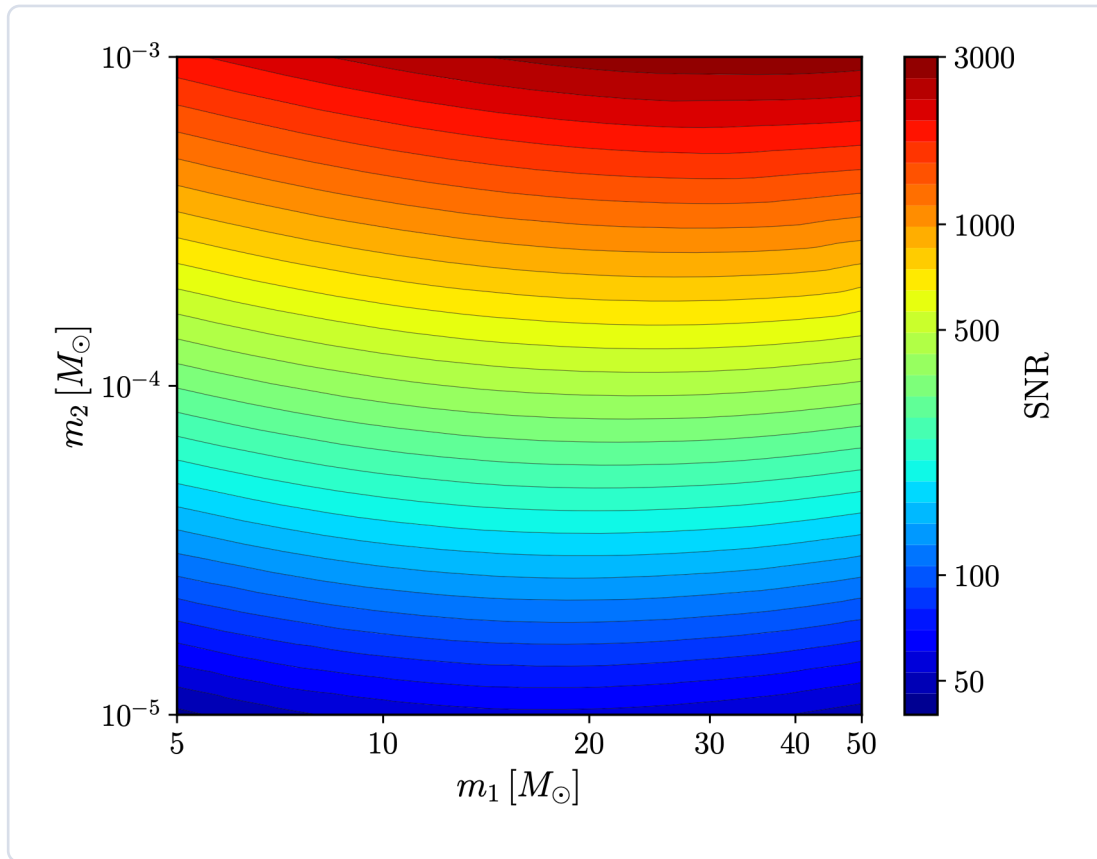
Characteristic strains: $m_1 = 5$ and $50 M_\odot$ ($m_2 = 10^{-4} M_\odot$, $r = 10^4$ pc), final 1000 s before ISCO.

SNR and $\Delta m_2/m_2$ across the mass plane

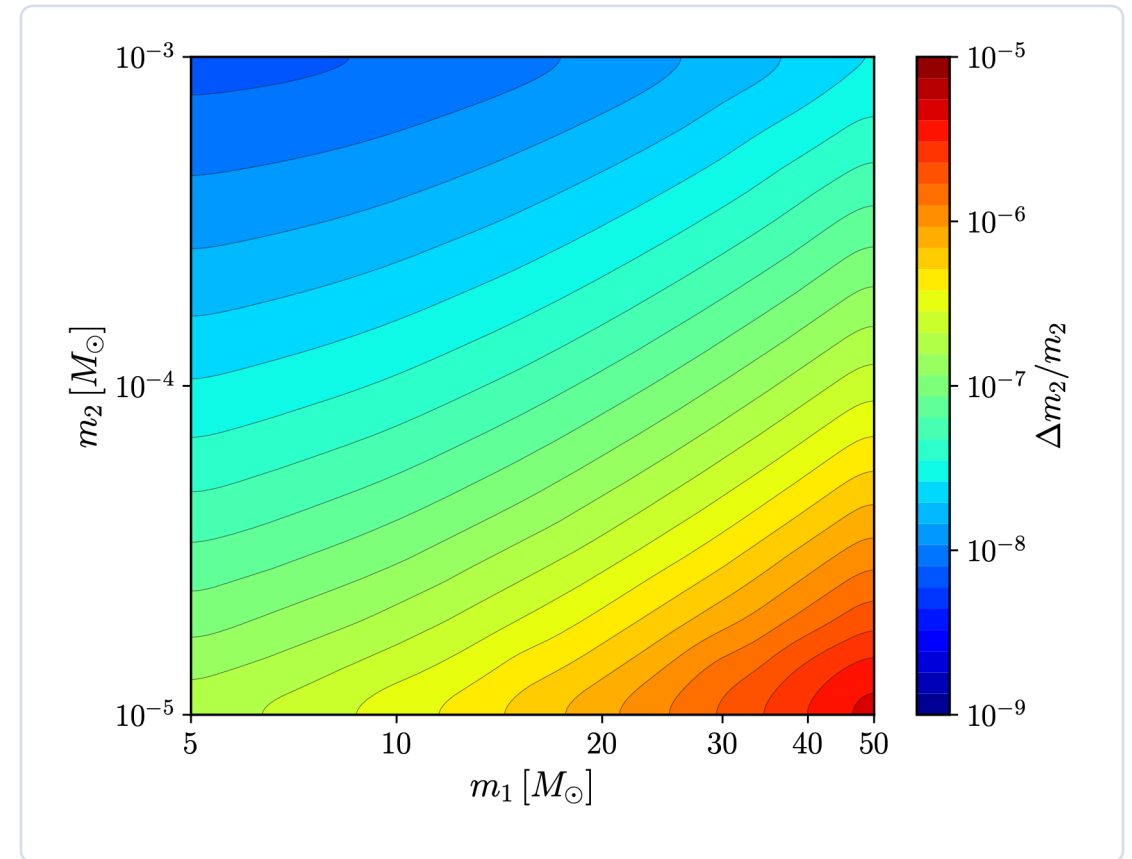
$$\frac{\Delta m_2}{m_2} \sim \frac{1}{\text{SNR} \cdot N_c}$$

$r = 10^4$ pc, $\text{SNR} > 40$ everywhere $\rightarrow$ the Fisher approach is justified.
Weak m_1 dependence (louder but narrower in band); SNR grows with m_2 .

Fractional uncertainty below 10^{-5} over all of the plane
 $\Delta m_2/m_2$ grows with m_1 (fewer cycles in band), falls with m_2 (louder).

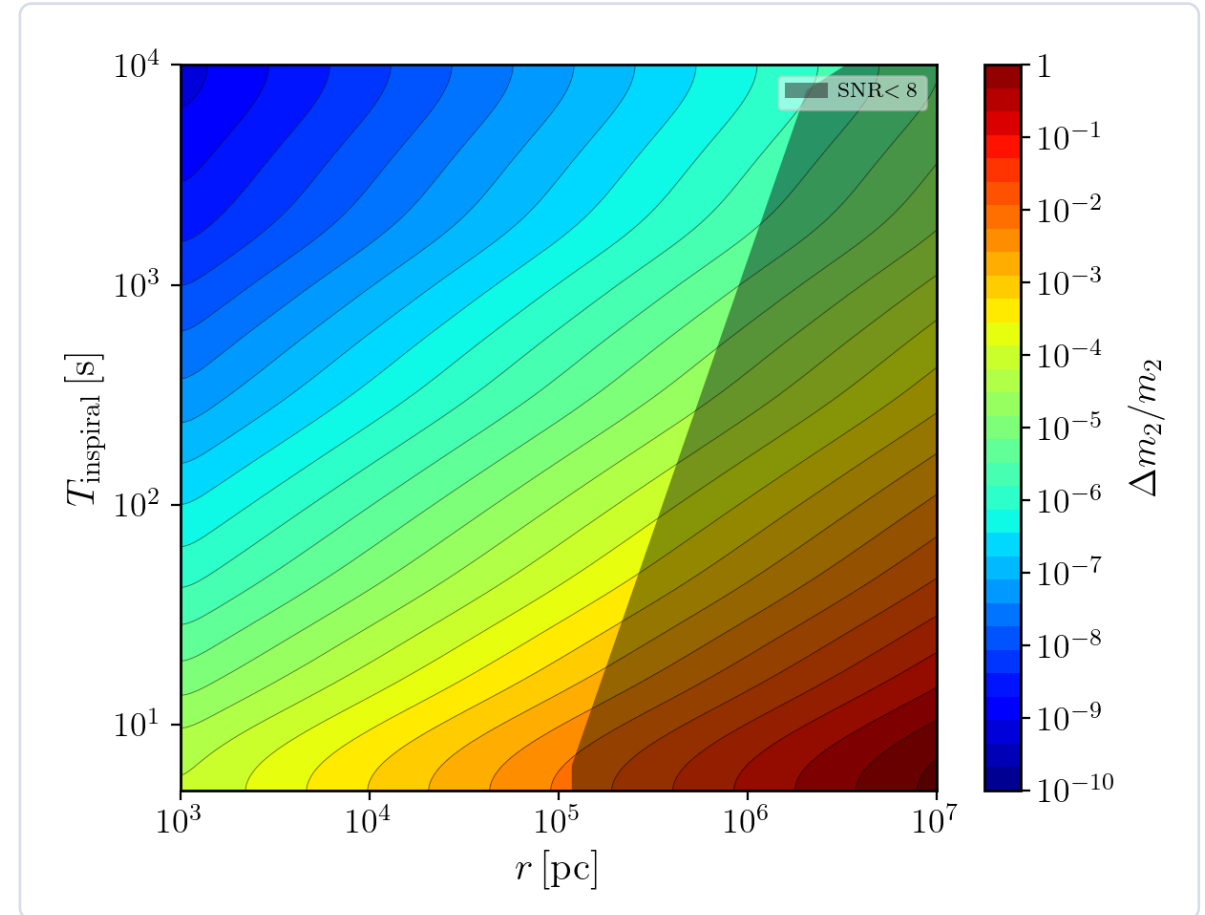


(a) SNR

(b) Fractional uncertainty in m_2

How far can we detect mini-EMRI?

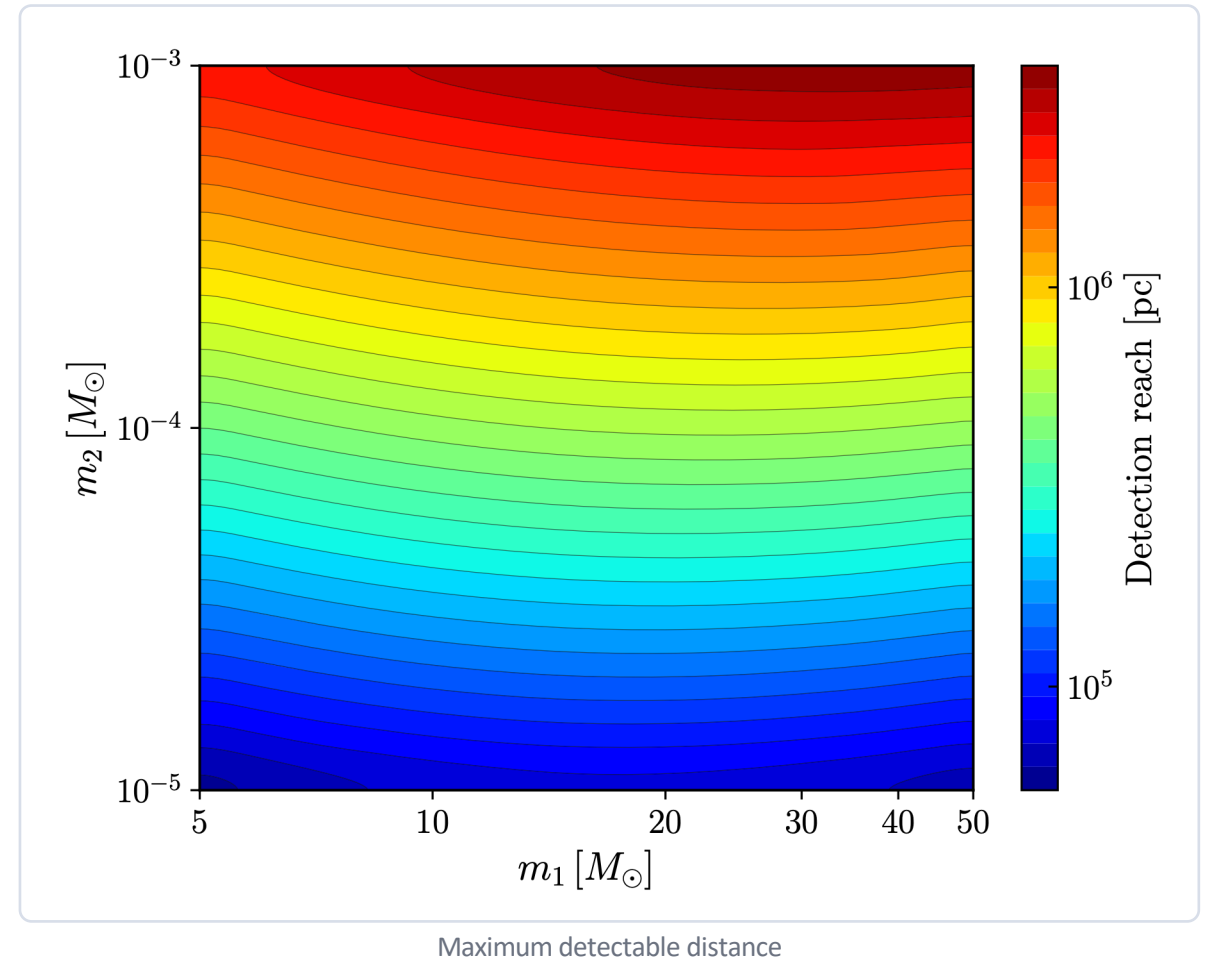
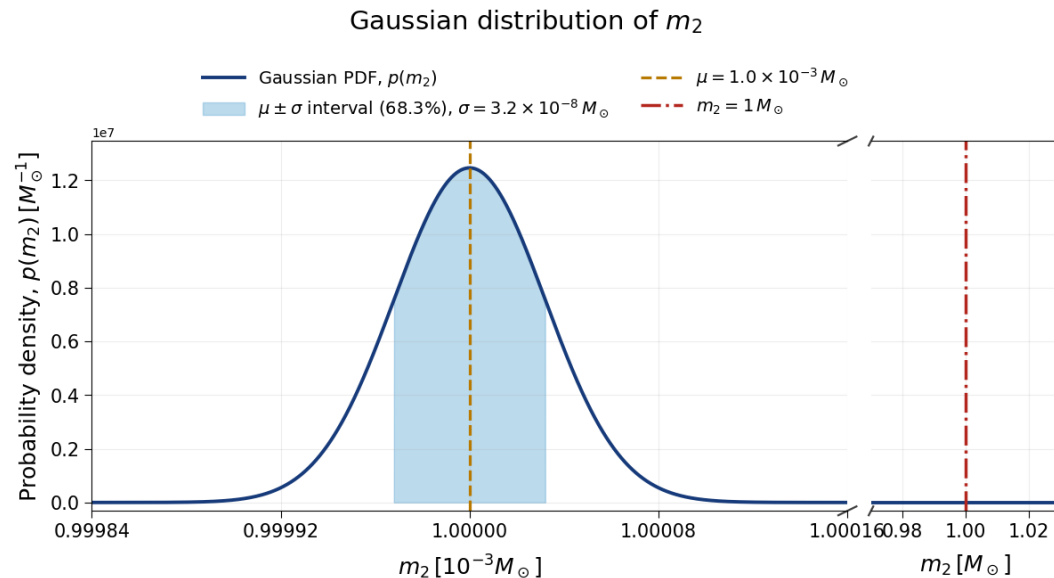
- Fix the system: $m_1 = 20M_\odot$, $m_2 = 10^{-4}M_\odot$; scan distance r and observation time T .
- $\Delta m_2/m_2$ grows with r (weaker signal) and falls with T (more cycles observed).
- Shaded region: $\text{SNR} < 8$ — not detectable, no estimation attempted.
- **Reading:** for a given observation time, the maximum distance at which the companion can still be confidently classified as subsolar.



$\Delta m_2/m_2$ vs distance and inspiral duration (FEW; $m_1 = 20 M_\odot$, $m_2 = 10^{-4} M_\odot$).

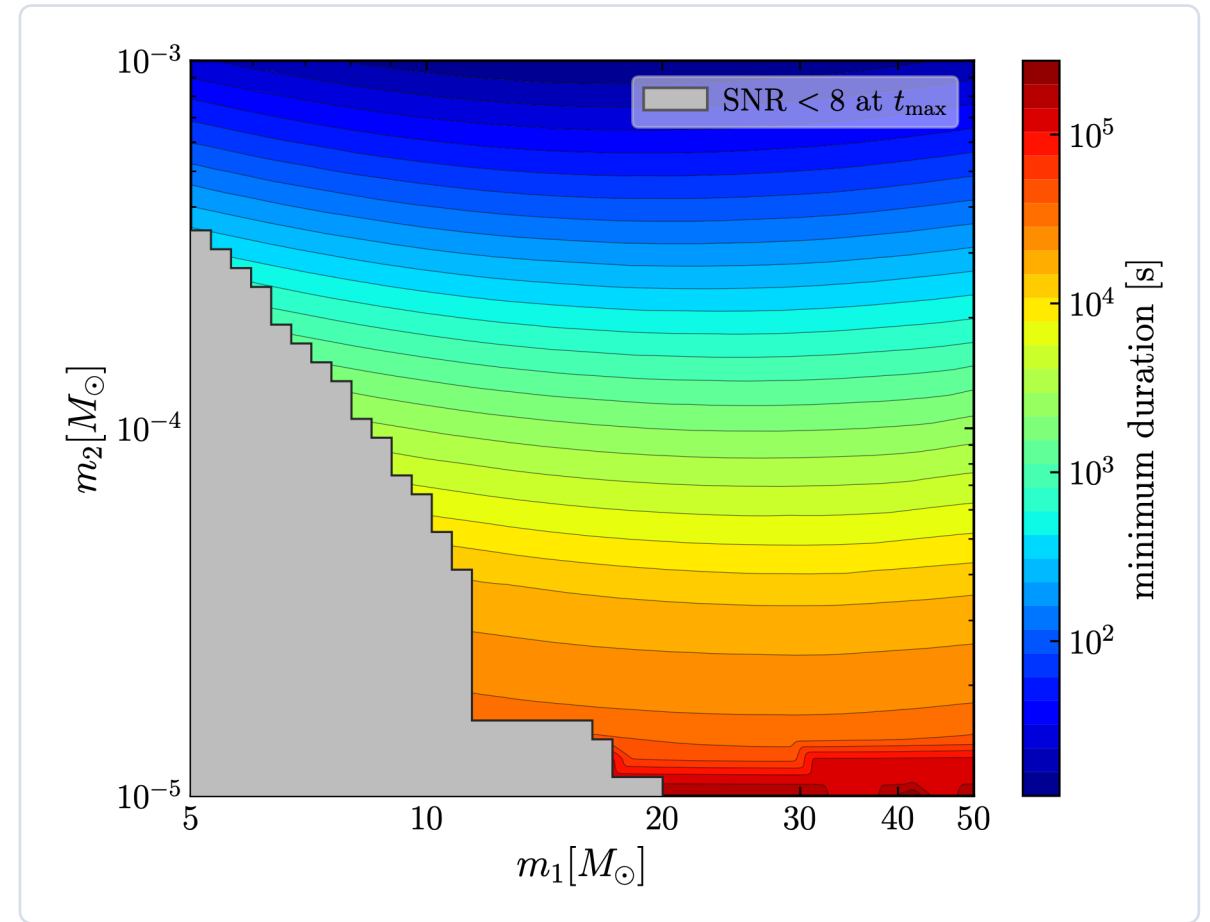
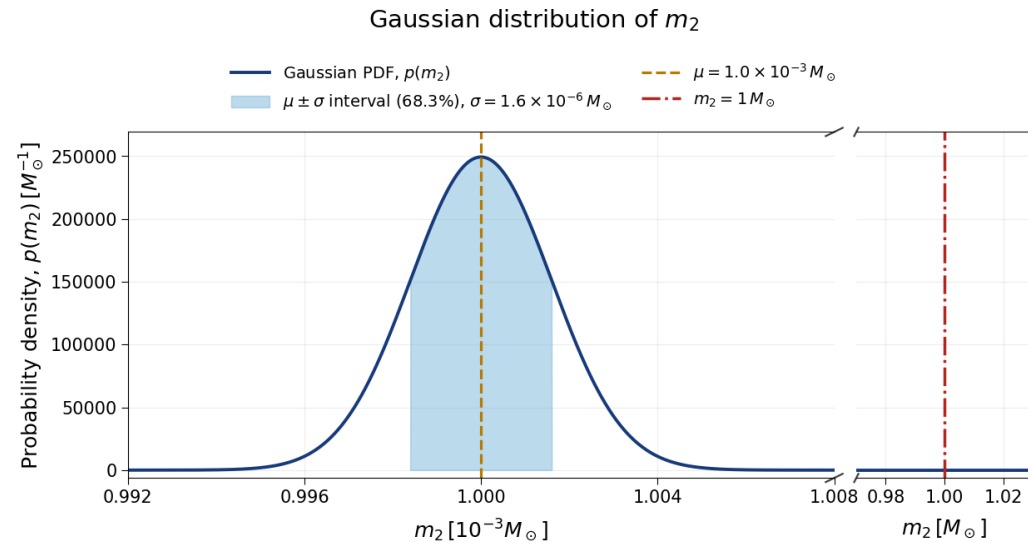
How far can we detect mini-EMRI?

- Detection threshold: $\text{SNR} = 8$. Final 1000 s before the ISCO.
- Detection reach grows strongly with m_2 and depends weakly on m_1 (driven by SNR).
- **Once the signal is detected, its secondary can already be confidently identified as a subsolar-mass object.**
- **Detectable out to Mpc.**



Time to identify a subsolar mass at Andromeda Galaxy

- At $r = 7.6 \times 10^5$ pc: minimum observation time such that $\text{SNR} = 8$
- Behaviour is SNR-driven — weak m_1 dependence, shorter times for larger m_2 .
- **Once the signal is detected, its secondary can already be confidently identified as a subsolar-mass object.**
- Grey: $\text{SNR} < 8$ even with the full inspiral (FEW domain $p_0 \leq 16$).



Minimum observation time for $\text{SNR} = 8$.

Summary & outlook

What we found

- Fisher-matrix forecast for mini-EMRIs
- The secondary mass is exceptionally well constrained: $\Delta m_2/m_2 \leq 10^{-5}$ at the Galactic distance for a 1000s observation.
- Detectability implies classifiability: once a signal reaches $\text{SNR} \geq 8$, the companion is immediately confirmed as subsolar (out to the Local Group).

Next steps

- Extend to the H1–L1 network (combined SNR).
- Include eccentricity and spins; .
- Goal: search-ready forecasts for upcoming mini-EMRI programs.

A composite image featuring a black hole, a solar system, and a particle accelerator. The background is a deep space scene with the Milky Way galaxy visible in the upper left. In the center, a large black hole with a glowing orange accretion disk is shown. To the right, a solar system with a yellow star and a blue planet is depicted with concentric orbital lines. In the bottom left, a particle accelerator facility with a long tunnel and a building is shown. The text "Thank you" is centered over the black hole.

Thank you