

# Multimessenger Tests of the Strong Equivalence Principle and Hidden Extra Dimensions with Gravitational-Wave Observations

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**Song**, Du<sup>†</sup>, and Xu<sup>†</sup>, *arXiv: 2609.xxxxx*

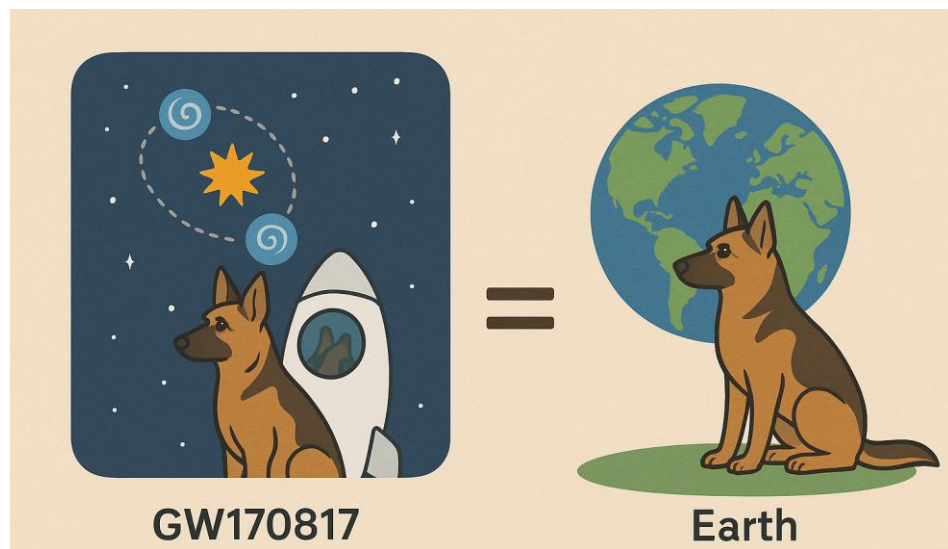
Collaborate with: Minghui Du<sup>†</sup> and Peng Xu<sup>†</sup>

**Zhu**<sup>\*</sup>, **Song**<sup>\*†</sup>, Lyu<sup>†</sup>, et. al, *arXiv: 2603.25413*

Collaborate with: Jie Zhu<sup>\*</sup>, Zhenwei Lyu<sup>†</sup>, Hao Li<sup>†</sup>, Peixiang Ji, Jun-Chen Wang, Haobo Yan, and Bo-Qiang Ma<sup>†</sup>

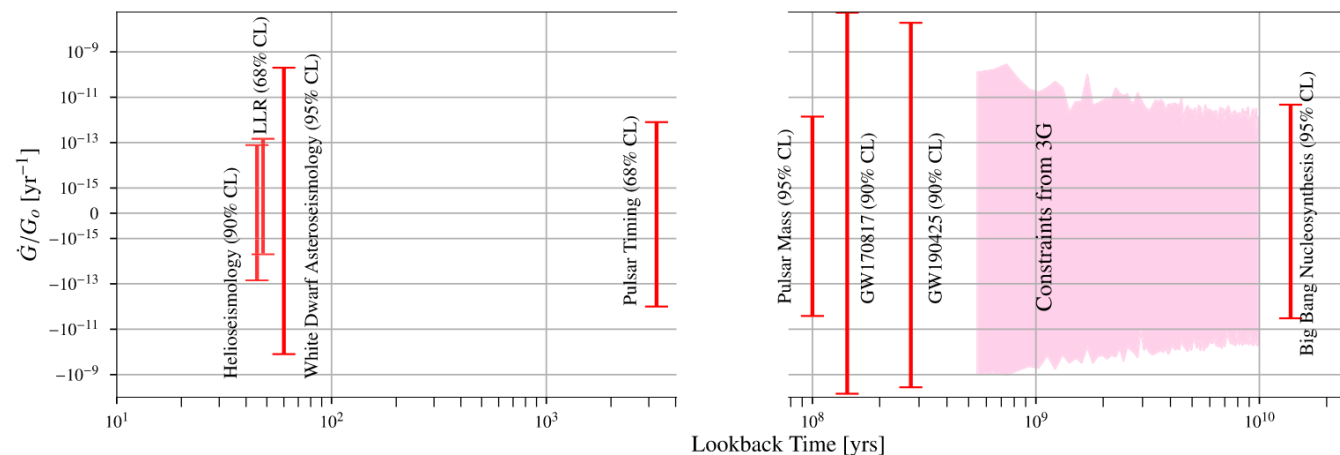
# Strong equivalence principle & varying-G

Vijaykumar et. al, 2021, *Phys. Rev. Lett.*



@OpenAI

## Experiment constraints



## Equivalence principles

- **Weak:** universality of free fall for test bodies (gravitational mass = inertial mass)
- **Einstein's:** WEP + local Lorentz invariance (LLI) + local position invariance (LPI) for non-gravitational experiments
- **Strong:** EEP + gravitational experiment LLI & LPI + applies to strongly self-gravitating bodies

- Helioseismology
- Lunar laser ranging
- Pulsar timing
- Pulsar mass
- Gravitational wave
- Big Bang Nucleosynthesis

**Varying-G violate the gravitational LPI, thus violate the SEP.**

# Varying-G's physical effects I: source frame neutron star

Sensitivity as the benchmark of non-GR effects

$$\frac{d}{dt}(m\mathbf{v}) = 0 = \dot{m}\mathbf{v} + m\frac{d}{dt}\mathbf{v} \quad \frac{d\mathbf{v}}{dt} \cong -\frac{\dot{m}}{m}\mathbf{v} = -\left[\frac{G}{m}\frac{\delta m}{\delta G}\right]\frac{\dot{G}}{G}\mathbf{v}$$

$$s = -\frac{G}{m}\frac{\delta m}{\delta G} = -\frac{\delta \ln m}{\delta \ln G}$$

Tolman-Oppenheimer-Volkoff equations

$$\begin{aligned} \frac{dm(r)}{dr} &= 4\pi r^2 \rho(r) \\ \frac{d\bar{m}_b(r)}{dr} &= 4\pi r^2 \epsilon_b(r) \left(1 - \frac{2Gm(r)}{r}\right)^{-1/2} \\ \frac{dP(r)}{dr} &= -\frac{Gm(r)\rho(r)}{r^2} \left(1 + \frac{P(r)}{\rho(r)}\right) \\ &\quad \times \left(1 + \frac{4\pi r^3 P(r)}{m(r)}\right) \left(1 - \frac{2Gm(r)}{r}\right)^{-1} \end{aligned}$$

G-free TOV equations via redefinition

$$\begin{aligned} \frac{dm^*}{dr^*} &= 4\pi r^{*2} \rho \\ \frac{d\bar{m}_b^*}{dr^*} &= 4\pi r^{*2} \epsilon_b \left(1 - \frac{2m^*}{r^*}\right)^{-1/2} \\ \frac{dP}{dr^*} &= -\frac{m^* \rho}{r^{*2}} \left(1 + \frac{P}{\rho}\right) \\ &\quad \times \left(1 + \frac{4\pi r^{*3} P}{m^*}\right) \left(1 - \frac{2m^*}{r^*}\right)^{-1} \end{aligned}$$

$m = G^{-\frac{3}{2}} m^*$   
 $\bar{m}_b = G^{-\frac{3}{2}} \bar{m}_b^*$   
 $r = G^{-\frac{1}{2}} r^*$

**Baryon mass is not effects when G varying!!!**

Numerically solve the relation between baryon mass and gravitational mass with given EOS

$$M^* = m^*(R^*) \quad \bar{M}^* = \bar{m}_b^*(R^*) \quad f : M^* \mapsto \bar{M}^* \quad g : \bar{M}^* \mapsto M^*$$

$$\bar{M} = G^{-3/2} \bar{M}^* = G^{-3/2} f(G^{3/2} M)$$

When G is varying

$$G_1 = G + \delta G \quad \bar{M} = \bar{M}_1 = G^{-3/2} \bar{M}^* = G_1^{-3/2} \bar{M}_1^*$$

$$\bar{M}_1^* = G_1^{3/2} \bar{M} = G_1^{3/2} G^{-3/2} f(G^{3/2} M)$$

$$M_1^* = g(G_1^{3/2} G^{-3/2} f(G^{3/2} M))$$

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$$M_1 = G_1^{-\frac{3}{2}} M_1^*$$

$$M_1 = G_1^{-3/2} g \left( G_1^{3/2} G^{-3/2} f(G^{3/2} M) \right)$$

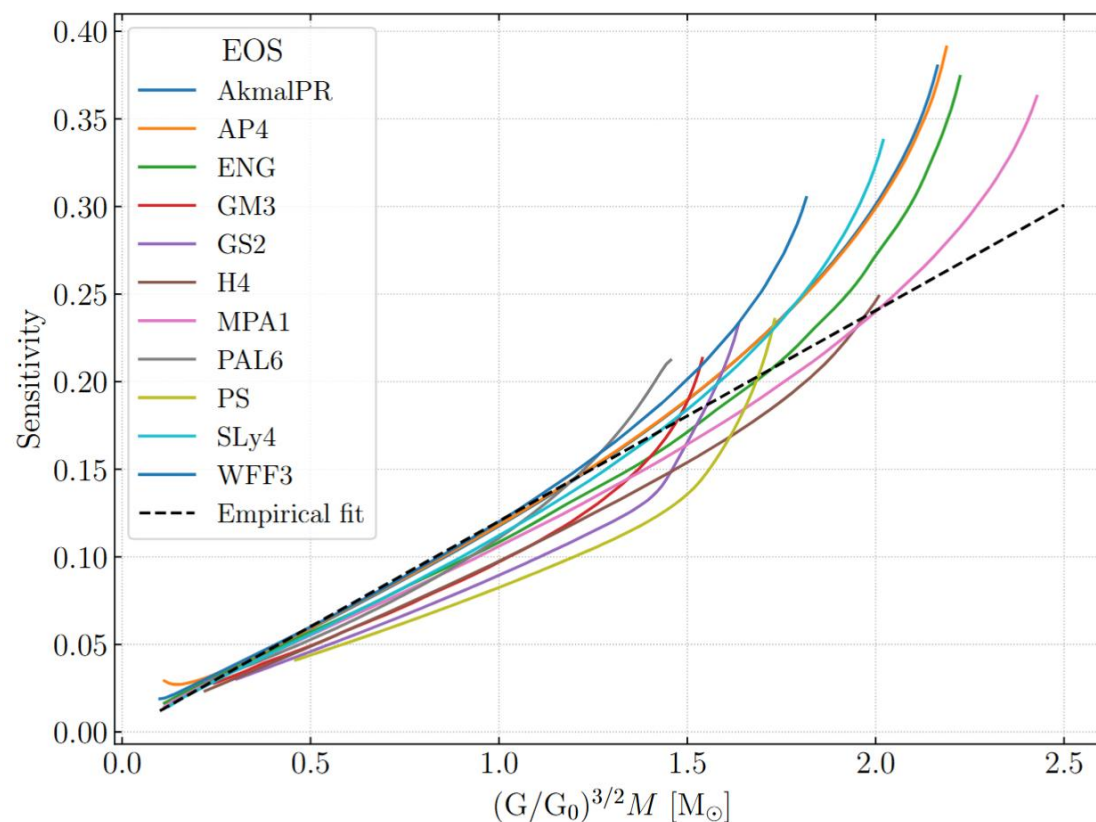
Finally, we obtain the sensitivity

$$\begin{aligned} s &= -\frac{G}{M} \lim_{\delta G \rightarrow 0} \frac{M_1 - M}{\delta G} \\ &= \frac{3}{2} \left( 1 - \frac{1}{G^{3/2} M} f(G^{3/2} M) g' \left( f(G^{3/2} M) \right) \right) \end{aligned}$$

# Varying-G's physical effects I: source frame neutron star

$$s = -\frac{G}{M} \lim_{\delta G \rightarrow 0} \frac{M_1 - M}{\delta G} = \frac{3}{2} \left( 1 - \frac{1}{G^{3/2} M} f(G^{3/2} M) g' \left( f(G^{3/2} M) \right) \right)$$

Neutron star sensitivities with different EOS



Zhu\*, Song<sup>\*†</sup>, Lyu<sup>†</sup>, et. al, [arXiv: 2603.25413](https://arxiv.org/abs/2603.25413)

Empirical results are based on the observation of PSR J1713+0747

# Varying-G's physical effects II: binary dynamics

Nordtvedt, 1990, *Phys. Rev. Lett.*

## Varying G induced anomalous acceleration

$$\frac{d}{dt}(m\mathbf{v}) = 0 = \dot{m}\mathbf{v} + m\frac{d}{dt}\mathbf{v}$$

$$\frac{d\mathbf{v}}{dt} \cong -\frac{\dot{m}}{m}\mathbf{v} = -\left[\frac{G}{m}\frac{\delta m}{\delta G}\right]\frac{\dot{G}}{G}\mathbf{v}$$

$$s = -\frac{G}{m}\frac{\delta m}{\delta G} = -\frac{\delta \ln m}{\delta \ln G}$$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = s\frac{\dot{G}}{G}\mathbf{v}$$

## Binary system orbital evolution

$$\dot{r} = \dot{r}_{\text{GW}} + \dot{r}_G$$

$$\dot{r}_{\text{GW}} = -\frac{64}{5}\frac{G^3\mu M^2}{r^3}$$

$$\dot{r}_G = -\left(\frac{\dot{G}}{G} + \frac{\dot{M}}{M} - 2\frac{\dot{j}}{j}\right)r$$

## Varying G effects on total mass and angular momentum of binary system

$$\dot{M} = -\frac{m_1 s_1 + m_2 s_2}{m_1 + m_2}\frac{\dot{G}}{G}M$$

$$\dot{j} = \frac{m_1 s_1 + m_2 s_2}{m_1 + m_2}\frac{\dot{G}}{G}j$$

## ppE parameters

$$a = -8$$

$$\alpha = \frac{5}{512}\eta^{3/5}\frac{\dot{G}}{G}[-7GM + (s_1 + s_2)GM] + 13(Gm_1 s_1 + Gm_2 s_2),$$

$$b = -13$$

$$\beta = -\frac{25}{851968}\eta^{3/5}\frac{\dot{G}}{G}[11GM + 3(s_1 + s_2)GM - 41(Gm_1 s_1 + Gm_2 s_2)]$$

# Varying-G's physical effects III: propagation in FLRW universe

Zhu\*, Song<sup>\*†</sup>, Lyu<sup>†</sup>, et. al, *arXiv: 2603.25413*

Full equation when G is varying

$$R_{\mu\nu} - \frac{2(\nabla_\mu G)(\nabla_\nu G)}{G^2} + \frac{\nabla_\nu \nabla_\mu G}{G} + \frac{g_{\mu\nu}(\nabla_\rho \nabla^\rho G)}{2G} - \frac{g_{\mu\nu}(\nabla_\rho G)(\nabla^\rho G)}{G^2} = 0$$

The linear order perturbation of FLRW metric

$$\begin{aligned} g_{\mu\nu} &= \text{diag}(-a(\eta), a(\eta), a(\eta), a(\eta)) \\ 0 &= -\frac{1}{2}\nabla_\nu \nabla_\mu h^\rho{}_\rho + \frac{1}{2}\nabla_\rho \nabla_\mu h_\nu{}^\rho + \frac{1}{2}\nabla_\rho \nabla_\nu h_\mu{}^\rho \\ &\quad - \frac{1}{2}\nabla_\rho \nabla^\rho h_{\mu\nu} + \frac{h_{\mu\nu}\nabla_\rho \nabla^\rho G}{2G} - \frac{\nabla_\mu h_{\nu\rho}\nabla^\rho G}{2G} \\ &\quad - \frac{\nabla_\nu h_{\mu\rho}\nabla^\rho G}{2G} + \frac{\nabla_\rho h_{\mu\nu}\nabla^\rho G}{2G} + \frac{g_{\mu\nu}\nabla_\rho h^\sigma{}_\sigma \nabla^\rho G}{4G} \\ &\quad - \frac{h_{\mu\nu}\nabla_\rho G \nabla^\rho G}{G^2} - \frac{g_{\mu\nu}\nabla^\rho G \nabla_\sigma h^\sigma{}_\rho}{2G} \\ &\quad + \frac{h_{\rho\sigma}g_{\mu\nu}\nabla^\rho G \nabla^\sigma G}{G^2} - \frac{h_{\rho\sigma}g_{\mu\nu}\nabla^\sigma \nabla^\rho G}{2G} \end{aligned}$$

TT gauge

$$\begin{aligned} h_{11} &= -h_{22} = a(\eta)h_+(\eta, z) \\ h_{12} &= h_{21} = a(\eta)h_\times(\eta, z) \end{aligned}$$

The reduced perturbation equation

$$\partial_\eta^2 h + \left(\frac{2a'}{a} - \frac{G'}{G}\right) \partial_\eta h - \partial_z^2 h + \left(\frac{2a''}{a} + \frac{2a'^2}{a^2} - \frac{4a'G'}{aG} - \frac{G''}{G} + \frac{2G'^2}{G^2}\right) h = 0$$

The plain wave approximation

$$h(\eta, z) = A(\eta)e^{ikz}$$

$$\begin{aligned} A'' + \left(\frac{2a'}{a} - \frac{G'}{G}\right) A' \\ + \left(k^2 + \frac{2a''}{a} + \frac{2a'^2}{a^2} - \frac{4a'G'}{aG} - \frac{G''}{G} + \frac{2G'^2}{G^2}\right) A = 0. \end{aligned}$$

$$h = \frac{\sqrt{G(\eta)}}{a(\eta)} \exp(ik(z \pm \eta))$$



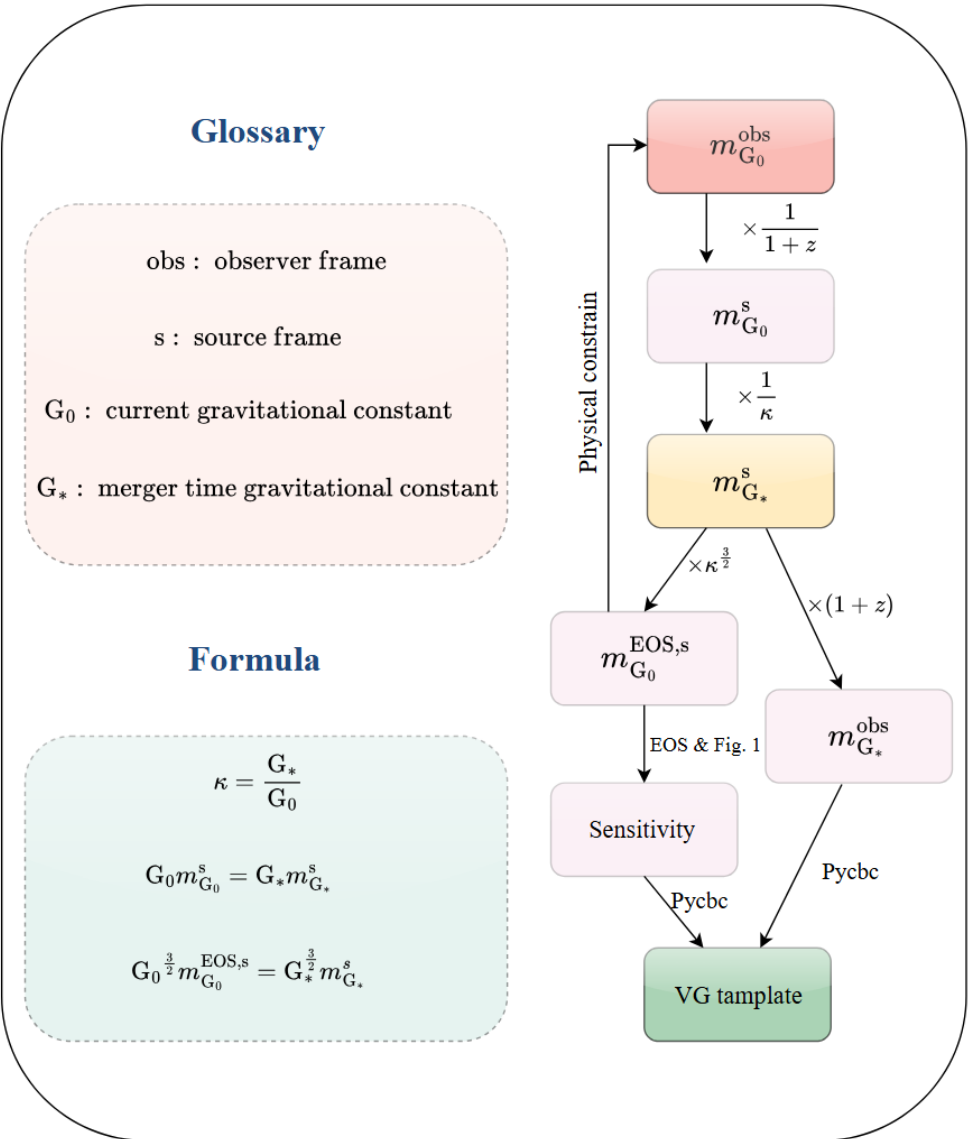
$$A_s a(\eta_s) / \sqrt{G_s} = A_o a(\eta_o) / \sqrt{G_o} \quad \longrightarrow \quad A_o = \frac{a(\eta_s)}{a(\eta_o)} \sqrt{\frac{G_o}{G_s}} A_s$$

The amplitude correction factor

$$h_o \propto \sqrt{\frac{G_o}{G_s}} h_s$$

# Full GW waveform of varying-G theory

Zhu\*, Song<sup>\*†</sup>, Lyu<sup>†</sup>, et. al, [arXiv: 2603.25413](#)



## The full waveform

$$\tilde{h}_{\text{VG}}(f) = \frac{1}{\sqrt{\kappa}} \tilde{h}_{\text{GR}} e^{i\beta_{\text{VG}} u^{b_{\text{VG}}}}$$

$$b_{\text{VG}} = -13$$

$$\beta_{\text{VG}} = -\frac{25}{851968} \eta^{3/5} \frac{(1-\kappa)}{T} [11 M_{G_*}^{\text{obs}} + 3(s_1 + s_2) M_{G_*}^{\text{obs}} - 41(m_{1,G_*}^{\text{obs}} s_1 + m_{2,G_*}^{\text{obs}} s_2)]$$

## The physical constraint from EOS

$$m_{G_0}^{\text{EOS},s} \in [m_{G_0,\text{min}}^{\text{EOS},s}, m_{G_0,\text{max}}^{\text{EOS},s}]$$

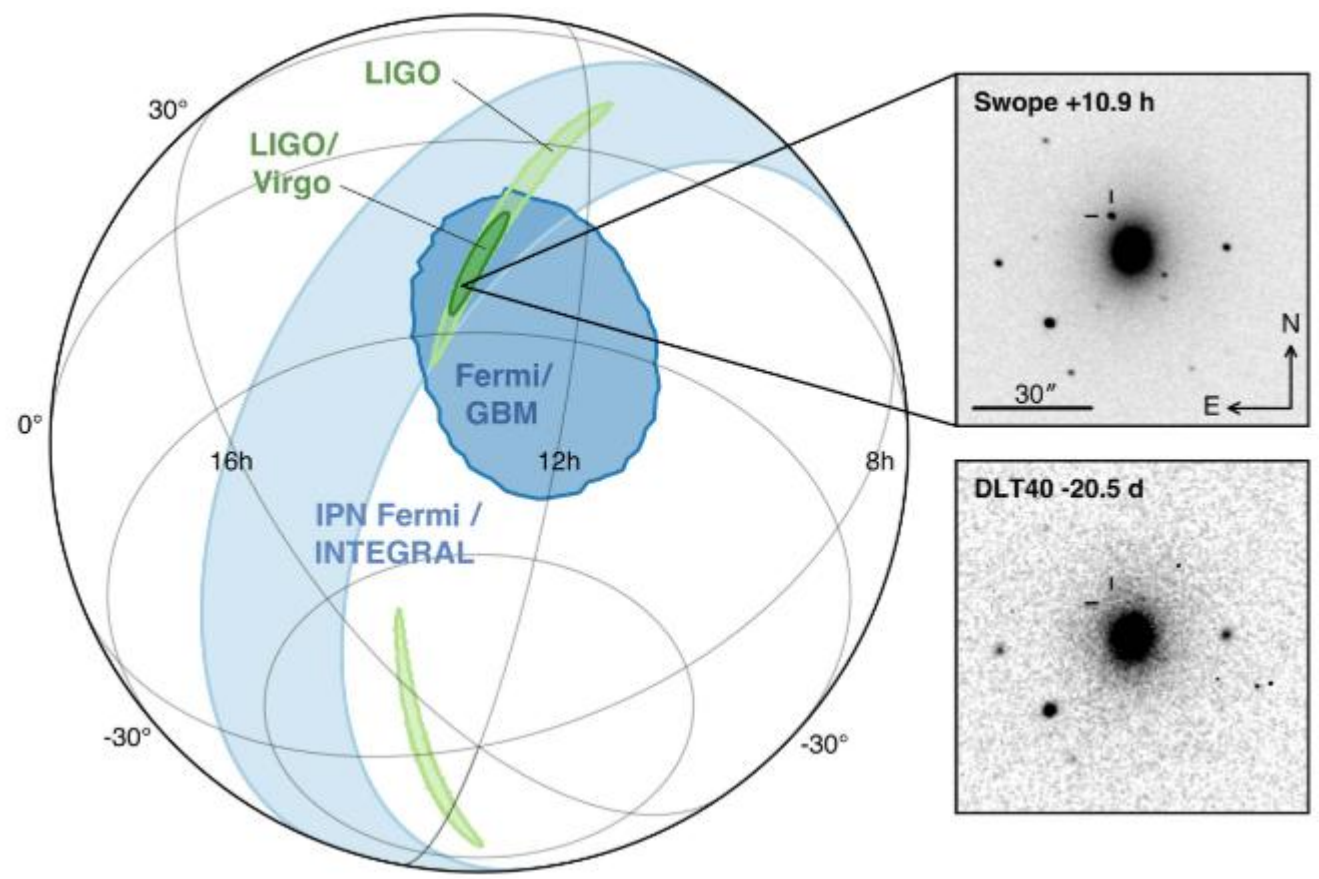
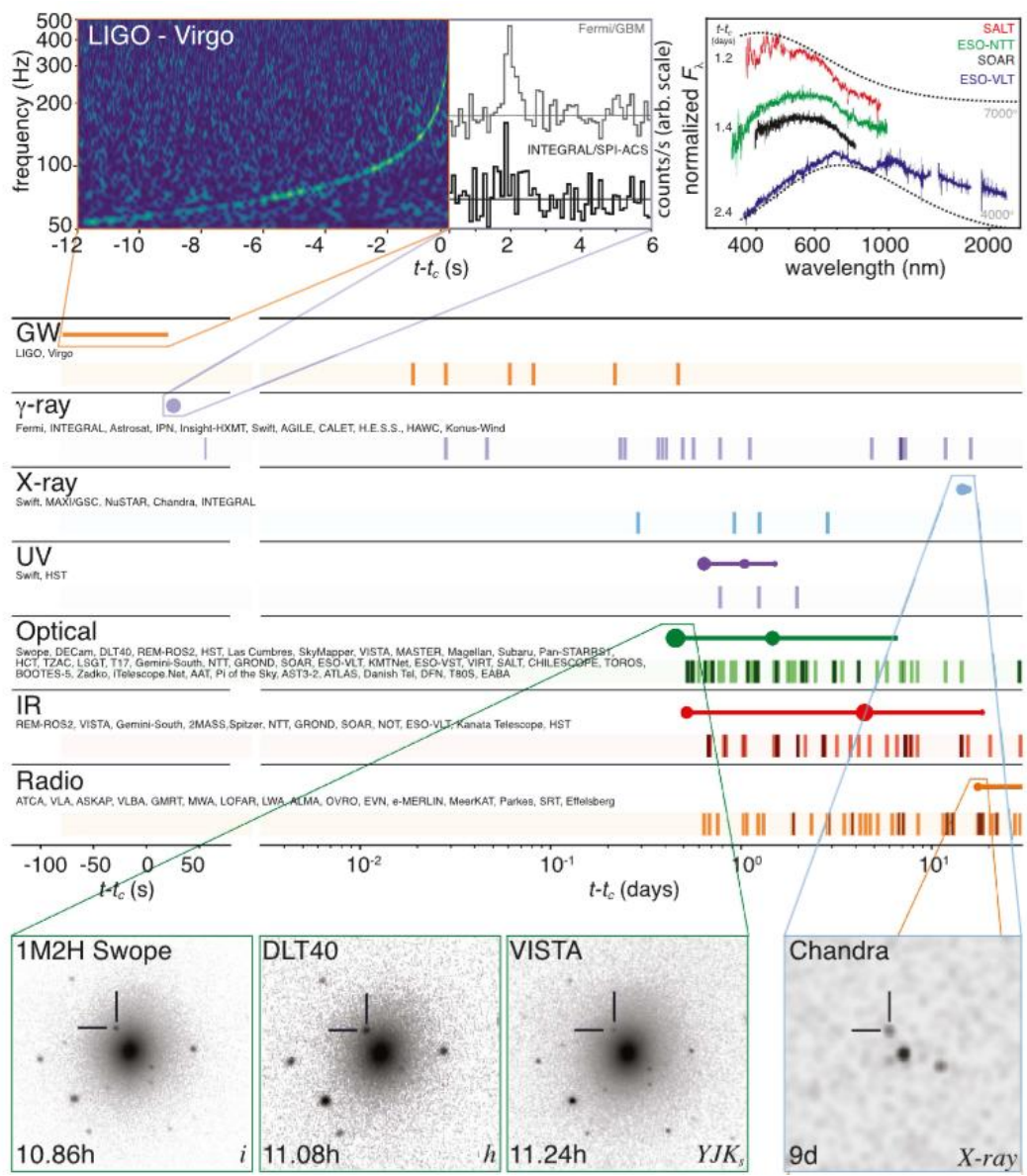
$$m_{G_0}^{\text{“H4”},s} \in [0.008, 2.01] \text{ M}_{\odot}$$

**Need to solve serious degeneracy among  $\kappa$ ,  $D_L$ , and  $\iota$  !!!**

Fortunately, we have a valuable multi-messenger GW event: GW170817

# Multimessenger observation of GW170817

Abbott et. al, 2017, *Astrophys. Lett.*



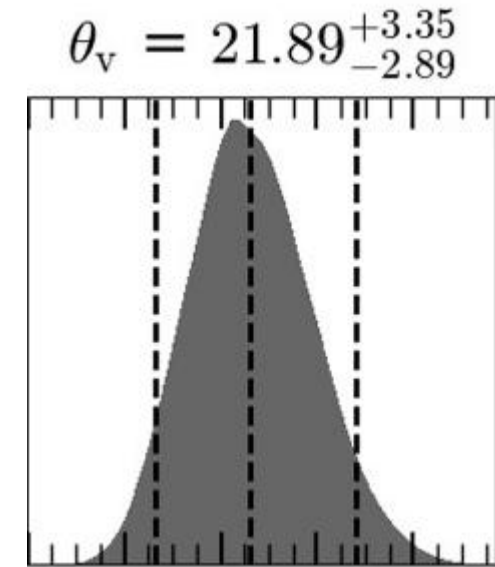
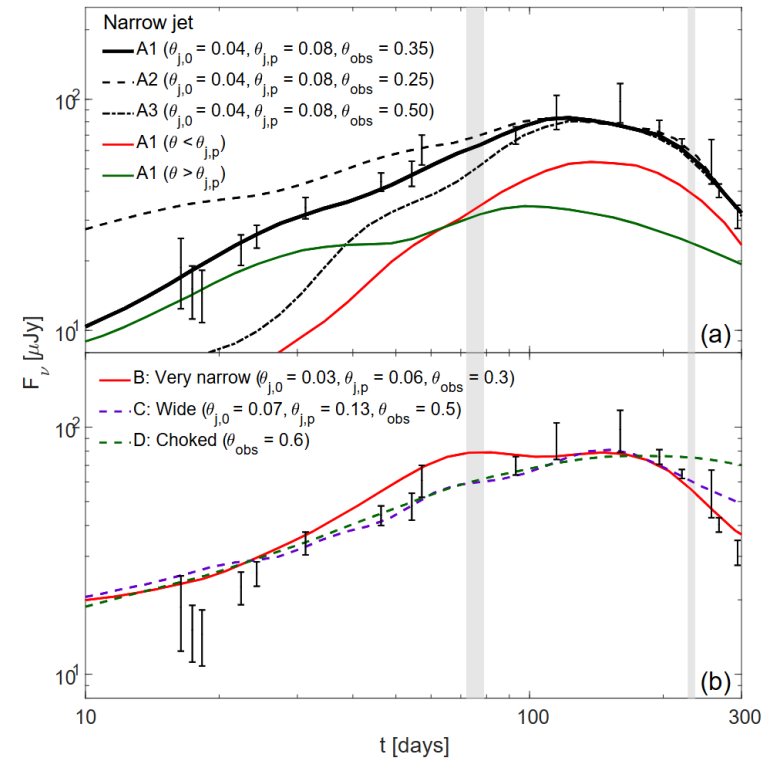
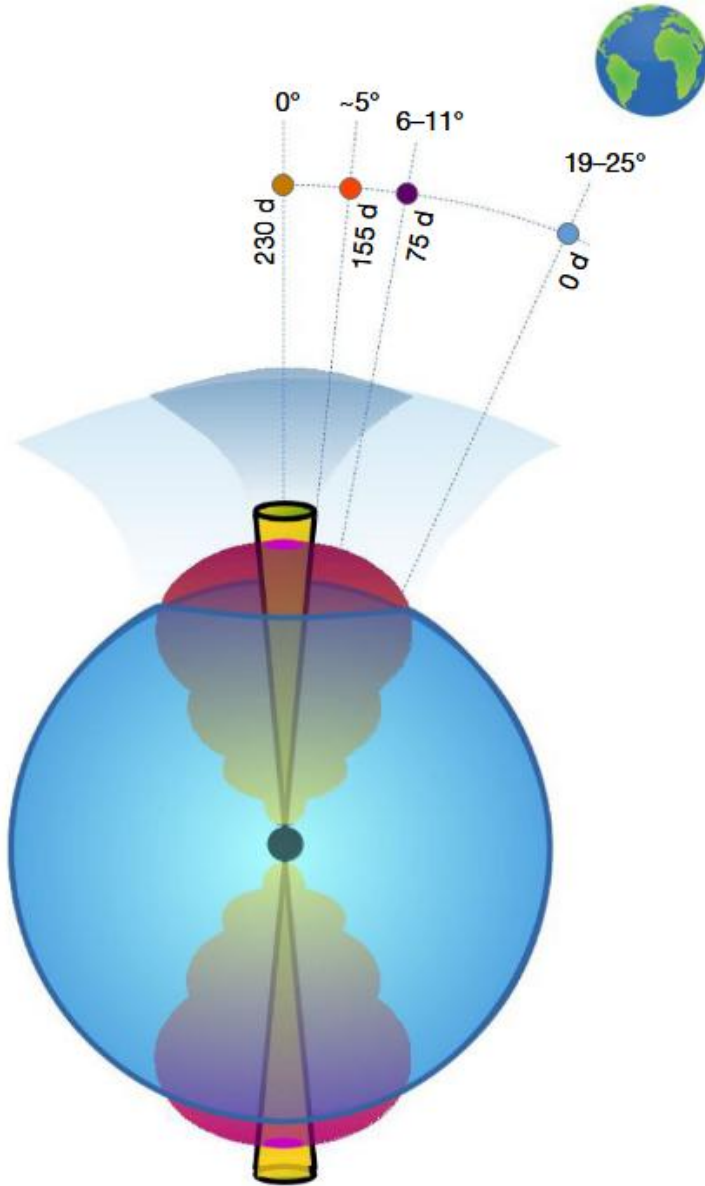
We can fix the prior of  $D_L$ , declination, and right ascension from EM observations

# Inclination bound form VLBI observation

Moley et. al, 2018, *Nature*

Moley et. al, 2019, *Nature*

Moley et. al, 2022, *Nature*



$$\iota \in \mathcal{U}[2.70, 2.81] \text{ rad}, \text{ where } \iota = \pi - \theta_v$$

# Results

Zhu\*, Song\*†, Lyu†, et. al, *arXiv: 2603.25413*

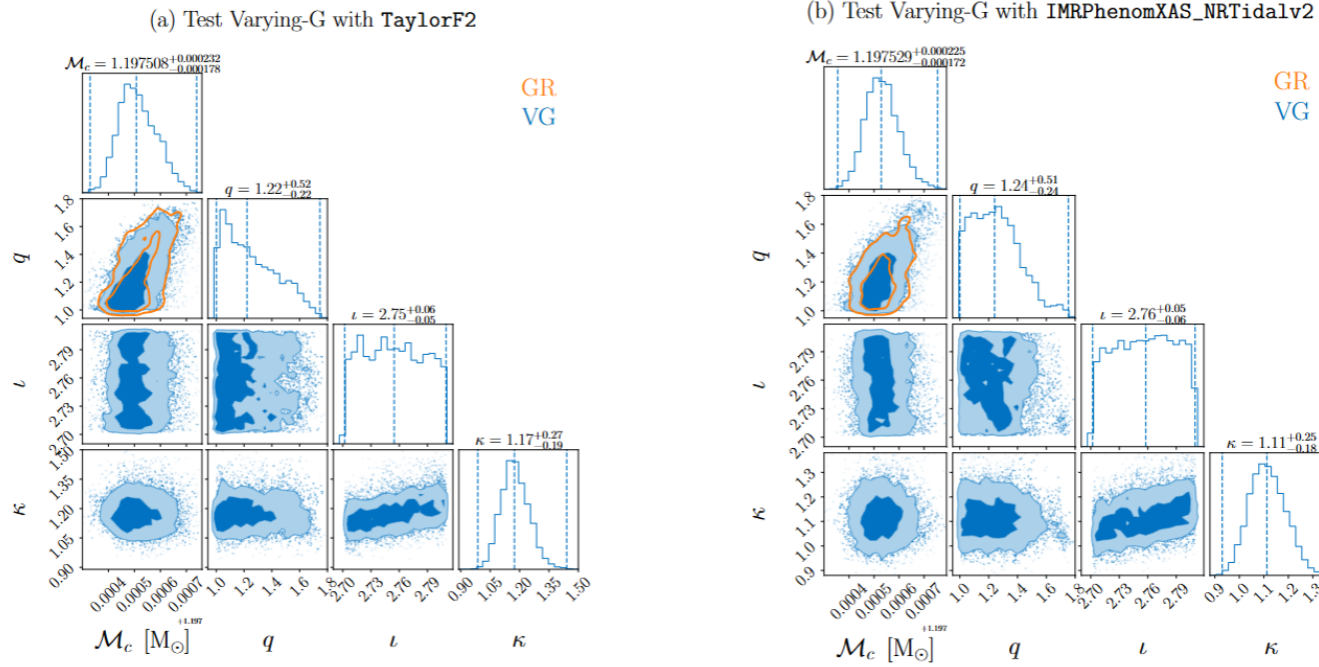
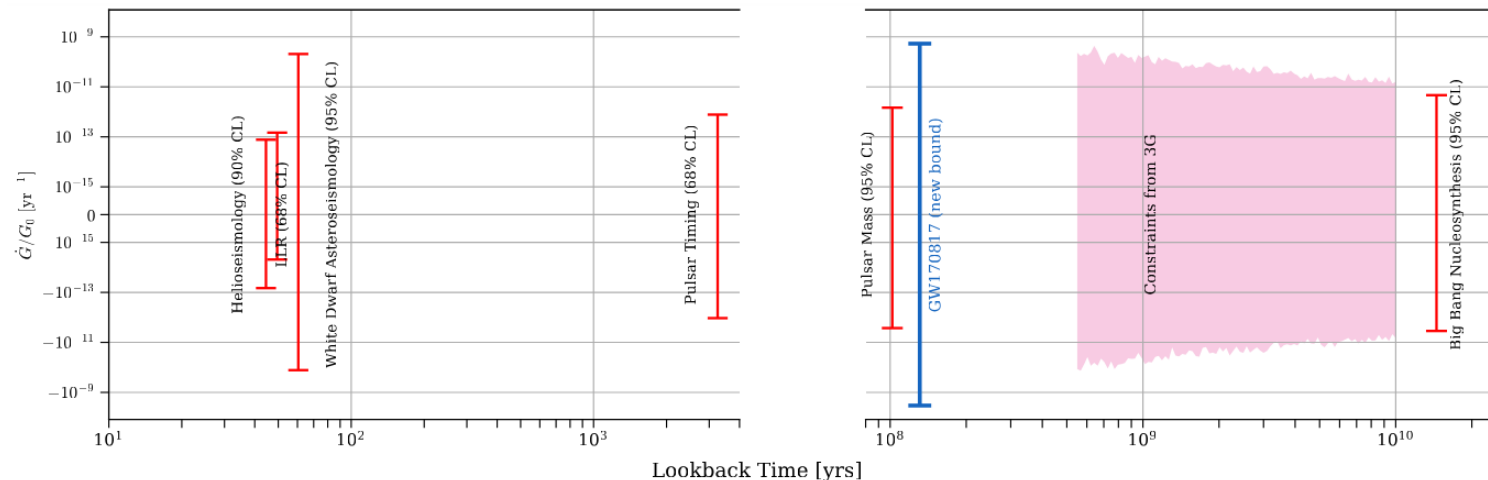


FIG. 3. Posterior distributions of key parameters for testing varying G theory using GW170817 with TaylorF2 (left panel) and IMRPhenomXAS\_NRTidalv2 (right panel). The 2D contours represent with different confidence levels, denoting the 50% and 90% regions, while the vertical lines indicate the  $3\sigma$  region for the 1D marginalized posterior distribution.



- The first Bayesian inference results and the most stringent constraint on varying-G theory with GW data

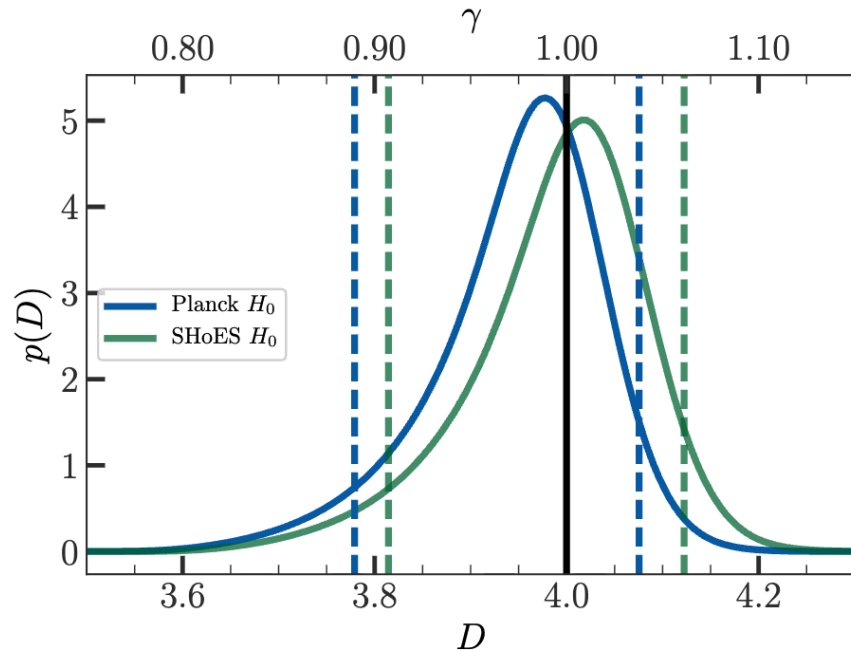
- Consider the varying-G generation, propagation and EOS constrain, along with the EM and GW multi-messenger observations we obtain the constraint of  $\dot{G}/G \sim [-3.36 \times 10^{-9}, 5.34 \times 10^{-10}] \text{ yr}^{-1}$

# Probe the number of spacetime with GW170817

## Phenomenologically model

Gravitational radiation can leak to extra dimension

$$h \propto \frac{1}{d_L \left[ 1 + \left( \frac{d_L}{R_c} \right)^{n(D-4)/2} \right]^{1/n}}$$



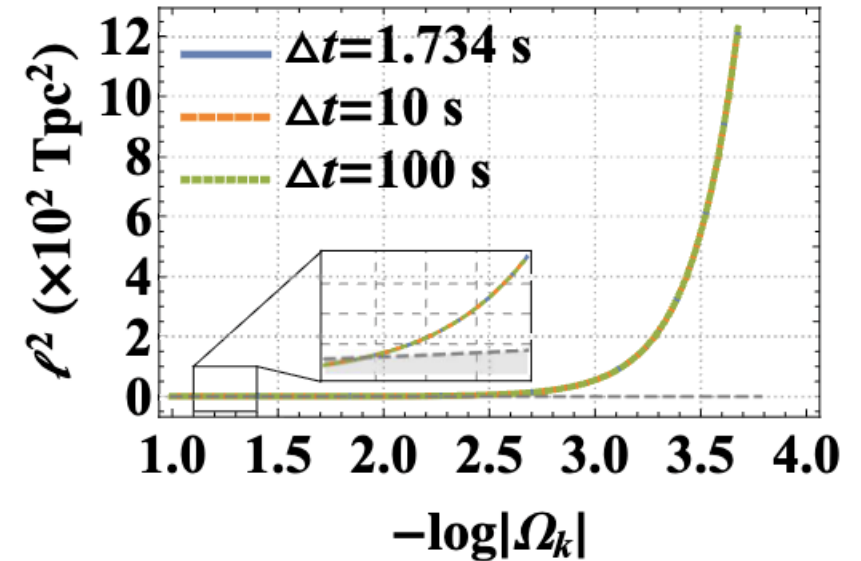
Pardo, et. al, 2018, *JCAP*

## 5D de Sitter spacetime

GW move on 5D bulk, while EM wave move on 4D brane

$$\tilde{r}_g \equiv a_B r_g = \int_{\frac{1}{1+z_A}}^1 \tilde{R}^{-2} \left( \frac{1}{s} + \frac{1}{\ell^2} - \tilde{k} \tilde{R}^{-2} \right)^{-\frac{1}{2}} d\tilde{R},$$

$$\tilde{r}_\gamma \equiv a_B r_\gamma = \int_{t_A}^{t_B} \frac{1}{\tilde{a}} dt,$$



Lin, Yu, and Liu, et. al, 2020, *PRD*

# Compactified Kaluza-Klein framework

Qiang, Zhao & Xu 2016, *EPJC*

Higher-dimensional extension of GR  
in a  $M^4 \times M^n$  manifold

$$R_{MN}(g) - \frac{1}{2}g_{MN}R(g) = 8\pi\mathbf{G}T_{MN}$$

Consider the ground state energy momentum tensor, the related metric is

$$\tilde{g}_{MN} = \begin{pmatrix} \eta_{\mu\nu} & \\ & \tilde{g}_{\alpha\beta}(y) \end{pmatrix}$$

The effective 4D compact binary system will perturb the spacetime

$$g_{MN} = \tilde{g}_{MN} + h_{MN}$$

$$\nabla_P \nabla^P \bar{h}_{MN} = -16\pi\mathbf{G}\Delta T_{MN} + \mathcal{O}(h^2)$$

Consider the Kaluza-Klein ansatz

- Physical tensor field are invariant under the isometries of compact internal space
- Lie derivative along the Killing vector field vanish
  - High order modes break isometries are not excited
  - Extra-dimensional components of velocity vanish

Solving leading-order equation and SPA to FD

$$\begin{aligned} \tilde{h}_+^{\text{HD}}(f) &= \frac{1 + \cos^2 \iota}{2} [1 + 8\kappa (\pi M f)^{2/3}] A(f) e^{i\Psi(f)}, \\ \tilde{h}_\times^{\text{HD}}(f) &= -i \cos \iota [1 + 8\kappa (\pi M f)^{2/3}] A(f) e^{i\Psi(f)}, \end{aligned}$$

$$\kappa \equiv \frac{n}{n+2}$$

Leading order: 1PN effect

Need to solve degeneracy among  $\kappa$ ,  $D$ , and  $\iota$  !

# Multimessenger MBHB event on Taiji

Song, Du, and Xu, *arXiv: 2609.xxxxx*

Mangiagli, et. al, 2022, *PRD*

## Three MBHB model

- Pop3: Light seeds from Pop III stellar collapse + merger delays
- Q3d: Heavy seeds from direct-collapse BHs + merger delays
- Q3nd: Heavy seeds without merger delays

## Observe the EM signal at different band

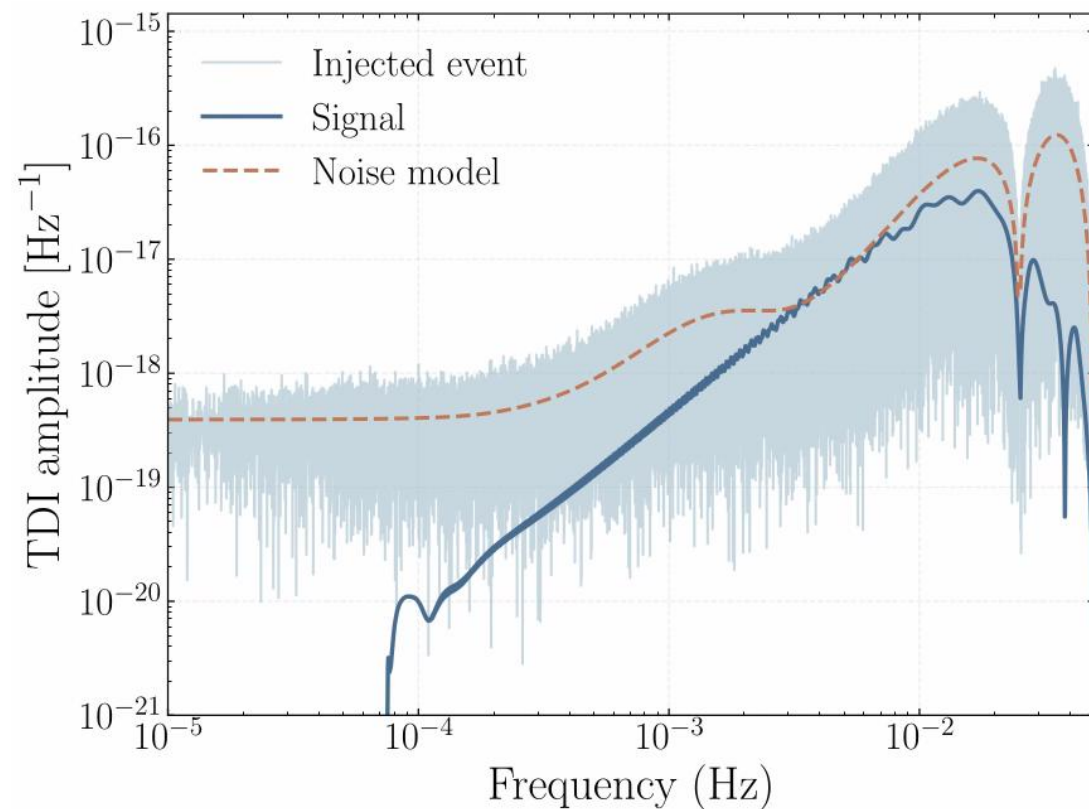
- Optical: Rubin observatory
- Radio: The SKA
- Soft X-ray: Advanced Telescope for High Energy Astrophysics (Athena)

## Event rate for LISA/Taiji (4 years observation)

- Pop3: 7 events
- Q3d: 15 events
- Q3nd: 21 events

## Project the MBHB event on Taiji

2<sup>nd</sup> TDI + instrument noise + confusion noise

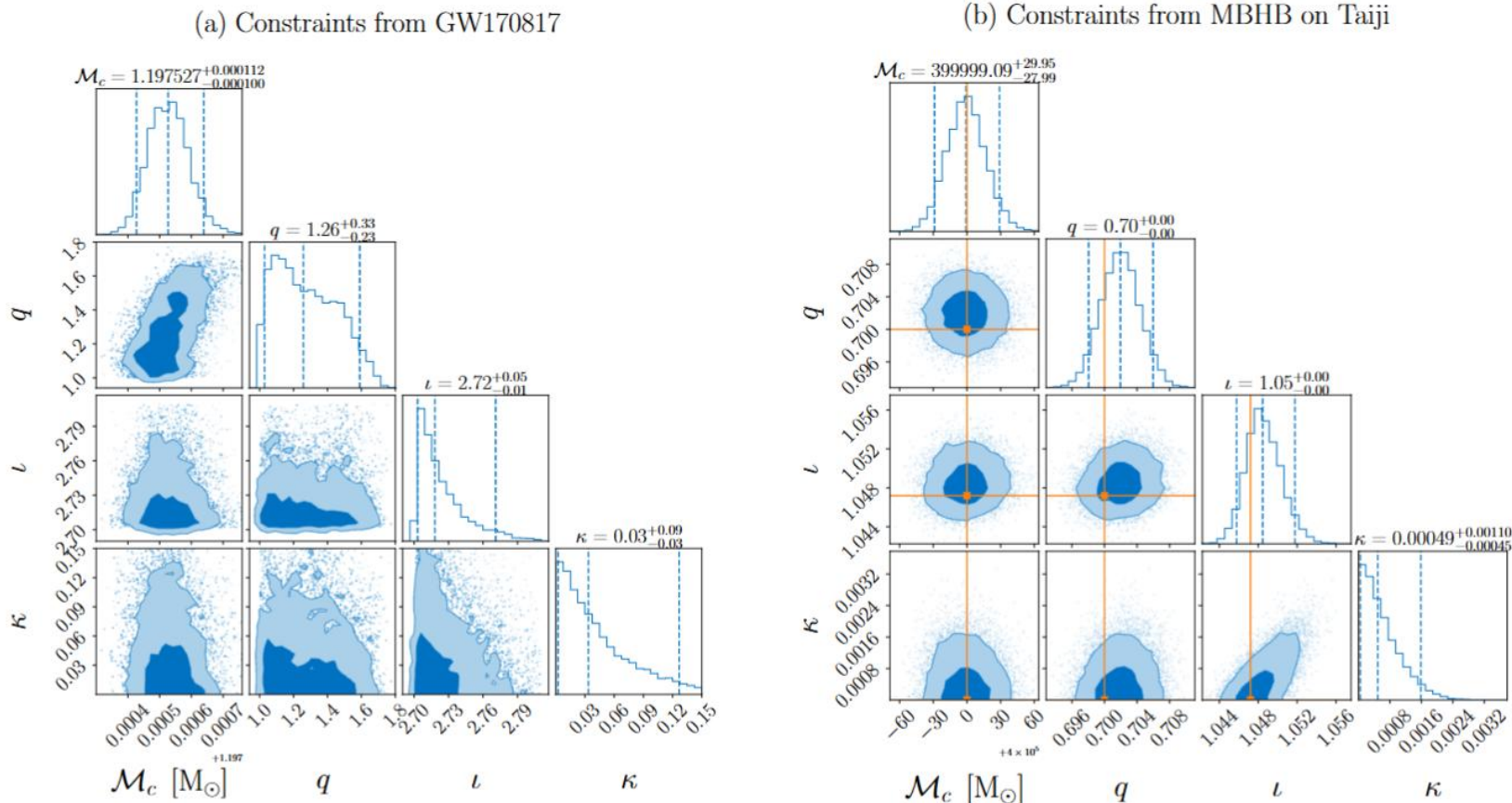


Perform data analysis with Triangle-BBH

Du et. al, 2026, *SCPMA*

# Results from GW170817 and MBHB on Taiji

Song, Du, and Xu, *arXiv: 2609.xxxxx*



Constraints with GW170817:  $n < 0.273$  (90% credible interval)  
Constraints with MBHB on Taiji:  $n < 0.003$  (90% credible interval)

# Summary

## Test the strong equivalent principle

- Consider 3 physical effects of varying-G
- Use data from GW170817 and GRB 170817A
- **Most stringent** bound for GW observation on  
 $\dot{G}/G \sim [-3.36 \times 10^{-9}, 5.34 \times 10^{-10}] \text{ yr}^{-1}$

Zhu\*, Song\*†, Lyu†, et. al, *arXiv: 2603.25413*

## Probe the extra dimension spacetime

- Leading order GW template from compactified  
Kaluza-Klein framework
- Multimessenger event from LVK and Taiji
- **Taiji shows the advantage on testing GR :  $n < 0.003$**   
(90% credible interval)

Song, Du, and Xu, *arXiv: 2609.xxxxx*

*Thanks for your attention !*