

Shaving off soft hairs and black hole image memory effect

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Introduction

- ▶ Gravitational memory effect:

the permanent change in the relative distance between test particles after the passage of gravitational waves

Introduction

► Gravitational memory effect:

the permanent change in the relative distance between test particles after the passage of gravitational waves

1. 1970's: Zel'dovich & Polnarev, Braginsky & Grishchuk, ...

The (linear) memory effect is related to the permanent change in the quadrupole moment of the source.

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► Gravitational memory effect:

the permanent change in the relative distance between test particles after the passage of gravitational waves

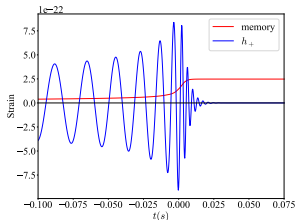
1. 1970's: Zel'dovich & Polnarev, Braginsky & Grishchuk, ...

The (linear) memory effect is related to the permanent change in the quadrupole moment of the source.

2. In 1990's: Chritodoulou, Blanchet & Damour, Wiseman & Will, Thorne, ...

The nonlinear memory effect is sourced by the energy flux of radiative degrees of freedom, including photons, neutrinos and gravitational wave itself.

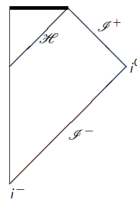
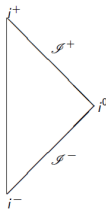
$$\Delta h_{jk}^{\text{TT}} = \frac{4}{r} \int_{-\infty}^u dt \left(\oint d\Omega' \frac{d^2 E_{\text{rad}}}{dt d\Omega'} \frac{n'_j n'_k}{1 - \vec{n}' \cdot \vec{N}} \right)^{\text{TT}}. \quad (1)$$



Introduction

New developments

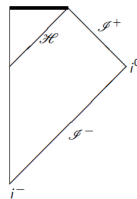
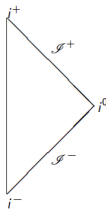
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 1. Connection with the asymptotic symmetry of the isolated system:
Strominger, Compère, Flanagan, ...



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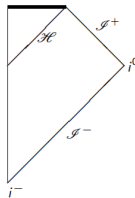
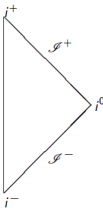


i Bondi-Metzner-Sachs symmetry = { supertranslations $\times$ Lorentz transformations }

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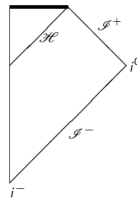
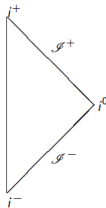
- i Bondi-Metzner-Sachs symmetry = { supertranslations $\times$ Lorentz transformations }
- ii Vacuum is degenerate at $\mathcal{I}$:

$$\text{vacuum} \xrightleftharpoons{\text{supertranslations}} \text{vacuum} \quad \Bigg| \quad \text{memory effect} \iff \text{vacuum transition}$$

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- iii The flux-balance law constrains the memory effect:

$$\Delta h \sim \int du \oint d\Omega \dot{h}^2$$

Introduction

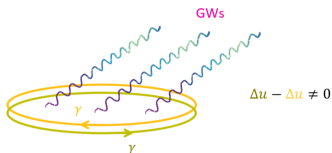
New developments

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 - 2. New memory effects:

Introduction

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 - a. Spin memory effect: Pasterski, Strominger, Zhiboedov



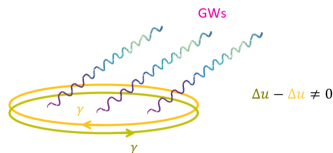
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2. New memory effects:

- a. Spin memory effect: Pasterski, Strominger, Zhiboedov



- b. Center-of-mass memory effect: Nichols

Spin and center-of-mass memories are also called the subleading displacement memory effect. The first ever discussed memory effect is specifically called the (leading) displacement memory effect.

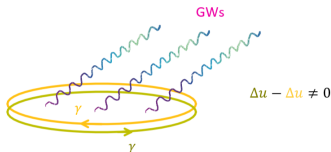
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Spin and center-of-mass memories are also called the subleading displacement memory effect.

The first ever discussed memory effect is specifically called the (leading) displacement memory effect.

3. Enlargement of the BMS symmetry:

$$\text{BMS} \longrightarrow \left\{ \begin{array}{l} \text{supertranslations} \times \text{local conformal transformations on } S^2 \\ \text{"extended BMS": Barnich, Troessaert, ...} \\ \text{supertranslations} \times \text{Diffeomorphisms on } S^2 \\ \text{"generalized BMS": Campiglia, Laddha, ...} \end{array} \right.$$

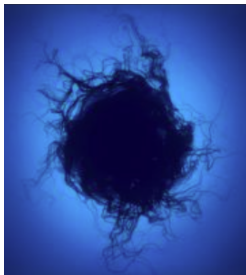
$$\text{Lorentz} \Longrightarrow \text{Super-Lorentz}$$

Introduction

Soft hairs

► Black hole memories, soft hairs, information paradoxes, ...:

Strominger, Hawking, Perry, Donnay, Wald, Flanagan, ...



How soft hairs impact the shadow?

Image memory effect?

Black hole soft hairs

Black hole soft hairs



知乎 @承玄以聞

No-hair theorem ~ “Three-hair theorem”

Black hole soft hairs



No-hair theorem $\sim$ “Three-hair theorem”

Conserved charges of (standard/extended/generalized) BMS symmetries

Asymptotically flat spacetimes

- Metric in Bondi-Sachs coordinates $(u, \varrho, \vartheta, \varphi)$

$$\begin{aligned} ds^2 = & \left[-\frac{\mathcal{R}}{2} + \frac{2M}{\varrho} + \mathcal{O}\left(\frac{1}{\varrho^2}\right) \right] du^2 + 2 \left[-1 + \frac{c_{AB}c^{AB}}{16\varrho^2} + \mathcal{O}\left(\frac{1}{\varrho^2}\right) \right] du d\varrho \\ & + 2 \left[\frac{\mathcal{D}^B c_{AB}}{2} + \frac{1}{\varrho} \left(\frac{2}{3} N_A - \frac{1}{8} c_{BC} \mathcal{D}_A c^{BC} \right) + \mathcal{O}\left(\frac{1}{\varrho^2}\right) \right] du d\vartheta^A \\ & + [\varrho^2 h_{AB} + \varrho c_{AB} + \mathcal{O}(\varrho^0)] d\vartheta^A d\vartheta^B. \end{aligned}$$

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- Asymptotic symmetry

$$x^\mu \rightarrow x^\mu + \xi^\mu$$

$$\mathcal{L}_\xi g_{\varrho\varrho} = 0, \quad \mathcal{L}_\xi g_{\varrho A} = 0, \quad \partial_\varrho (g^{AB} \mathcal{L}_\xi g_{AB}) = 0, \quad \text{and the falloff}$$

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resulting in

$$\begin{aligned} \xi^u &= \tau \equiv T + uW, \\ \xi^A &= Y^A - \frac{\mathcal{D}^A \tau}{\varrho} + \mathcal{O}\left(\frac{1}{\varrho^2}\right), \\ \xi^\varrho &= -\varrho W + \frac{1}{2} \mathcal{D}^2 \tau + \mathcal{O}\left(\frac{1}{\varrho}\right), \\ \partial_u T &= \partial_u W = \partial_u Y^A = 0. \end{aligned}$$

Asymptotically flat spacetimes

Noether charges

► Charges

$$Q_T = \int_{\mathbb{S}} d^2\vartheta \sqrt{h} T \left(M + \frac{1}{4} \mathcal{D}_A \mathcal{D}_B c^{AB} \right),$$

$$Q_W = Q_{T=0} W + 4 \int_{\mathbb{S}} d^2\vartheta \sqrt{h} W \varsigma_2,$$

$$Q_Y = \int_{\mathbb{S}} d^2\vartheta \sqrt{h} Y^A \left(N_A + 4 \mathcal{D}_A \varsigma_2 - \frac{1}{4} c_{AB} \mathcal{D}_C c^{BC} \right),$$

$$\varsigma_2 = -\frac{c_{AB} c^{AB}}{32}.$$

In general,

$$Q_\xi = Q_T + Q_W + Q_Y.$$

Soft-haired Kerr black hole
&
the image

Bald Kerr black hole

► Bald metric

$$\begin{aligned} ds^2 &= -\frac{\Delta\Sigma}{\Xi} dt^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2 + \frac{\Xi \sin^2 \theta}{\Sigma} (d\phi - \omega dt)^2 \\ &= -\left[1 - \frac{2\dot{M}}{\varrho} + \mathcal{O}\left(\frac{1}{\varrho^3}\right)\right] du^2 - 2\left[1 + \mathcal{O}\left(\frac{1}{\varrho^4}\right)\right] du d\varrho \\ &\quad - \left[\frac{4\dot{M}a \sin^2 \vartheta}{\varrho} + \mathcal{O}\left(\frac{1}{\varrho^2}\right)\right] du d\varphi + \left[\varrho^2 + \mathcal{O}\left(\frac{1}{\varrho}\right)\right] (d\vartheta^2 + \sin^2 \vartheta d\varphi^2) \\ &\quad + \mathcal{O}\left(\frac{1}{\varrho^2}\right) du d\vartheta + \mathcal{O}\left(\frac{1}{\varrho^3}\right) d\vartheta d\varphi. \end{aligned}$$

► Conserved charges, e.g.,

Time translation $Q_t = 4\pi\dot{M}$

Rotation $Q_\phi = -4\pi\dot{M}a$

Soft-haired Kerr black hole

Generalized BMS transformation and the metric

- Generalized BMS transformation $\subset$ Weyl BMS

$$\tilde{u} = \mathcal{G} - \frac{e^{-\mathcal{W}}}{2r} \mathcal{D}_A \mathcal{G} \mathcal{D}^A \mathcal{G} + \mathcal{O}\left(\frac{1}{r^2}\right),$$

$$\tilde{\varrho} = e^{-\mathcal{W}} r + \frac{e^{-2\mathcal{W}}}{2} \mathcal{D}^2 \mathcal{G} + \mathcal{O}\left(\frac{1}{r}\right),$$

$$\tilde{\vartheta}^A = \Upsilon^A(\vartheta) - \frac{e^{-\mathcal{W}}}{r} \mathcal{J}_B^A \mathcal{D}^B \mathcal{G} + \mathcal{O}\left(\frac{1}{r^2}\right),$$

$$\mathcal{G} = \mathcal{T} + u\mathcal{W}, \quad e^{2\mathcal{W}} = |\det(\mathcal{J}_A^B)| \sqrt{\frac{\tilde{h}(\Upsilon(\vartheta))}{h(\vartheta)}}, \quad \mathcal{J}_A^B \equiv \frac{\partial \Upsilon^B}{\partial \vartheta^A}.$$

Finite supertranslation $\mathcal{T}$

Finite superrotation Υ^A with $\mathcal{W} = 0$

Finite super-boost Υ^A with $\mathcal{W} \neq 0$

- Soft-haired metric

$$\begin{aligned} ds^2 = & \left[-\frac{\tilde{\mathcal{R}}}{2} + \frac{2\tilde{\mathcal{M}}}{\tilde{\varrho}} + \mathcal{O}\left(\frac{1}{\tilde{\varrho}^2}\right) \right] d\tilde{u}^2 + 2 \left[-1 + \frac{\tilde{c}_{AB} \tilde{c}^{AB}}{16\tilde{\varrho}^2} + \mathcal{O}\left(\frac{1}{\tilde{\varrho}^2}\right) \right] d\tilde{u} d\tilde{\varrho} \\ & + 2 \left[\frac{\tilde{\mathcal{D}}^B \tilde{c}_{AB}}{2} + \frac{1}{\tilde{\varrho}} \left(\frac{2}{3} \tilde{N}_A - \frac{1}{8} \tilde{c}_{BC} \tilde{\mathcal{D}}_A \tilde{c}^{BC} \right) + \mathcal{O}\left(\frac{1}{\tilde{\varrho}^2}\right) \right] d\tilde{u} d\tilde{\vartheta}^A \\ & + [\tilde{\varrho}^2 \tilde{h}_{AB} + \tilde{\varrho} \tilde{c}_{AB} + \mathcal{O}(\tilde{\varrho}^0)] d\tilde{\vartheta}^A d\tilde{\vartheta}^B, \end{aligned}$$

Soft-haired Kerr black hole

Conserved charges

- Conserved charges, e.g.,

Time translation

$$\tilde{Q}_t = 4\pi \dot{M} + \int d^2\tilde{\vartheta} \sqrt{\tilde{h}} \left(\frac{1}{4} \tilde{\mathcal{G}}_A \tilde{\mathcal{G}}_B \tilde{c}^{AB} - \frac{1}{8} \tilde{c}_{AB} \tilde{N}^{AB} \right)$$

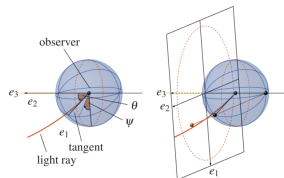
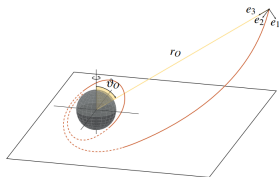
Rotation

$$\begin{aligned} \tilde{Q}_\phi = & -4\pi \dot{M} a + \int d^2\tilde{\vartheta} \sqrt{\tilde{h}} \left\{ \tau_\varphi \left(\tilde{M} + \frac{1}{4} \tilde{\mathcal{G}}_A \tilde{\mathcal{G}}_B \tilde{c}^{AB} \right) \right. \\ & \left. + 4\tilde{\zeta}_2 \partial_\varphi \mathcal{W} + \mathcal{J}_\varphi^A \left(4\partial_A \tilde{\zeta}_2 - \frac{1}{4} \tilde{c}_{AB} \tilde{\mathcal{G}}_C \tilde{c}^{BC} \right) \right\} \end{aligned}$$

- And more soft hairs...

Soft-haired Kerr black hole

Adapted tetrads



PRD 89,124004(2014)

► Adapted tetrads

Bald Kerr

$$\sigma_{\hat{0}} = \partial_t + \mathcal{O}\left(\frac{1}{r}\right)$$

$$\sigma_{\hat{1}} = \partial_r + \mathcal{O}\left(\frac{1}{r}\right) \quad \text{radially outward}$$

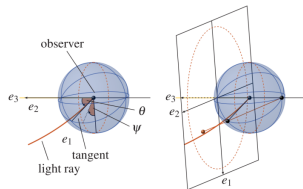
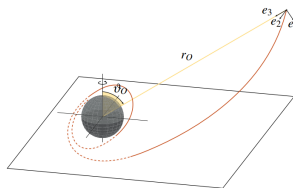
$$\sigma_{\hat{i}} = \frac{\mathcal{K}_{\hat{i}}^A}{r} \partial_A + \mathcal{O}\left(\frac{1}{r^2}\right) \quad \text{angular directions}$$

Soft-haired Kerr

$$\begin{aligned} \sigma_{\hat{0}} &= \frac{1}{2} \tilde{\sigma}_{\hat{0}} \left(\tilde{\mathcal{D}}^2 + 1 + \frac{\tilde{\mathcal{R}}}{2} \right) e^{\mathcal{W}} \\ &\quad + \frac{1}{2} \tilde{\sigma}_{\hat{1}} \left(\tilde{\mathcal{D}}^2 - 1 + \frac{\tilde{\mathcal{R}}}{2} \right) e^{\mathcal{W}} \\ &\quad - \tilde{\sigma}_{\hat{i}} \tilde{\mathcal{K}}_{\hat{A}}^{\hat{i}} \tilde{\mathcal{D}}^{\hat{A}} e^{\mathcal{W}}, \\ \sigma_{\hat{i}} &= (\tilde{\sigma}_{\hat{0}} + \tilde{\sigma}_{\hat{1}}) \tilde{\mathcal{K}}_{\hat{i}}^{\hat{A}} \tilde{\mathcal{D}}_{\hat{A}} e^{-\mathcal{W}} + \mathcal{R}_{\hat{i}}^{\hat{j}} \tilde{\sigma}_{\hat{j}}. \end{aligned}$$

Soft-haired Kerr black hole

Celestial coordinates



PRD 89,124004(2014)

► Bald Kerr

$$x^{\hat{2}} \approx \pm \frac{\sqrt{q + a^2 \cos^2 \theta_o - \lambda^2 \cot^2 \theta_o}}{r_o}, \quad x^{\hat{3}} \approx \frac{\lambda}{r_o \sin \theta_o}.$$

► Soft-haired Kerr

$$\tilde{x}^{\hat{i}} = e^{\mathcal{W}_o} \mathcal{R}_{\hat{j}}^{\hat{i}}(\tilde{\vartheta}_o) x^{\hat{j}} + S^{\hat{i}}(\tilde{u}_o, \tilde{\vartheta}_o) + \mathcal{O}\left(\frac{1}{\tilde{\varrho}_o^2}\right), \quad \hat{i}, \hat{j} = \hat{1}, \hat{2}$$

$$\mathcal{R}_{\hat{j}}^{\hat{i}}(\tilde{\vartheta}) = \text{rotation matrix}$$

$$e^{\mathcal{W}(\tilde{\vartheta})} = \text{dilation factor}$$

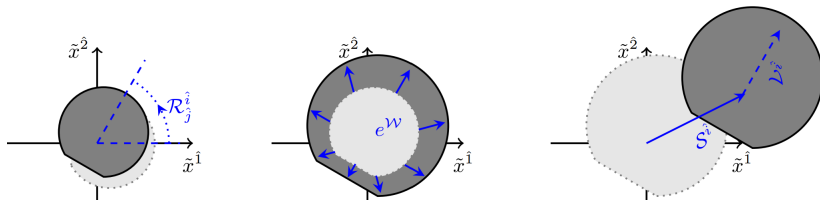
$$S^{\hat{i}}(\tilde{u}, \tilde{\vartheta}) = \frac{1}{\tilde{\varrho}} \tilde{\mathcal{K}}_{\hat{A}}^{\hat{i}} [\tilde{\vartheta}^{\hat{A}} \mathcal{T} + (\tilde{u} - \mathcal{T}) \tilde{\vartheta}^{\hat{A}} \mathcal{W}]$$

Soft-haired Kerr black hole

The image

Soft-haired Kerr image

$$\tilde{x}^{\hat{i}} = e^{\mathcal{W}_o} \mathcal{R}_{\hat{j}}^{\hat{i}}(\tilde{\vartheta}_o) x^{\hat{j}} + \mathcal{S}^{\hat{i}}(\tilde{u}_o, \tilde{\vartheta}_o) + \mathcal{O}\left(\frac{1}{\tilde{\varrho}_o^2}\right), \quad \hat{i}, \hat{j} = \hat{1}, \hat{2}$$



1. rotated by $\mathcal{R}_{\hat{j}}^{\hat{i}}(\tilde{\vartheta})$
2. dilated by $e^{\mathcal{W}(\tilde{\vartheta})}$
3. shifted by $\mathcal{S}^{\hat{i}}(\tilde{u}, \tilde{\vartheta})$ at a speed $\dot{\mathcal{S}}^{\hat{i}}(\tilde{\vartheta}) = \tilde{\mathcal{K}}_A^{\hat{i}} \tilde{\mathcal{Q}}^A \mathcal{W} / \tilde{\varrho}$

An astronomical black hole is not exactly stationary.

An astronomical black hole is not exactly stationary.

Image memory effect

Image memory effect

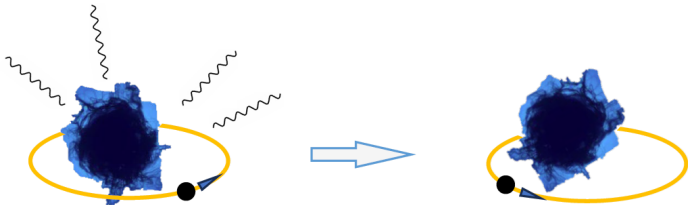
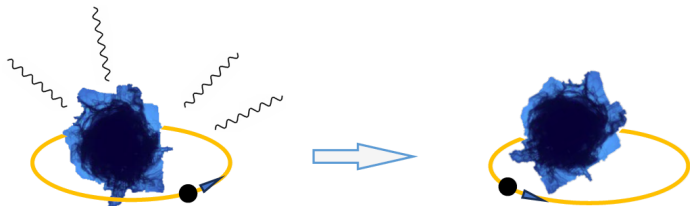


Image memory effect



$$\tilde{\mathcal{G}}^2(\tilde{\mathcal{G}}^2 + 2)\Delta\mathcal{T} \approx 8\Delta\tilde{M} + \int_{\tilde{u}_i}^{\tilde{u}_f} \underbrace{(\tilde{N}_{AB}\tilde{N}^{AB} + 32\pi T_{\tilde{u}\tilde{u}}^{\mathcal{M}})}_{\text{null energy flux}} d\tilde{u},$$

Image memory effect

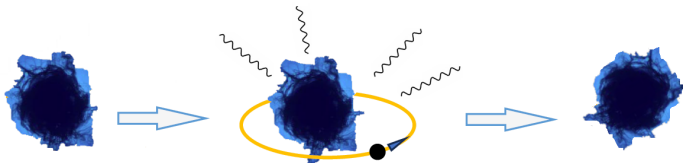


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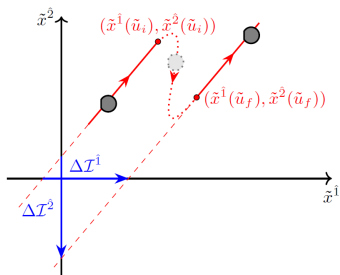
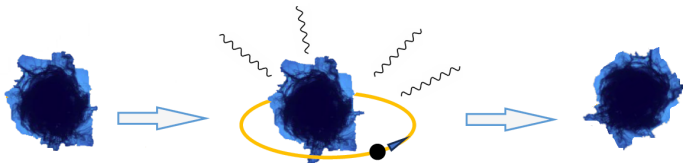
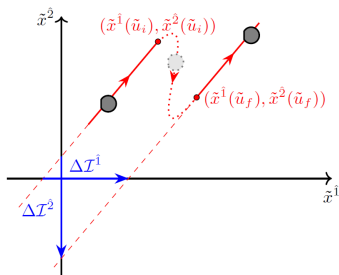
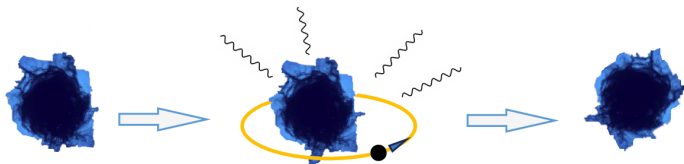
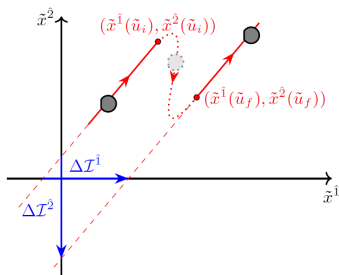
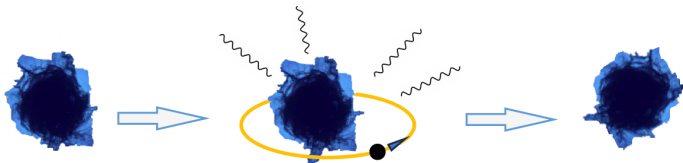


Image memory effect



$$\Delta \mathcal{I}^{\hat{i}} = \frac{1}{\tilde{\varrho}} \tilde{\mathcal{K}}^{\hat{i}}_A \tilde{\mathcal{D}}^A \Delta \mathcal{T}$$

Image memory effect



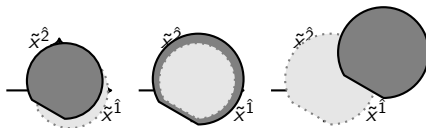
$$\Delta \mathcal{I}^{\hat{i}} = \frac{1}{\tilde{\varrho}} \tilde{\mathcal{K}}^{\hat{i}}_A \tilde{\mathcal{D}}^A \Delta \mathcal{T}$$

IMRI

- ▶ $M = 10^{12} M_{\odot}$
- ▶ $a = 0.8$
- ▶ $m = 10^{10} M_{\odot}$
- ▶ $D_L = 1 \text{ Gpc}$
- ▶ $|\vec{\Delta \mathcal{I}}| \sim 4 \times 10^{-4} \mu\text{as}$
during the last 10 yrs.

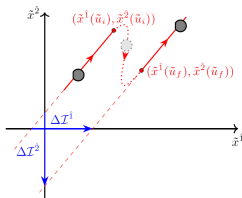
Summary

- ▶ Soft-haired Kerr black hole is related to the bald counterpart by
 - ▶ a supertranslation $\mathcal{T}(\theta)$
 - ▶ a super-Lorentz transformation $\Upsilon^A(\theta)$
- ▶ Soft hair modified black hole image gets
 1. rotated
 2. dilated
 3. drifted at a constant velocity



- ▶ The emission of the null radiation causes the change of soft hairs, leading to the image memory effect

the shift of straight line trajectory



Thank you!