



Relativistic frequency shifts in gravitational waves from axion clouds

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Phys. Rev. D 114, 044008, [arXiv:2604.21239]

2nd New Horizon for Psi Workshop, Aug 27, 2026

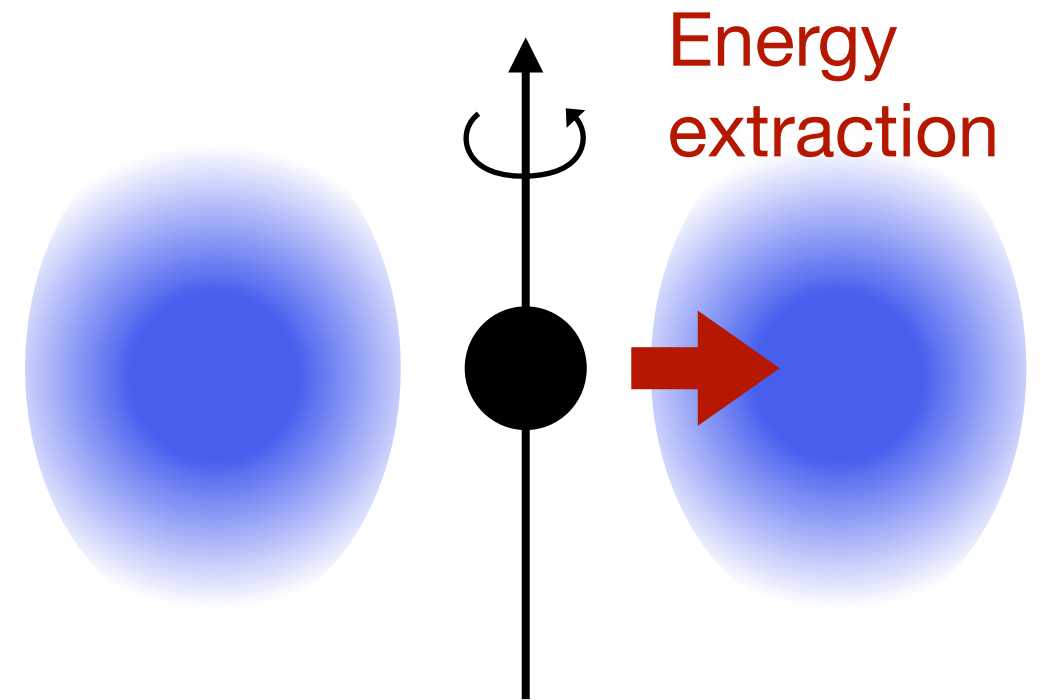
Black Hole Superradiance

What is an axion?

- Predicted in theories such as string theory
- A light scalar particle

Black hole superradiance

- Energy can be extracted from a rotating black hole, leading to the formation of an **axion cloud**.
- Gravitational waves emitted from the cloud can be used to probe axions.

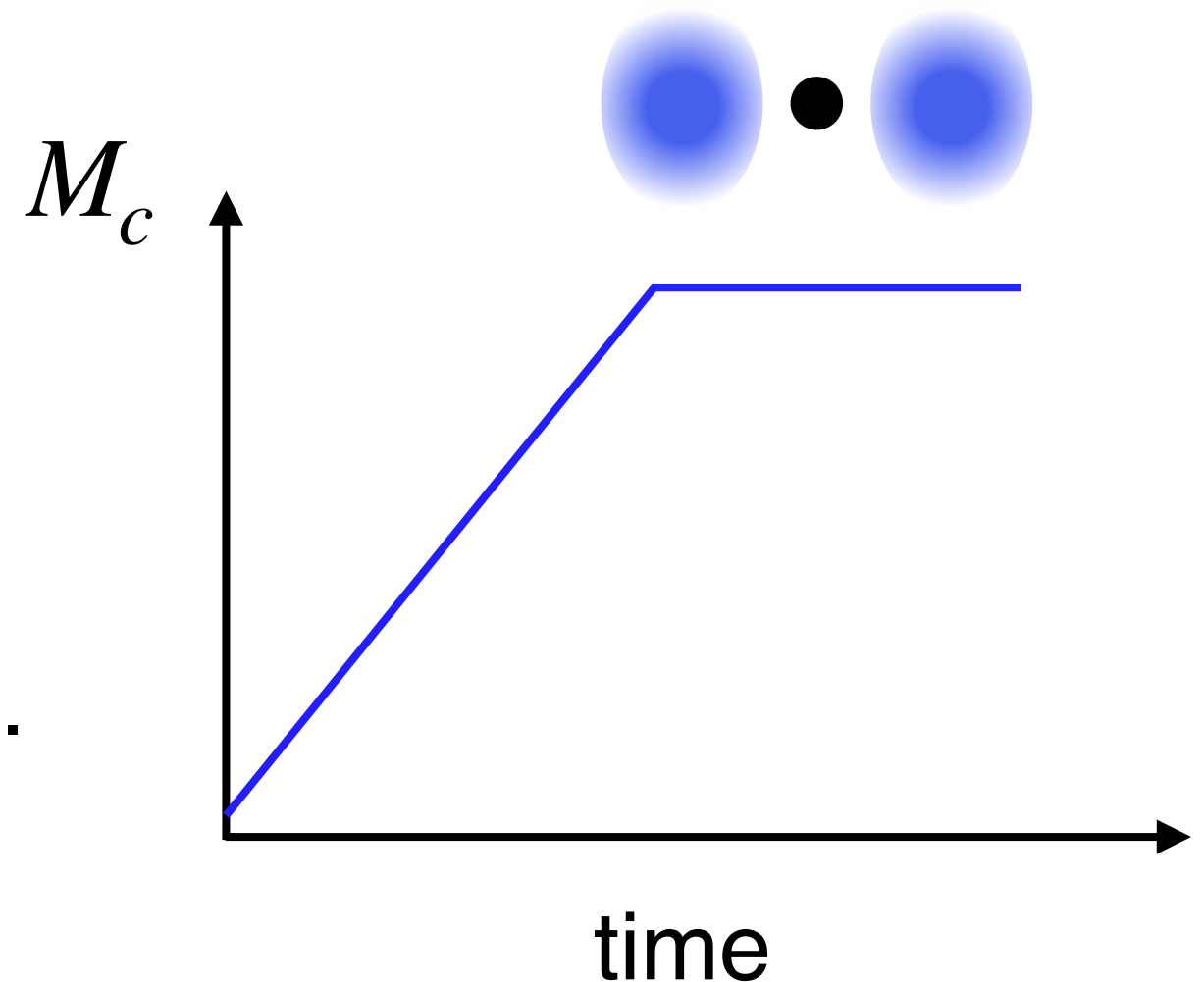


$$\frac{r_{\text{BH}}}{\lambda_c} = \frac{GM/c^2}{\hbar/\mu c} \simeq 0.2 \left(\frac{M}{30M_{\odot}} \right) \left(\frac{\mu}{10^{-12}\text{eV}} \right)$$

Evolution of the Cloud

Without interactions,
the cloud grows until the
BH spins down sufficiently.

The cloud then emits GWs.

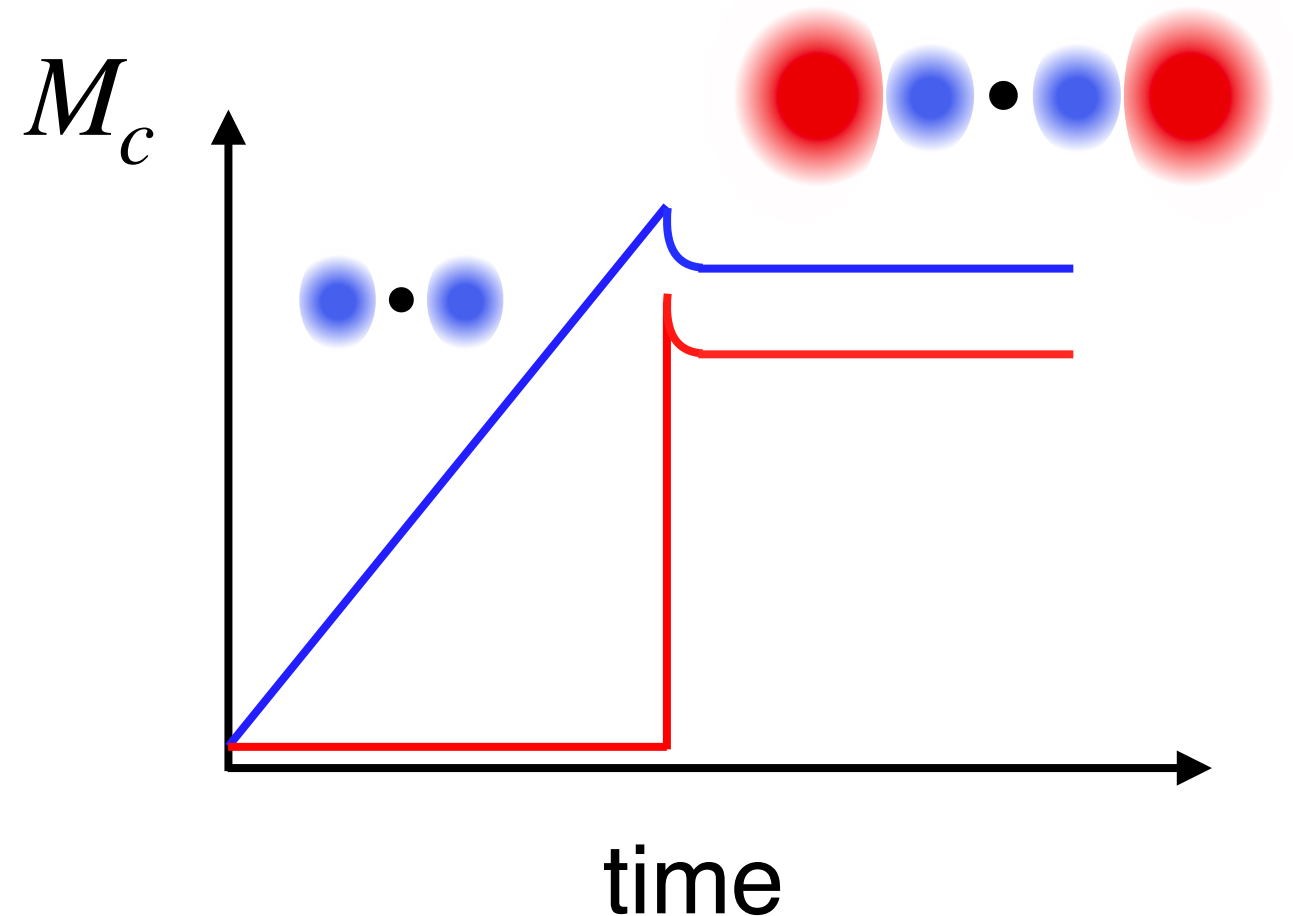


Evolution of the Cloud

Axions generically have **self-interactions**.

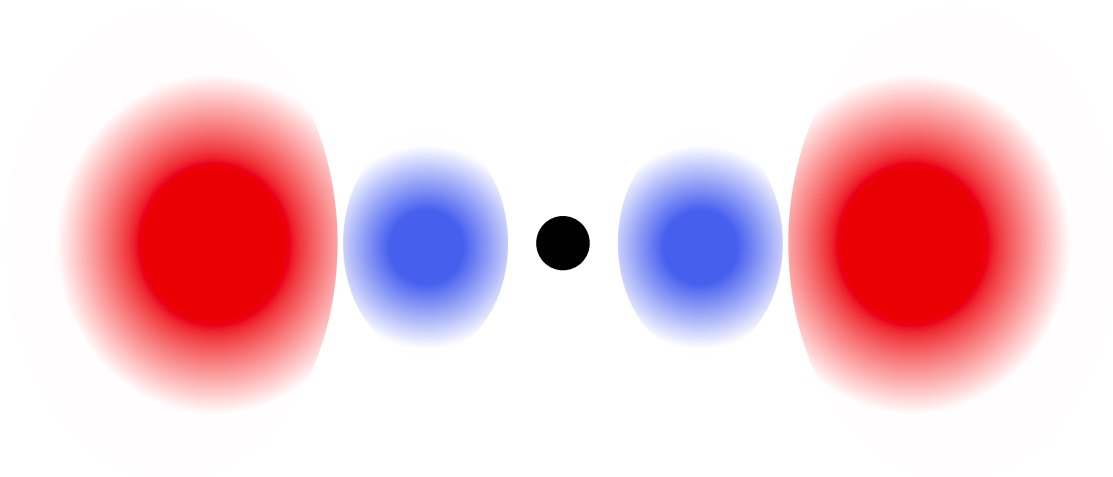
$$S = \int d^4x \sqrt{-g} \left(-\frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - V(\phi) \right)$$

$$V(\phi) \sim \frac{1}{2} \mu^2 \phi^2 - \frac{1}{4!} \mu^2 F_a^2 \left(\frac{\phi}{F_a} \right)^4$$



As the cloud grows, self-interactions becomes significant and excite multiple modes.

Gravitational Waves from the Cloud

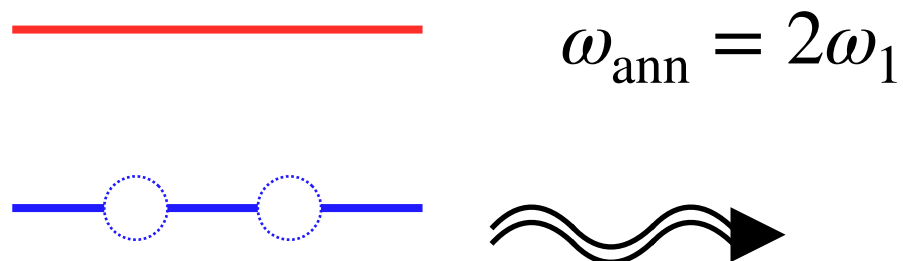


$$\phi = e^{-i(\omega_1 t - m_1 \varphi)} \psi_1 + e^{-i(\omega_2 t - m_2 \varphi)} \psi_2 + \dots$$

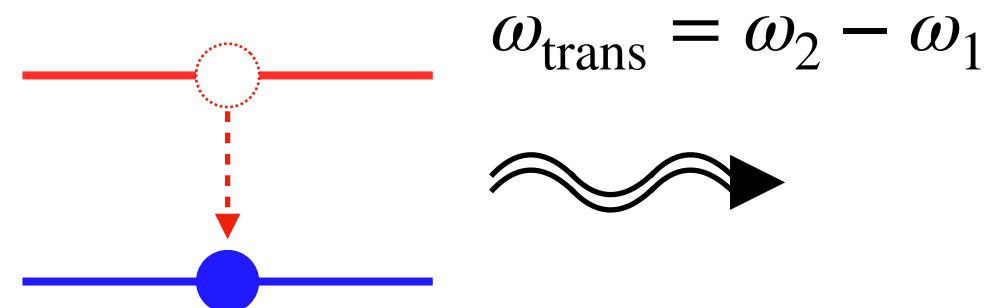
$$T_{\mu\nu} \sim \partial_\mu \phi \partial_\nu \phi$$

- Multiple GW emission channels from coexisting modes

Pair annihilation



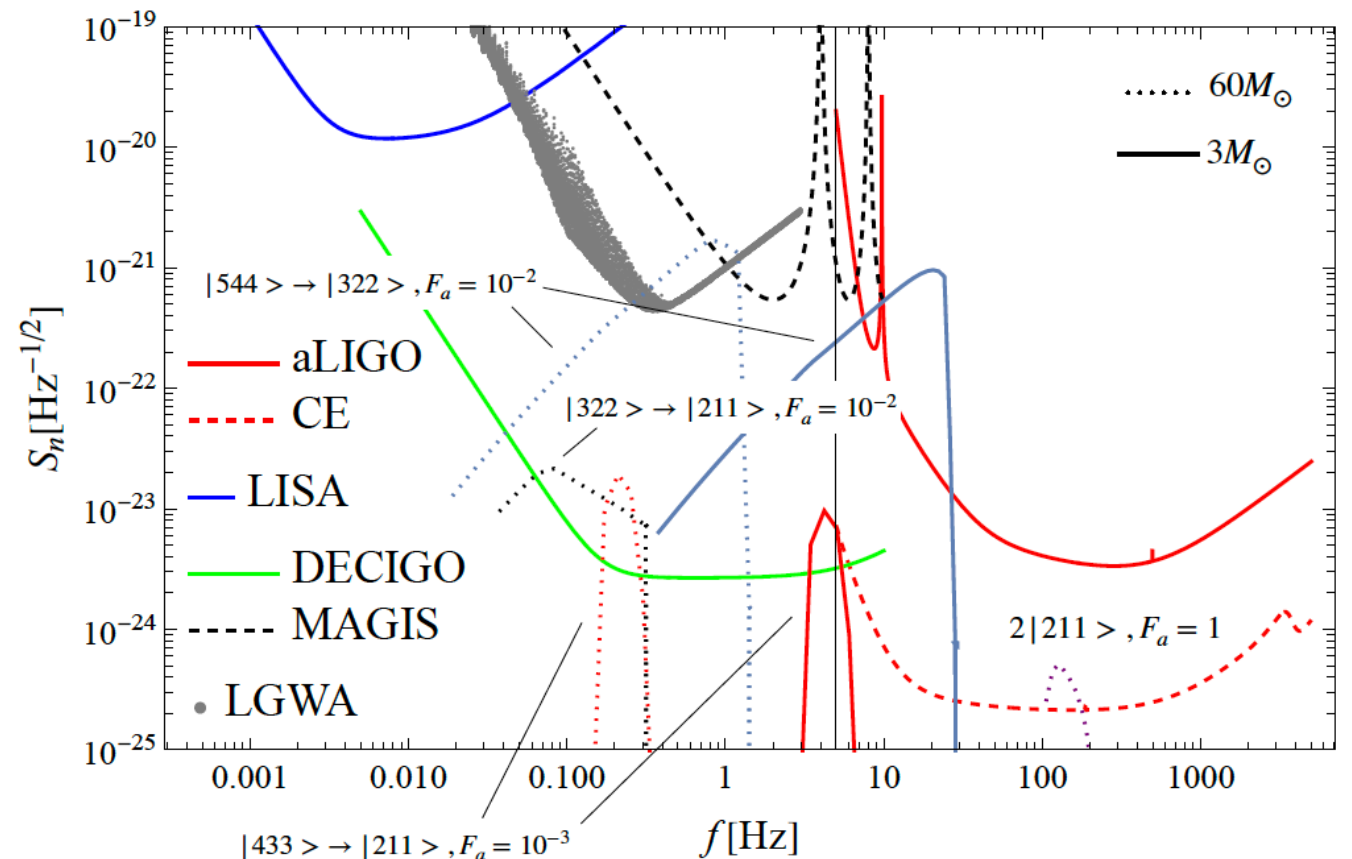
Level transition



Towards Continuous GW Searches

- Accurate predictions of GW frequencies are essential for signal detection.
- Frequency correction from non-linear interaction must be calculated.

$$\omega_i \rightarrow \omega_i + \delta\omega_i$$



- Most previous studies have focused on the pair-annihilation process.
- The non-relativistic approximation has typically been employed.

Bilinear Product

- Unperturbed background: $\mathcal{L} = -\frac{1}{2}\sqrt{-g} (g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi + \mu^2\phi^2)$
- Mode function: $\Phi_i = e^{-i(\omega_i t - m_i \varphi)} R_i(r) S_i(\theta)$

The bilinear product is defined by

$$\langle\langle\Phi_i, \Phi_j\rangle\rangle = \int_{\Sigma_t} d\Sigma \, n^\mu (\mathcal{J}\Phi_i \nabla_\mu \Phi_j - \Phi_j \nabla_\mu \mathcal{J}\Phi_i)$$

$$\mathcal{J}\Phi_i = e^{+i(\omega_i t - m_i \varphi)} R_i^*(r) S_i^*(\theta)$$

$$(i) \quad \langle\langle\Phi_i, \Phi_j\rangle\rangle = \langle\langle\Phi_j, \Phi_i\rangle\rangle \qquad (ii) \quad \langle\langle\mathcal{L}_t\Phi_i, \Phi_j\rangle\rangle = \langle\langle\Phi_i, \mathcal{L}_t\Phi_j\rangle\rangle$$

Relativistic Perturbation Theory

- Canonical conjugate momentum: $\pi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}}$
- Hamiltonian density: $\mathcal{H} = \pi \dot{\phi} - \mathcal{L}$

EoM: $(\square_g - \mu^2) \phi = 0 \quad \Leftrightarrow \quad \dot{\phi} = \frac{\partial \mathcal{H}}{\partial \pi} \quad \text{and} \quad \dot{\pi} = -\frac{\partial \mathcal{H}}{\partial \phi}$

$\Leftrightarrow \quad \mathcal{L}_t F = H F \quad \text{with} \quad F = \begin{pmatrix} \phi \\ \pi \end{pmatrix}$

Under the perturbation

$$H \rightarrow H + \delta H \quad \Rightarrow$$

$$\delta \omega_i = \frac{i \langle \langle F_i, \delta H F_i \rangle \rangle}{\langle \langle F_i, F_i \rangle \rangle}$$

Application

- **Self-interaction**

$$V(\phi) \sim \frac{1}{2}\mu^2\phi^2 - \frac{1}{4!}\mu^2 F_a^2 \left(\frac{\phi}{F_a}\right)^4$$

$$\phi^3 \supset \frac{1}{2\sqrt{2}}(3|\Phi_i|^2\Phi_i + 6|\Phi_j|^2\Phi_i + \dots)$$

$$\implies \delta H_j^{\text{SI}} = \begin{pmatrix} 0 & 0 \\ \sqrt{-g} \frac{1}{2(1+\delta_{ij})} \frac{\mu^2}{F_a^2} |\Phi_j|^2 & 0 \end{pmatrix}$$

- **Self-gravity**

$$g_{\mu\nu} \rightarrow g_{\mu\nu} + h_{\mu\nu}$$

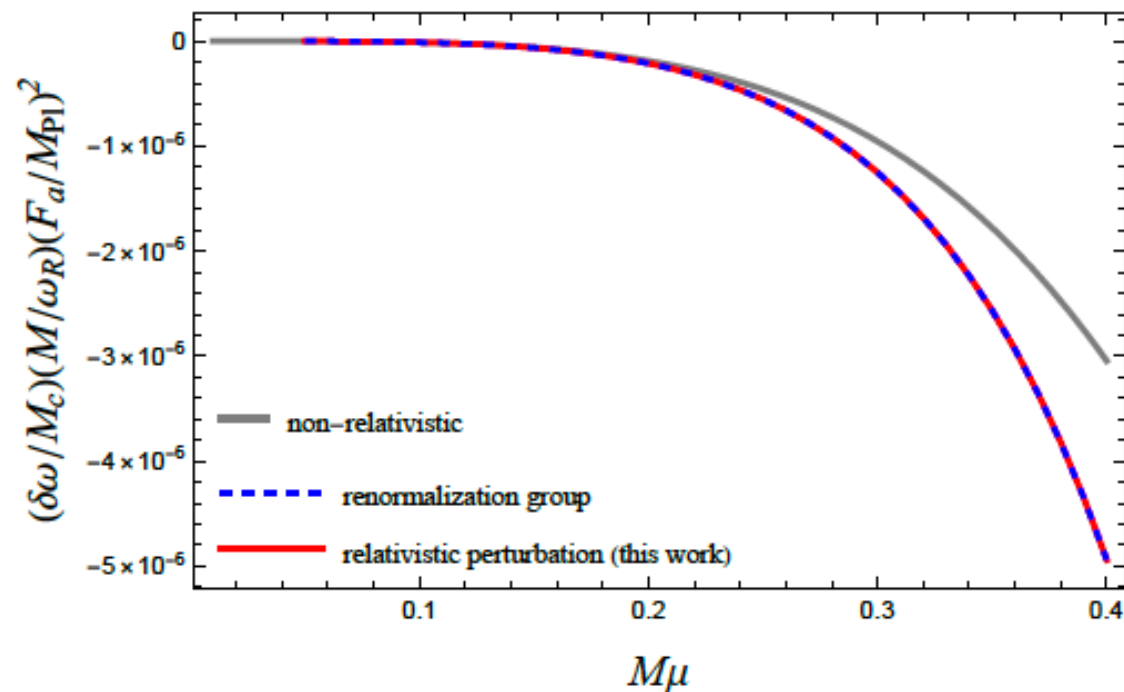
Semi-Newton approximation:

$$\nabla^2 U = 4\pi T_{tt}$$

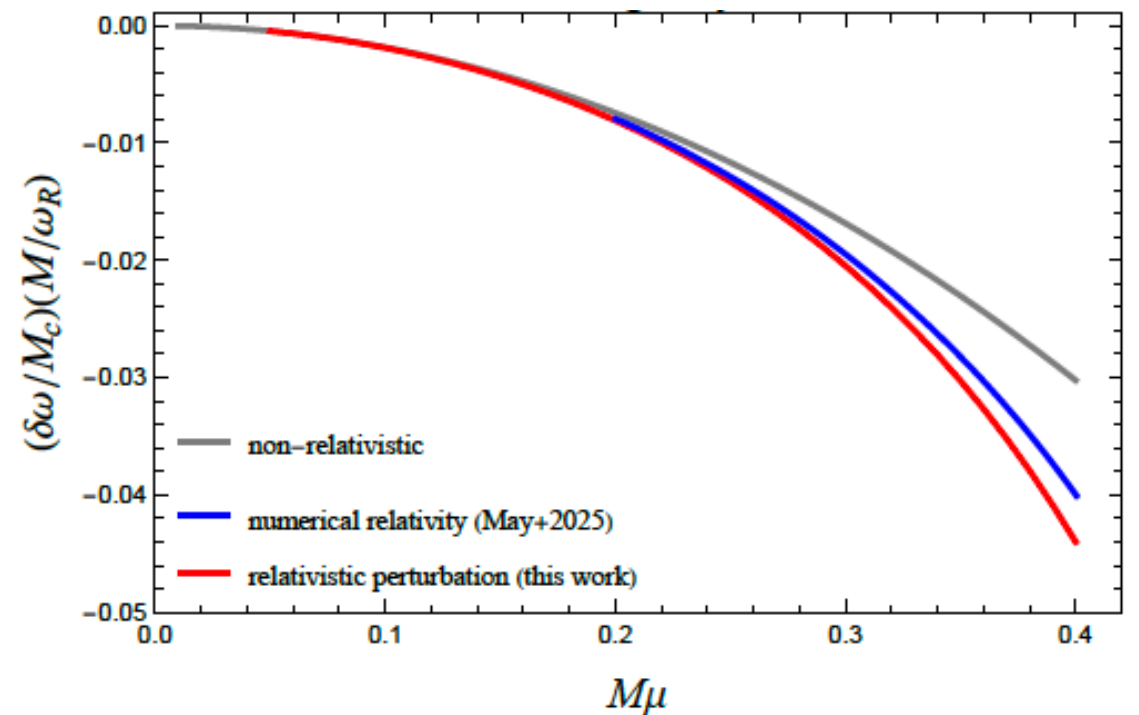
$$\implies \delta H_j^{\text{SG}} = \begin{pmatrix} 0 & \frac{1}{\sqrt{-g}} 4U_j \\ 2\sqrt{-g}\mu^2 U_j & 0 \end{pmatrix}$$

Result 1: Comparison

Self-interaction

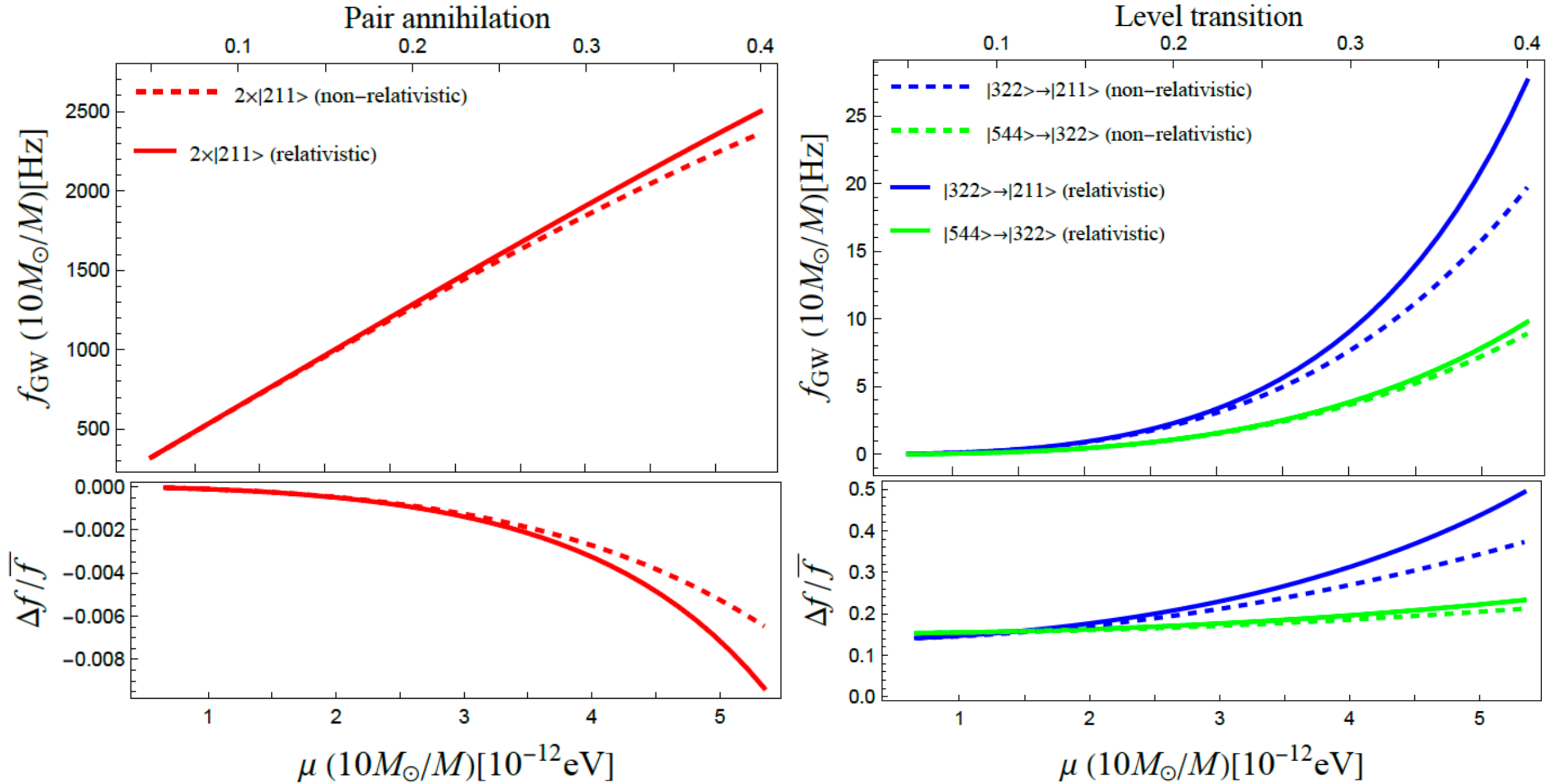


Self-gravity



- Frequency shift of the $(n,l,m)=(2,1,1)$ mode
- The self-interaction result is in excellent agreement with the renormalization group analysis of Omiya et al. (2021).
- The self-gravity contribution differs from numerical relativity simulations (May et al. 2025) by at most about 10%.

Result 2: GW Frequency



$$M_c = 0.1M \quad F_a = 10^{-2}M_{\text{Pl}} \quad a/M = 0.99$$

Summary

- Gravitational waves from axion clouds are important observational targets.
- Self-interactions lead to various emission channels across multiple frequency bands.
- Frequency corrections are calculated using relativistic perturbation theory.
- This method applies to various emission channels and multiple simultaneously excited modes.
- This provides a basis for waveform modeling for continuous GW searches.