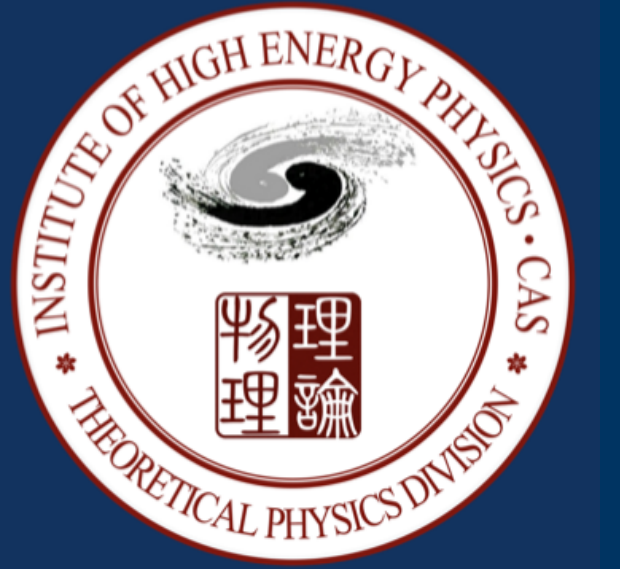




Causal Green function decomposition for quantum black hole seismology

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2026.8.25

Beijing

Based on: Arxiv: [2607.26010](https://arxiv.org/abs/2607.26010)

Collaboration with Jing Ren

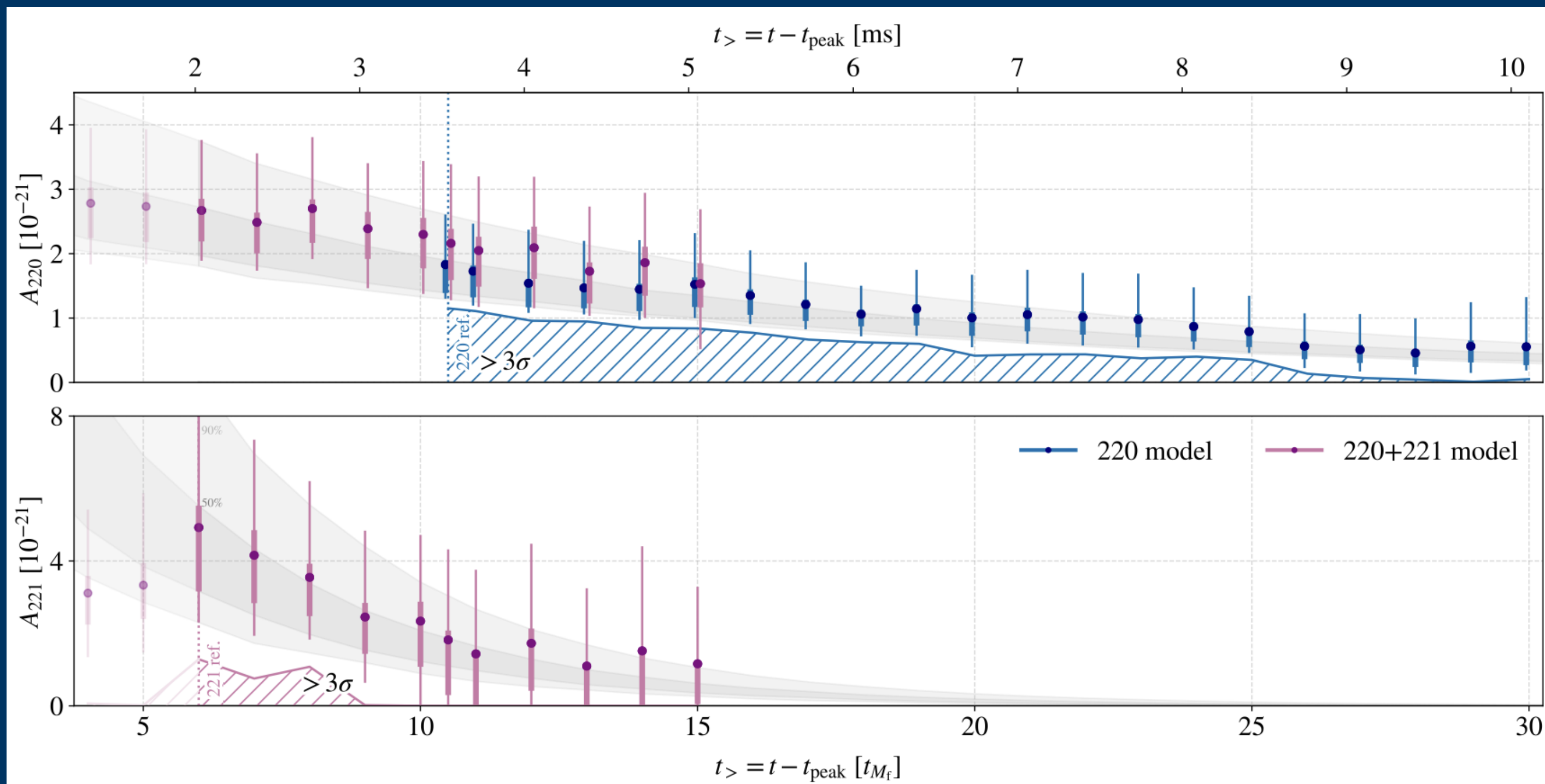
2nd New Horizons for Psi Workshop

I. Background and Motivation

Black Hole Spectroscopy

★ *GW250114*: test the Kerr nature of black holes

- High ringdown SNR enables precise black hole spectroscopy
- Confident detection of the fundamental mode (220) and its first overtone (221)
- Consistency with Kerr frequencies to 30%

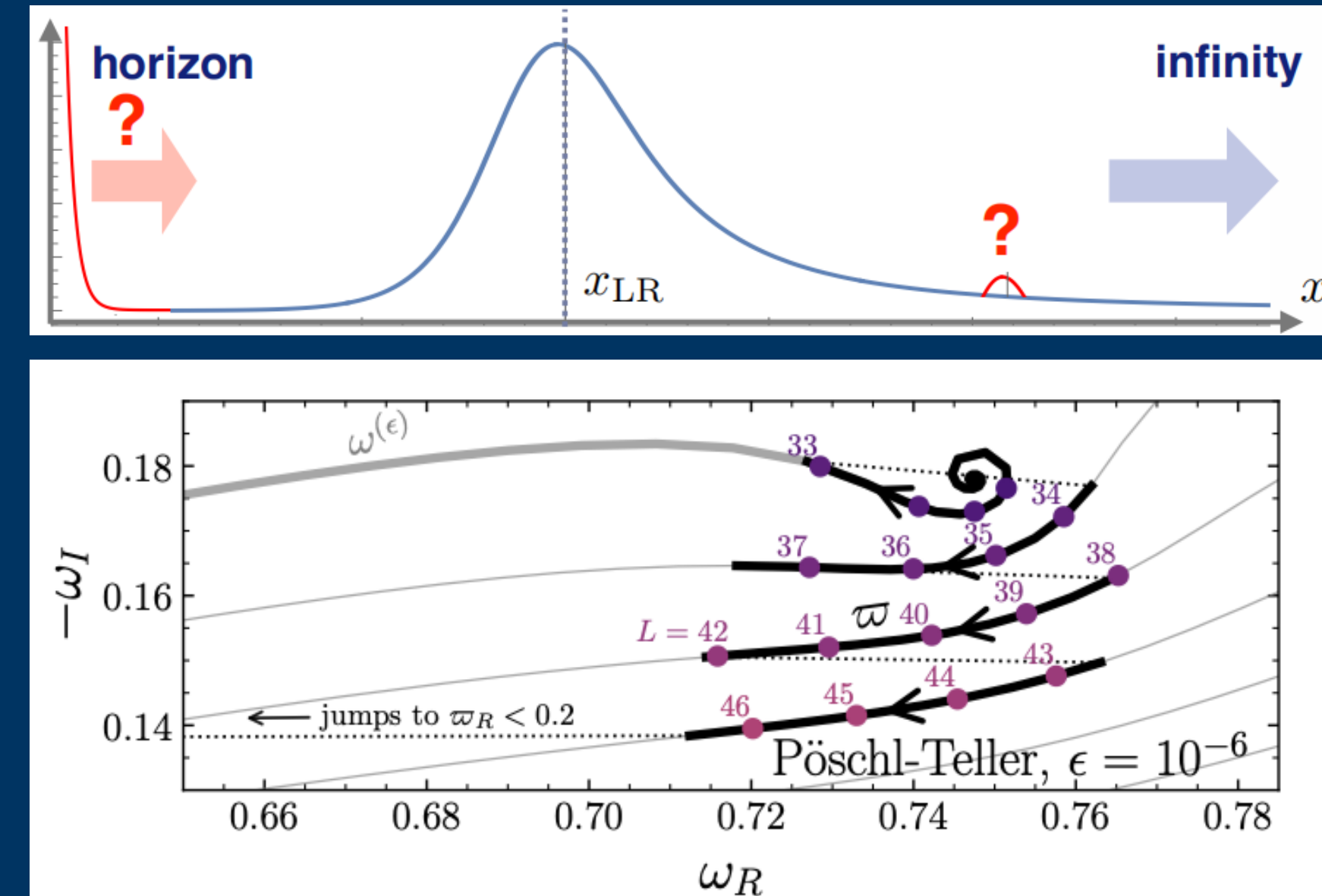


- Even with a complete and precisely measured QNM spectrum, can ringdown establish the existence of a horizon?

Spectral Instability vs Waveform Stability

★Spectral instability in FD

- **Astrophysical environment** may produce small bump on the potential at $x \gg x_{LR}$
- **Quantum black holes** near-horizon corrections may change the potential or ingoing boundary condition at $x \ll x_{LR}$
- Both of them can cause a large drift of the BH QNM spectrum.



[Cheung et al.PRL 128(2022)]

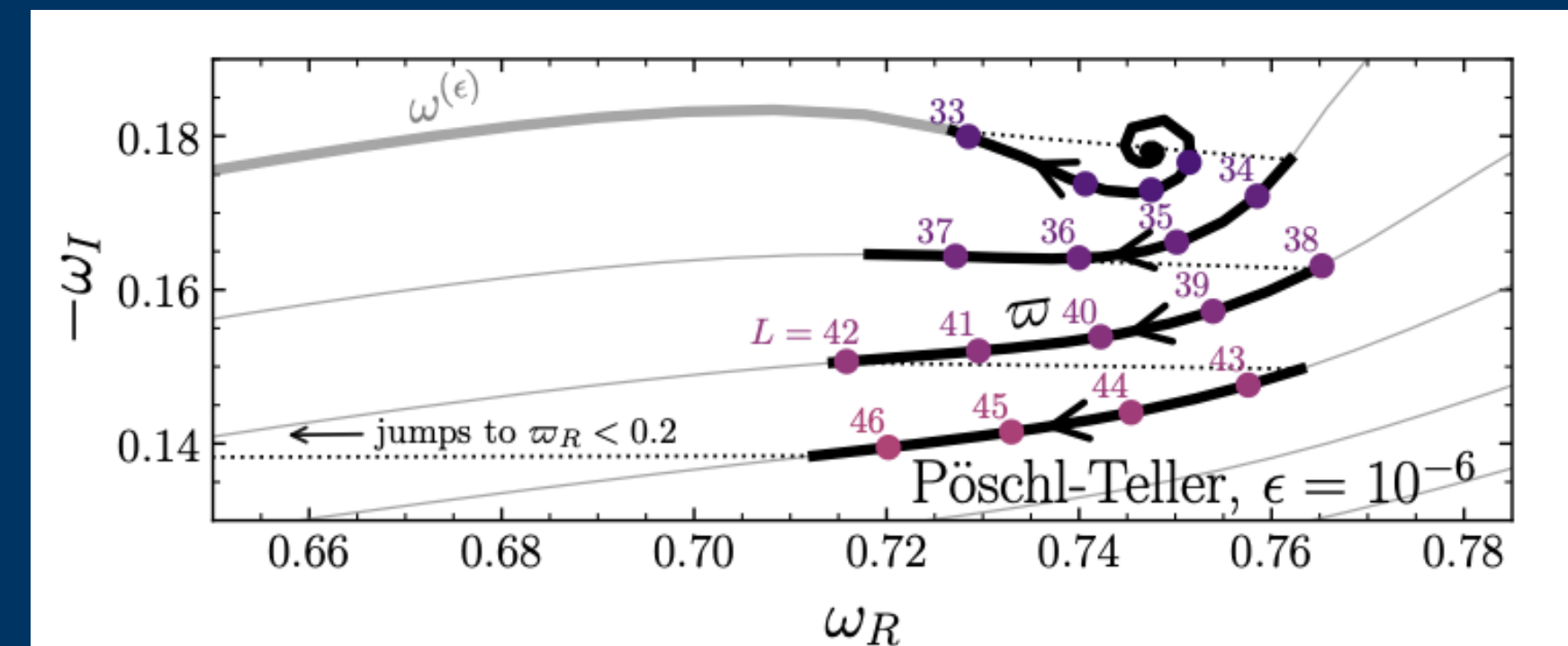
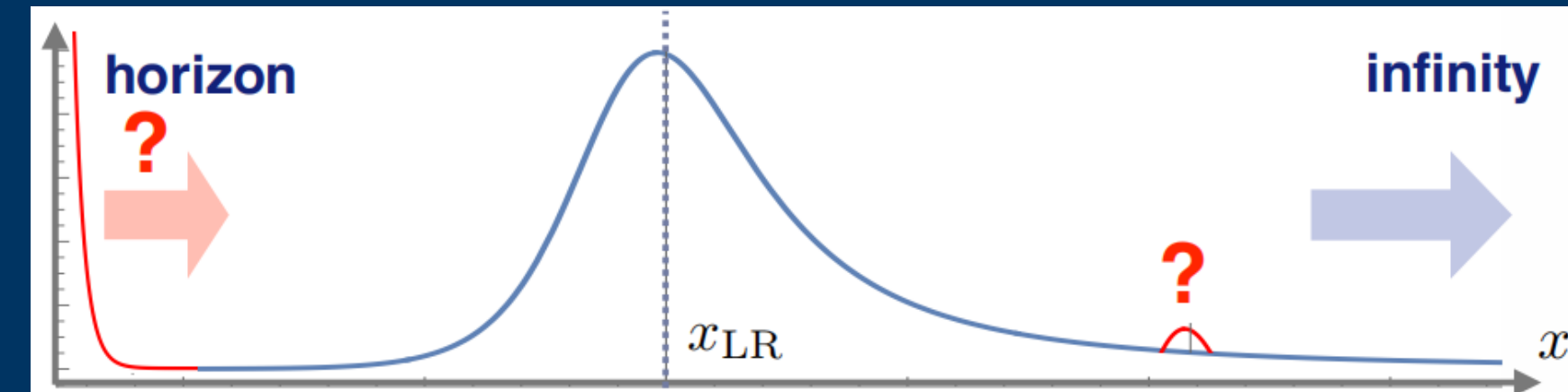
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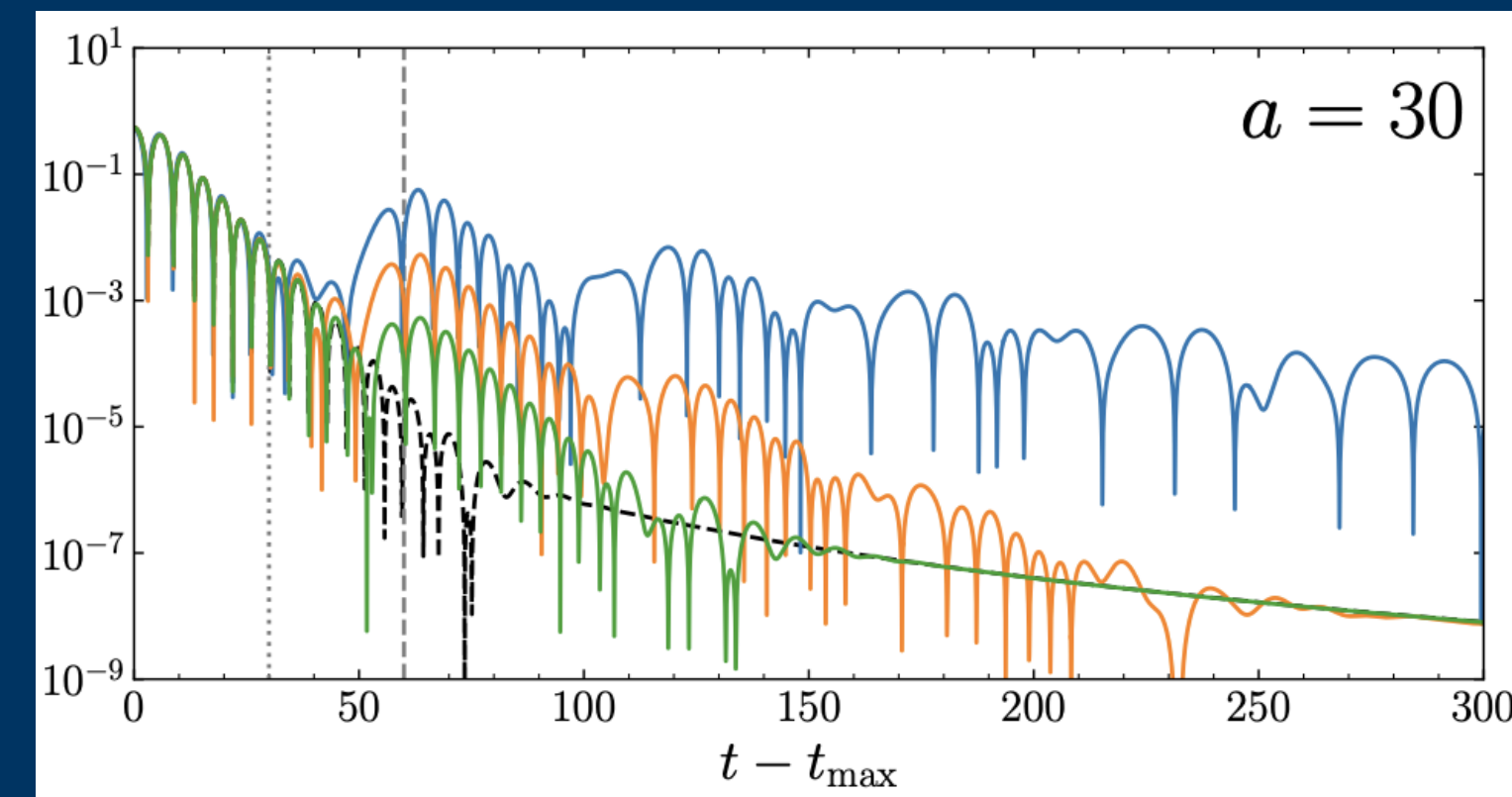
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★Waveform stability in TD

- The prompt signal closely resembles the vacuum black-hole response.
- **Causality:** it takes time to see corrections to vacuum BH potential



[Cheung et al. PRL 128(2022)]

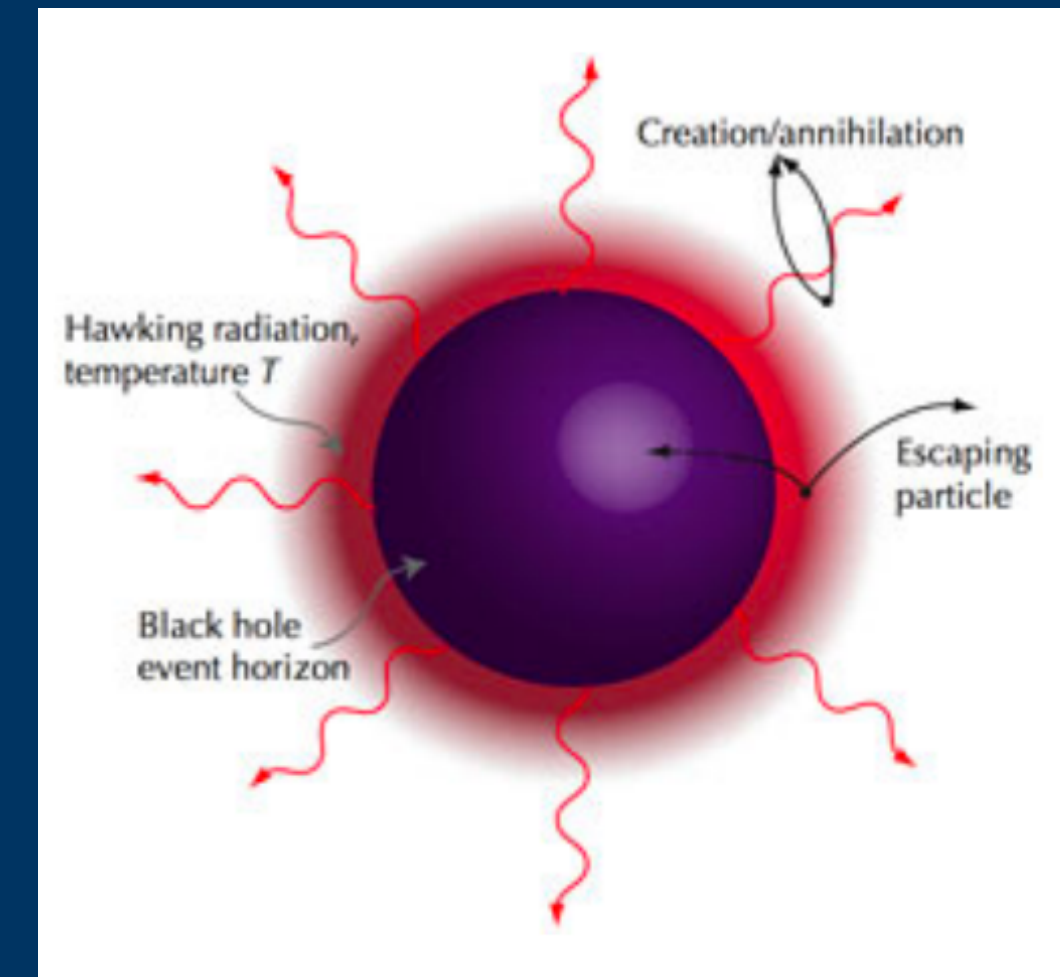


[E. Berti et al. PRD 106 (2022)]

Quantum Black Hole Seismology

★ Why Quantum Black Hole?

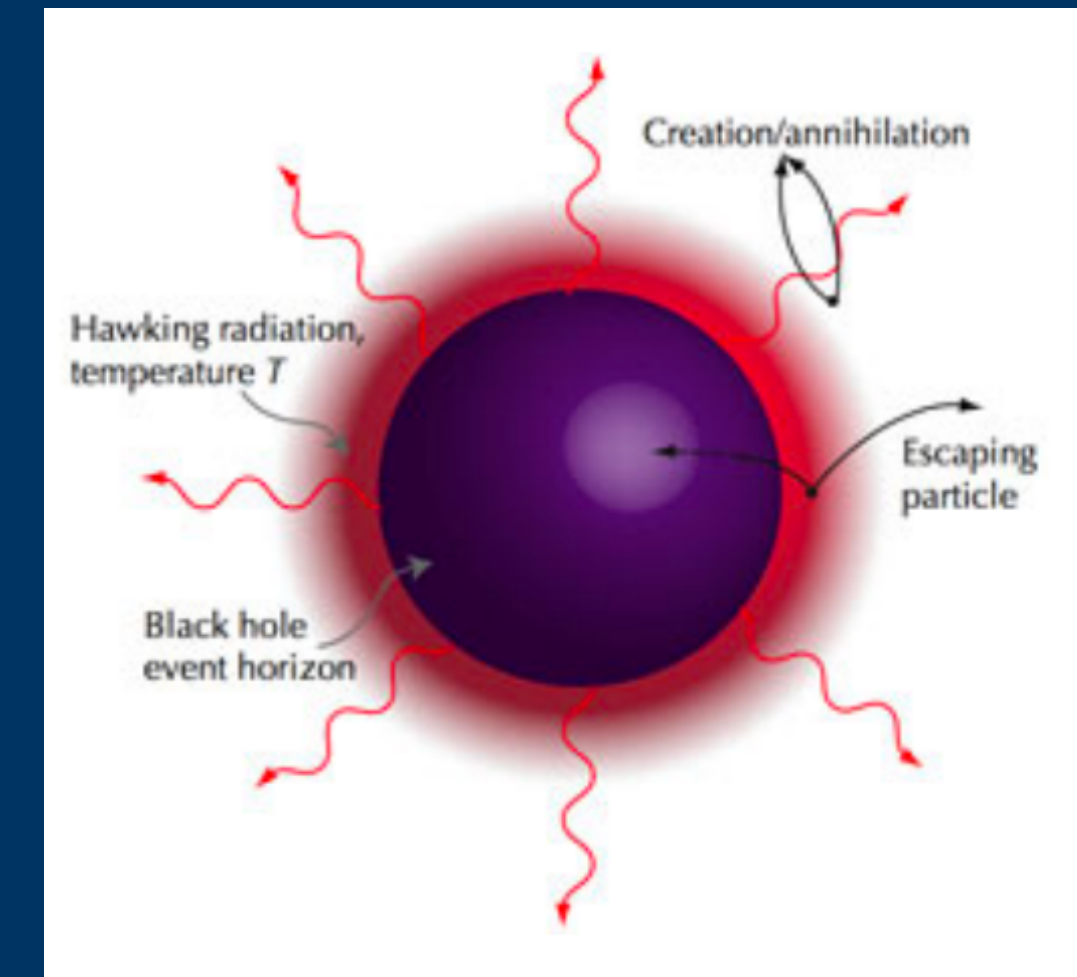
- Black hole thermodynamics (i.e. entropy area law), and information loss problems for evaporating black holes suggest the new physics
- **Quantum black holes** may feature strong deviations around horizon, or even be *horizonless* ultracompact objects (UCOs)



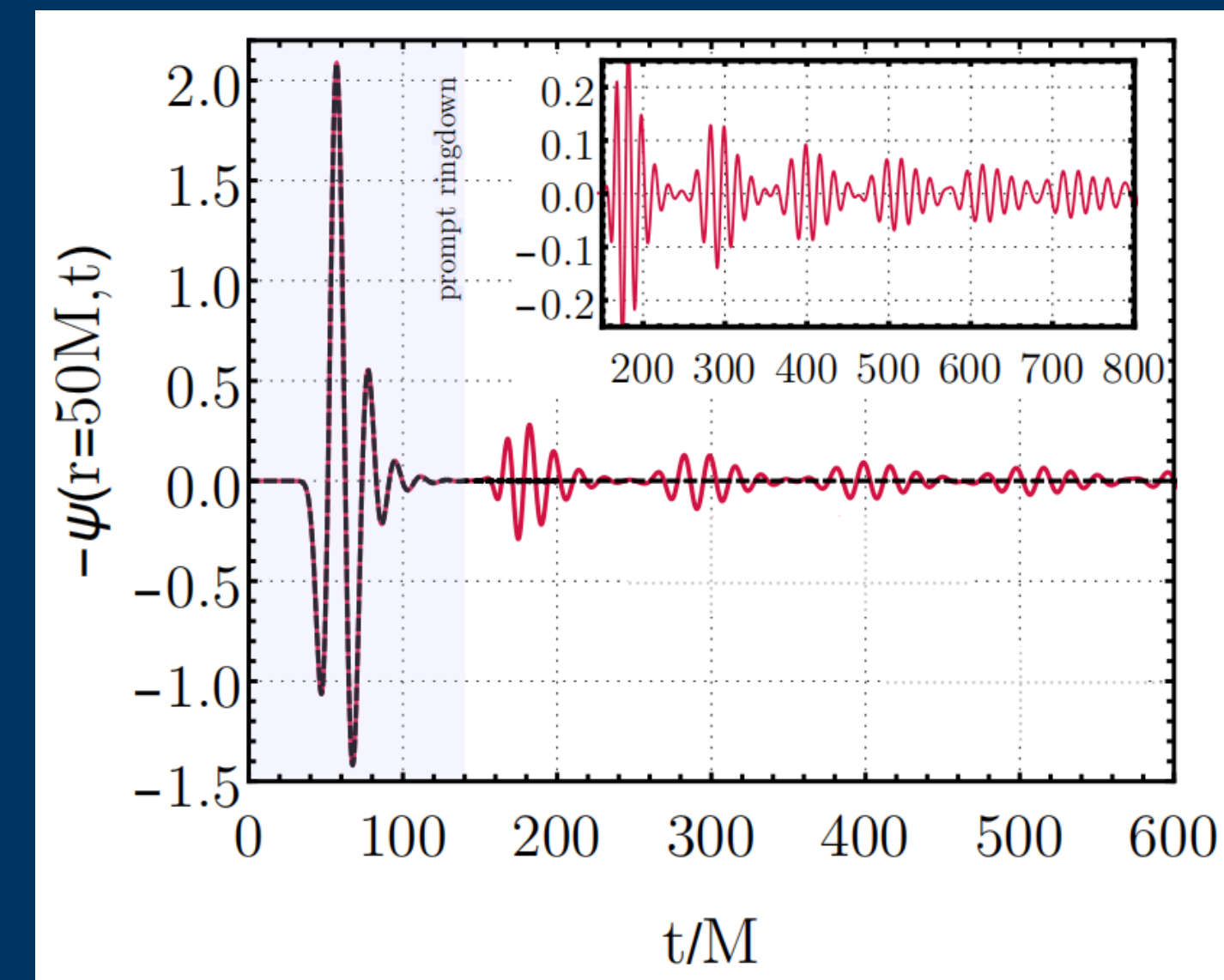
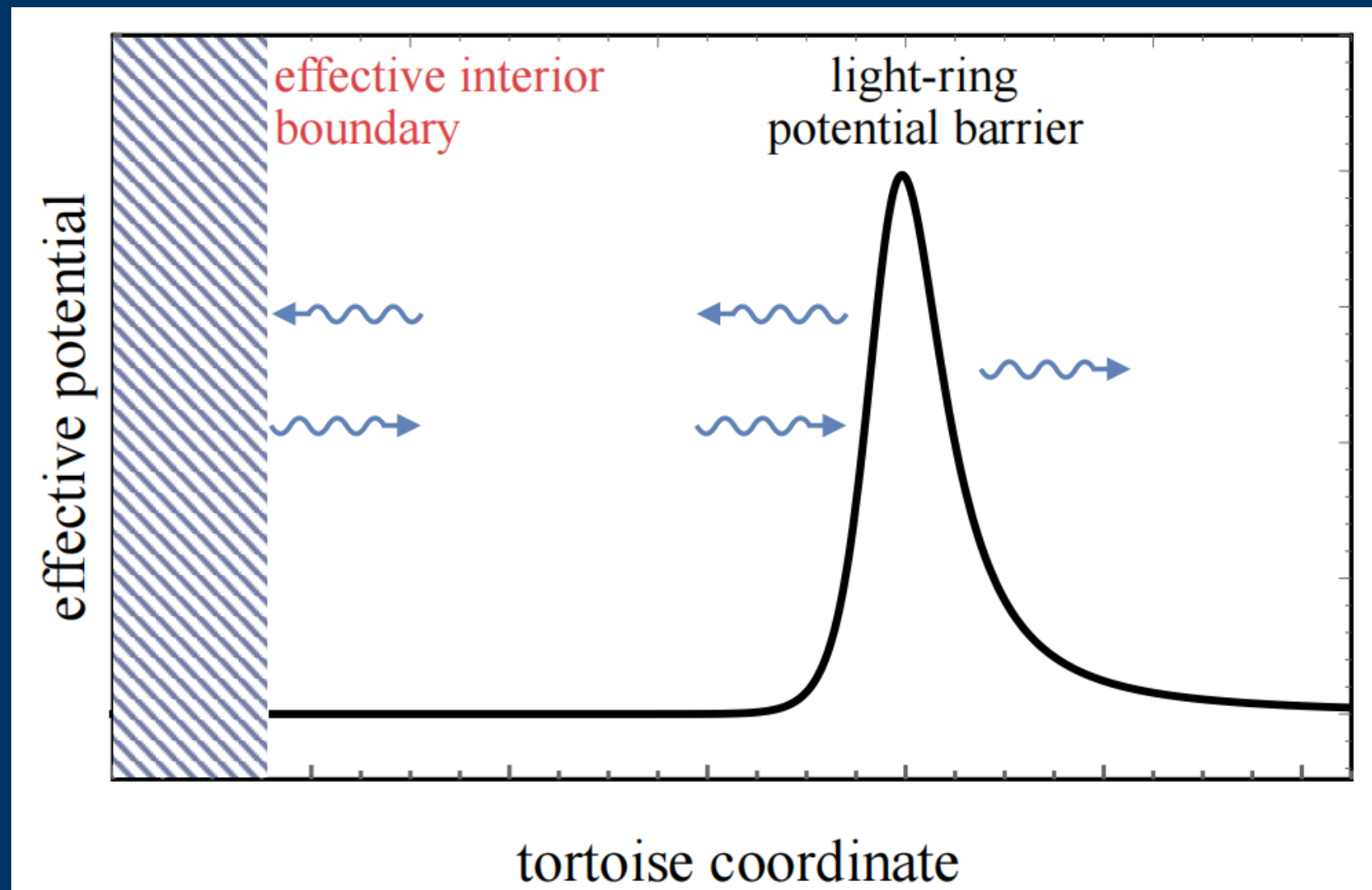
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★Gravitational Wave Echo



[V. Cardoso et al. LRR 22(2019)]

★**Puzzle:** • Why does the signal before echos stay BH prompt ringdown?

II. Causal decomposition for Green Function

Black Hole Perturbation Theory

★Regge-Wheeler Equation

$$\left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial t^2} - V(r) \right) \psi(t, x) = 0$$

★Green Function

$$\tilde{G}(\omega, x, x') = \frac{1}{W(\omega)} X^+(\omega, x) X^-(\omega, x')$$

$$G(t, x, x') = \frac{1}{2\pi} \int_{-\infty+i\epsilon}^{\infty+i\epsilon} \tilde{G}(\omega, x, x') e^{-i\omega t} d\omega$$

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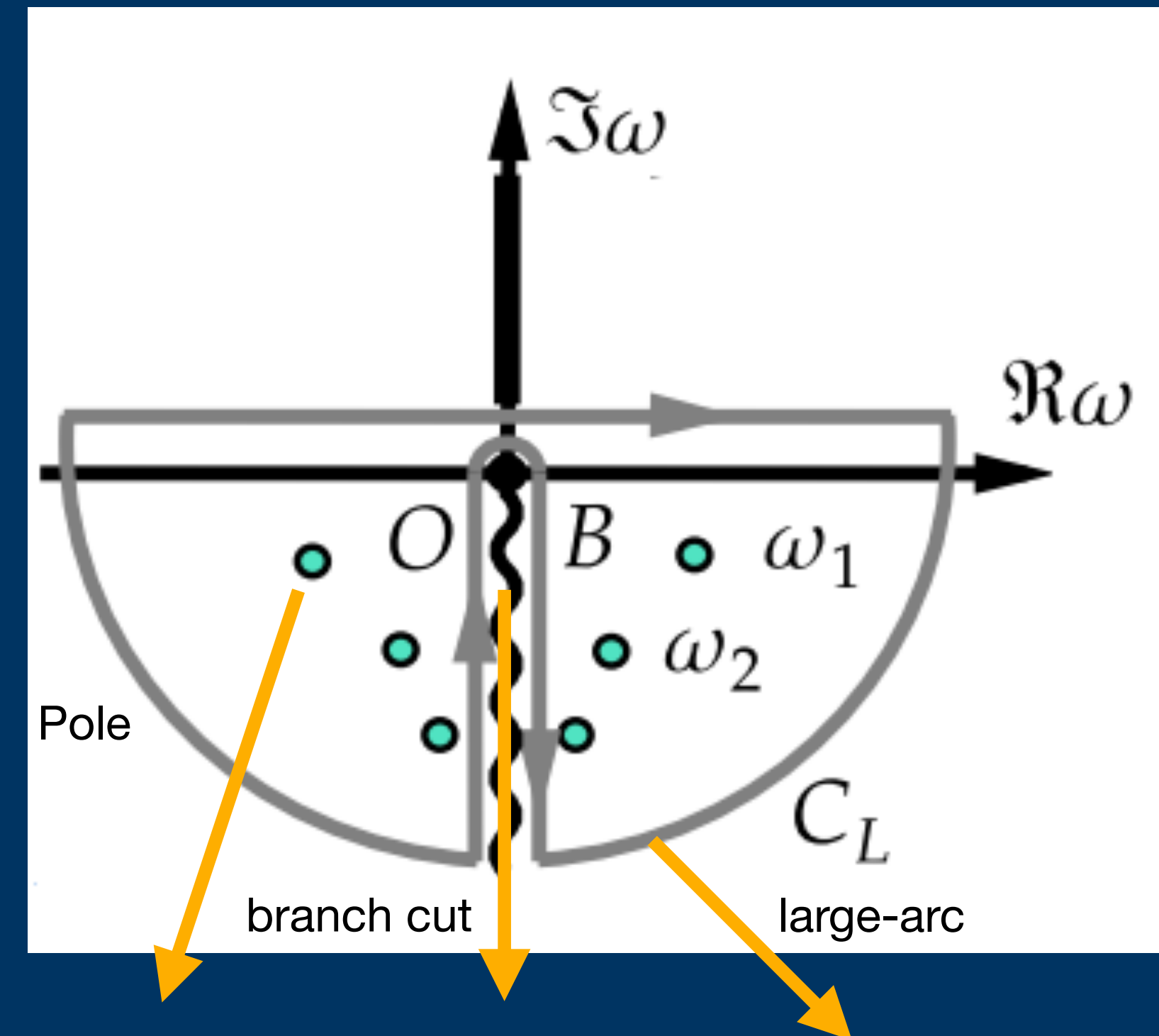
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★Contour Integration

$$G(t, x, x') = \frac{1}{2\pi} \oint_C \tilde{G}(\omega, x, x') e^{-i\omega t} d\omega = G_{\text{QNM}}(t, x, x') + G_{\text{tail}}(t, x, x') + G_{\text{dir}}(t, x, x').$$

[E.Leaver PRD 34(1986)]



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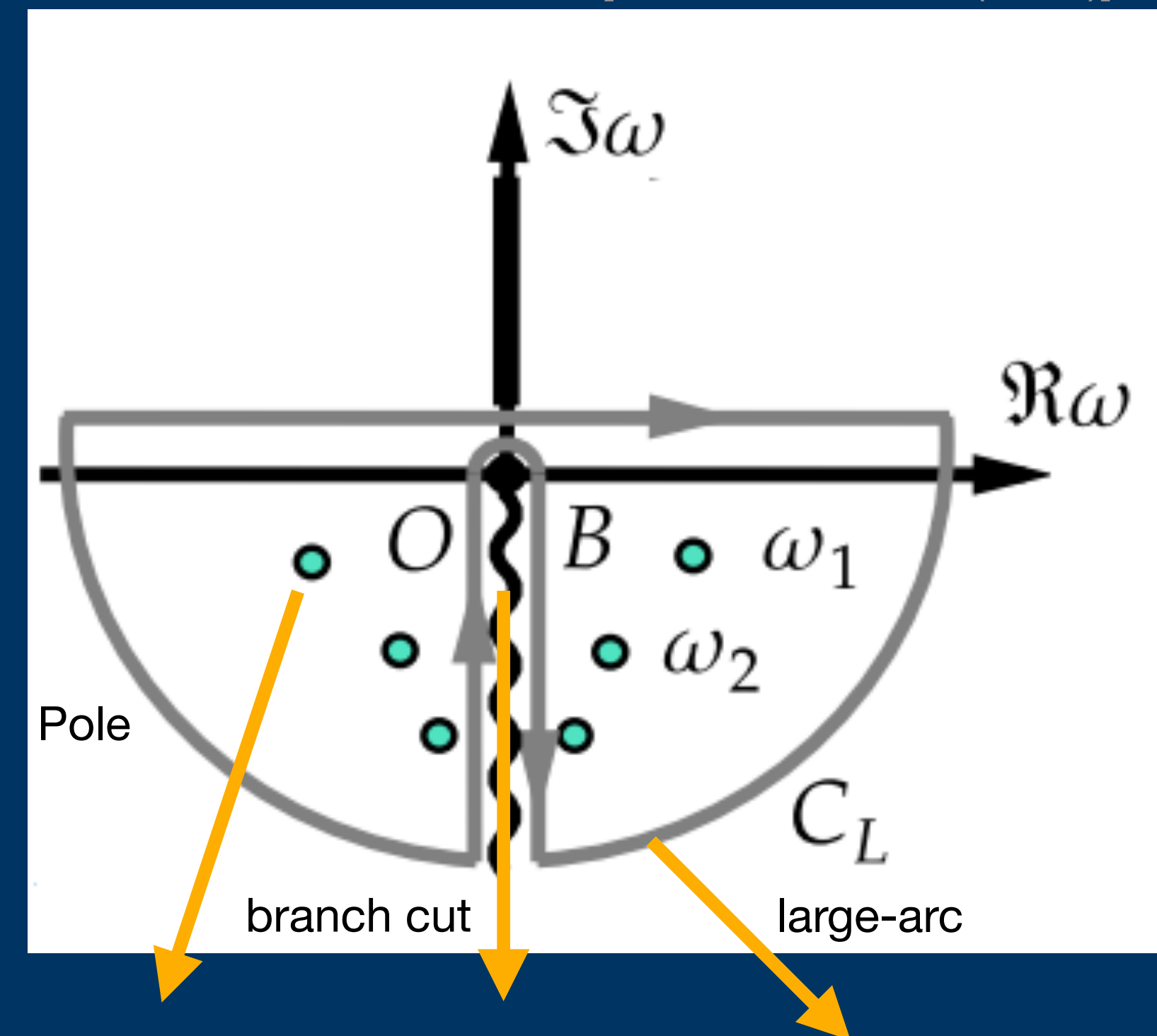
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★Problem of Leaver's original formulation

- **Direct part:** large-arc is difficult to compute in convergence limit
- Divergence of **QNMs** contributions at early time

[E.Leaver PRD 34(1986)]



Causal decomposition for BH Green function

★New formulation for BH Green Function

Source outside the light ring [Su et al.PRD 113(2026)]

$$\tilde{G}_{BH}(\omega, x, x') = \tilde{G}_{BH}^{-}(\omega, x, x') + \tilde{G}_{BH}^{+}(\omega, x, x')$$

$$\tilde{G}_{BH}^{-}(\omega, x, x') = \frac{X_{BH}^{+}(\omega, x)X_{BH}^{+}(\omega, x')}{2i\omega} \rightarrow e^{i\omega(x-x')}$$

$$\tilde{G}_{BH}^{+}(\omega, x, x') = \frac{A_{out}^{BH}(\omega)}{A_{in}^{BH}(\omega)} \frac{X_{BH}^{+}(\omega, x)X_{BH}^{+}(\omega, x')}{2i\omega} \rightarrow e^{i\omega(x+x')}$$

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$$e^{-i\omega[t-(x-x')]} : \begin{cases} \text{close in the UHP,} & t < x - x', \\ \text{close in the LHP,} & t > x - x'. \end{cases}$$

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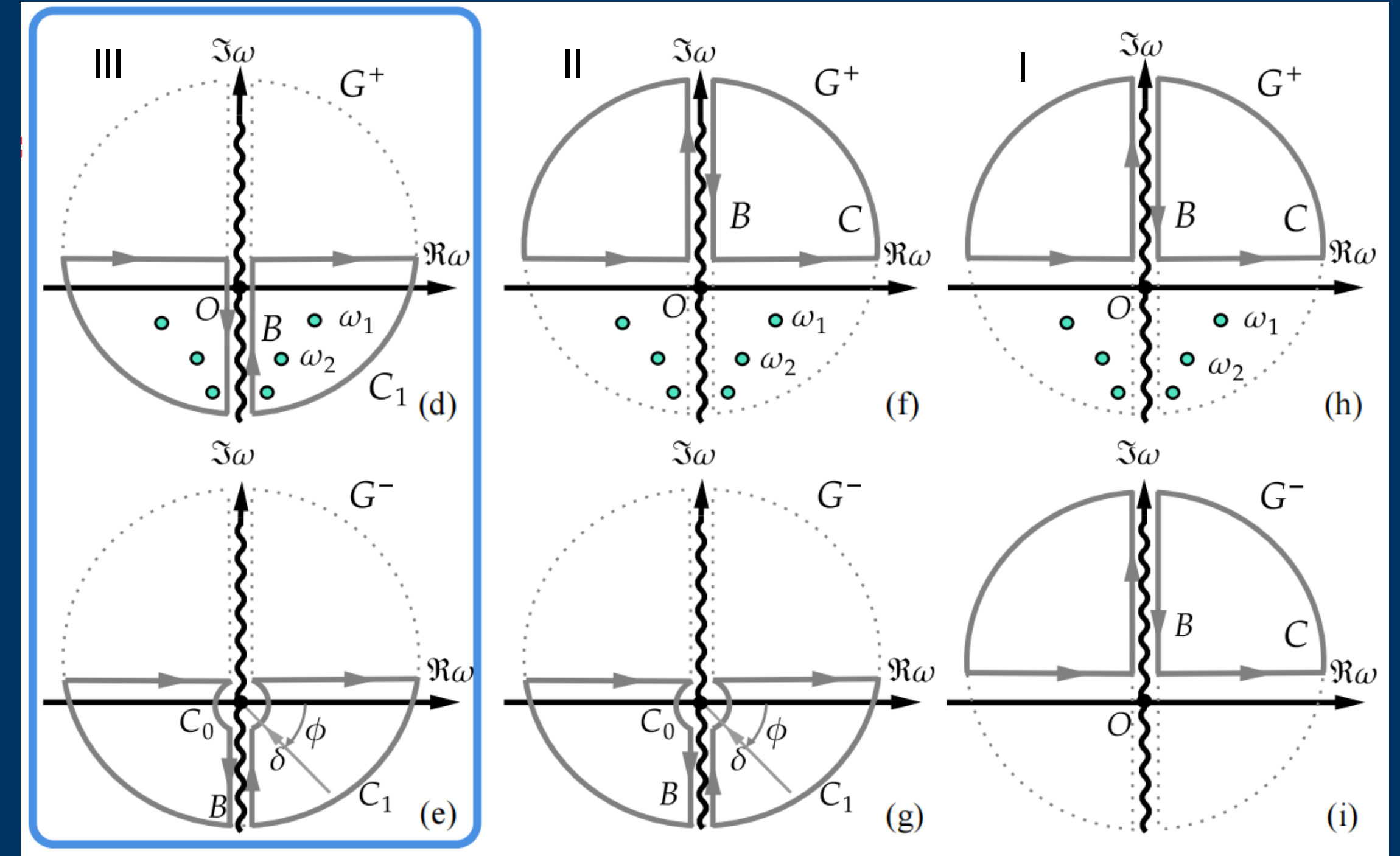
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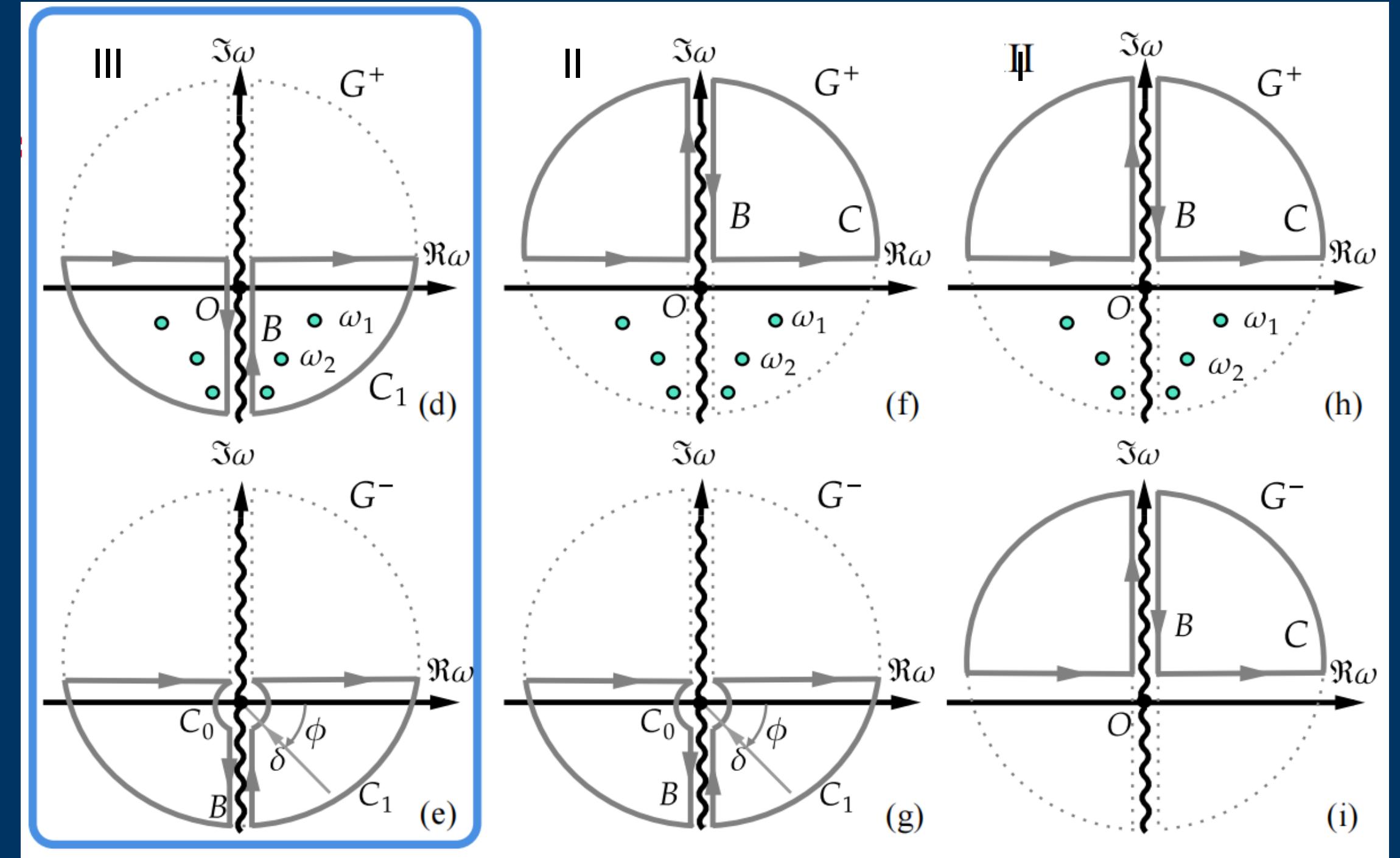
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Stage I ($x - x' > t$): No signal

Stage II ($x - x' < t < x + x'$): direct part from brunch cut

Stage III ($t > x + x'$): QNMs and tail from $\tilde{G}_{BH}$

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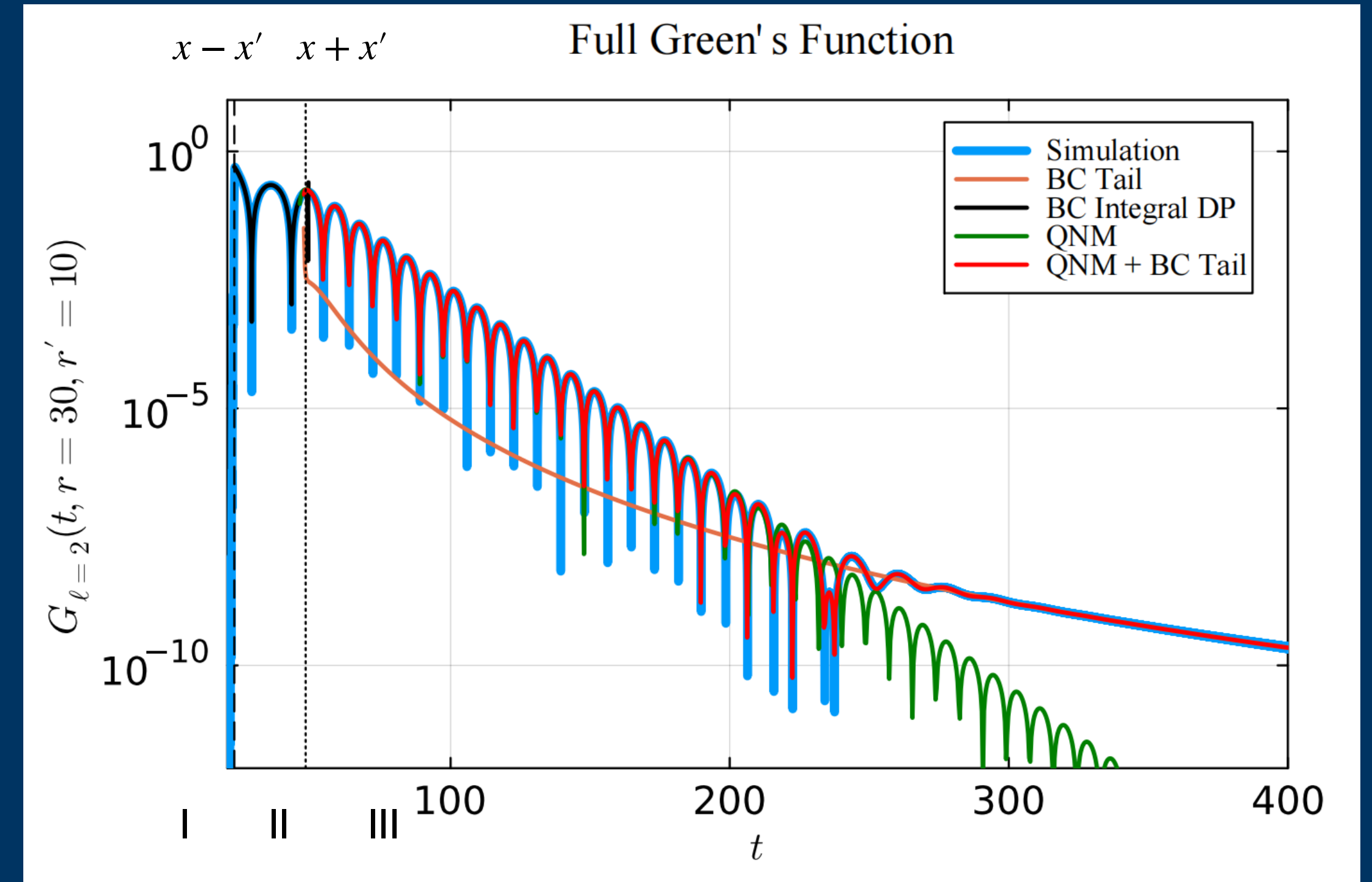
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UCO Waveform Reconstruction

★Frequency-domain UCO Green function

$$\tilde{G}_{\text{UCO}}(\omega, x, x') = \frac{1}{W_{\text{UCO}}(\omega)} X_{\text{BH}}^+(\omega, x) X_{\text{UCO}}^-(\omega, x'), \quad W_{\text{UCO}} = 2i\omega_n A_{\text{in}}^{\text{UCO}} \propto 1 - R_{\text{wall}} R^{\text{BH}} e^{i\omega_n t_d}$$

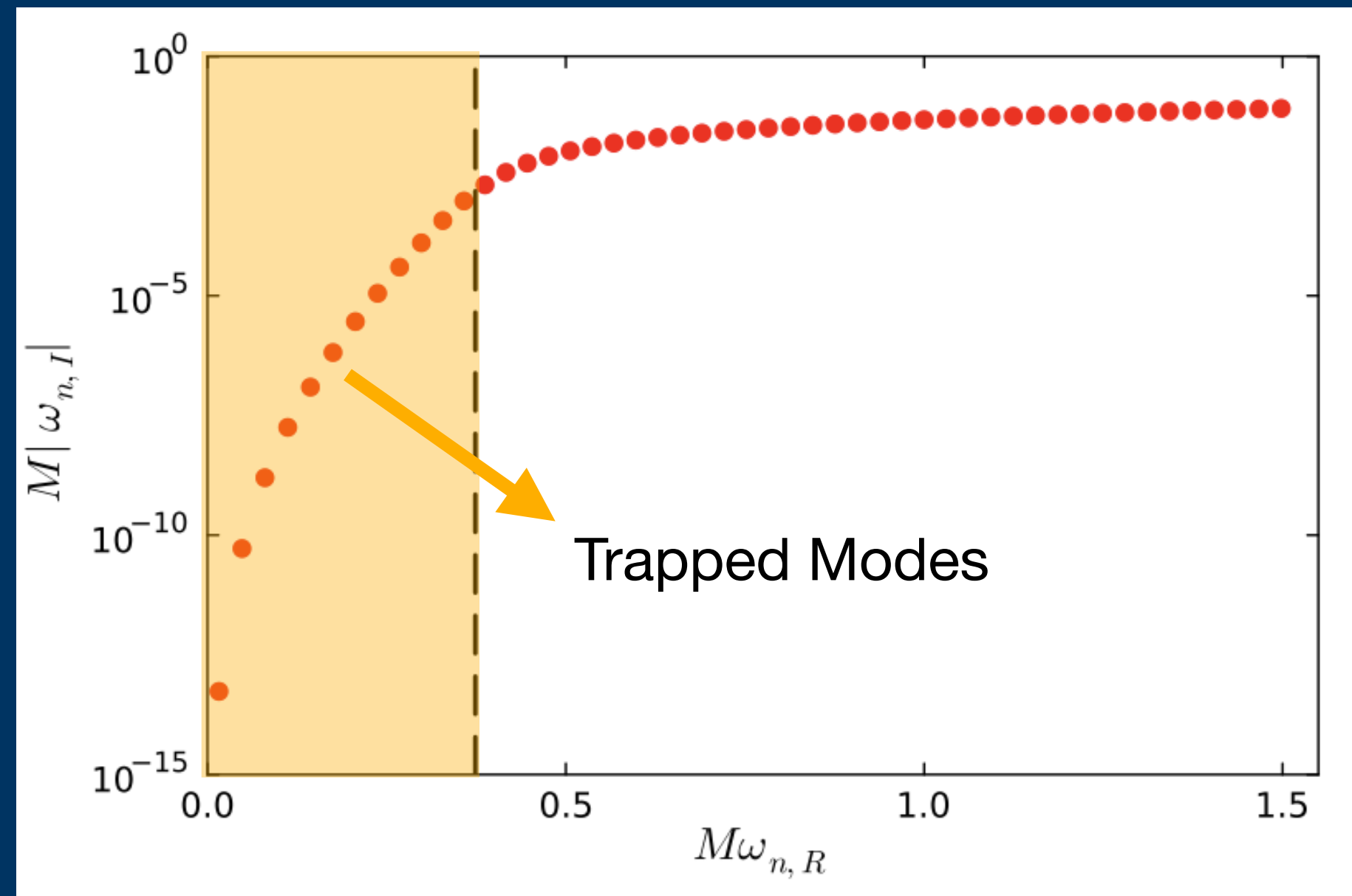
UCO QNM condition: $W_{\text{UCO}}(\omega_n) = 0 \rightarrow 1 - R_{\text{wall}} R_{\text{BH}} e^{i\omega_n t_d} = 0$

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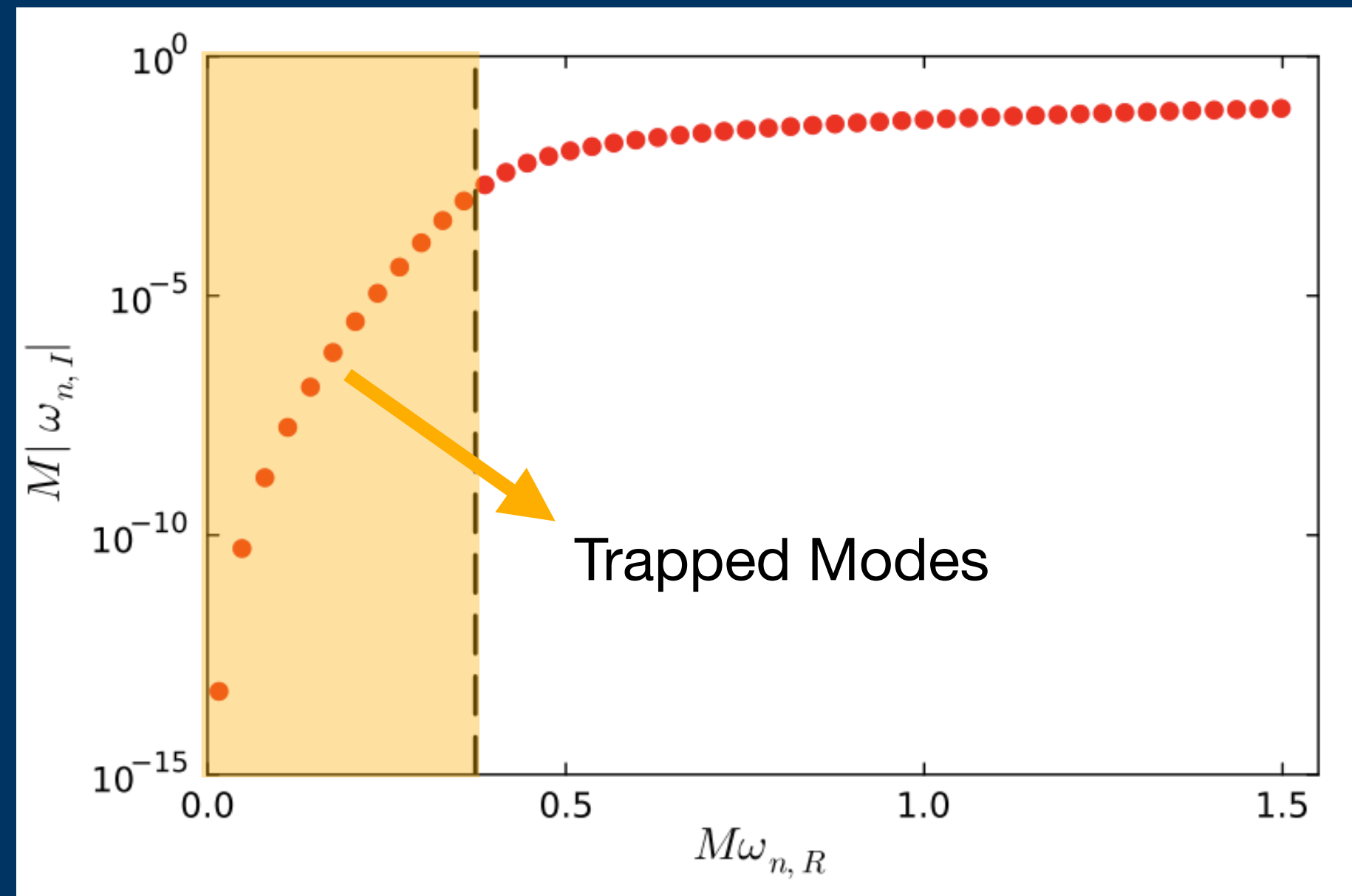
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UCO QNM condition: $W_{\text{UCO}}(\omega_n) = 0 \rightarrow 1 - R_{\text{wall}} R_{\text{BH}} e^{i\omega_n t_d} = 0$



★QNM reconstruction

$$G_{\text{QNM}}(t, x, x') = 2\Re \sum_n B_n e^{-i\omega_n(t-t_{\text{ref}})}$$

• The time-domain Green function is reconstructed by summing the residues of the UCO QNM poles.

- $\Re\omega_n$ increases approximately linearly with n
- $\Im\omega_n$ is small at low n , then increases roughly linearly at high n

Time-domain Green's function for UCOs

(a). Source outside the light ring ($x' \gg x_{LR}$)

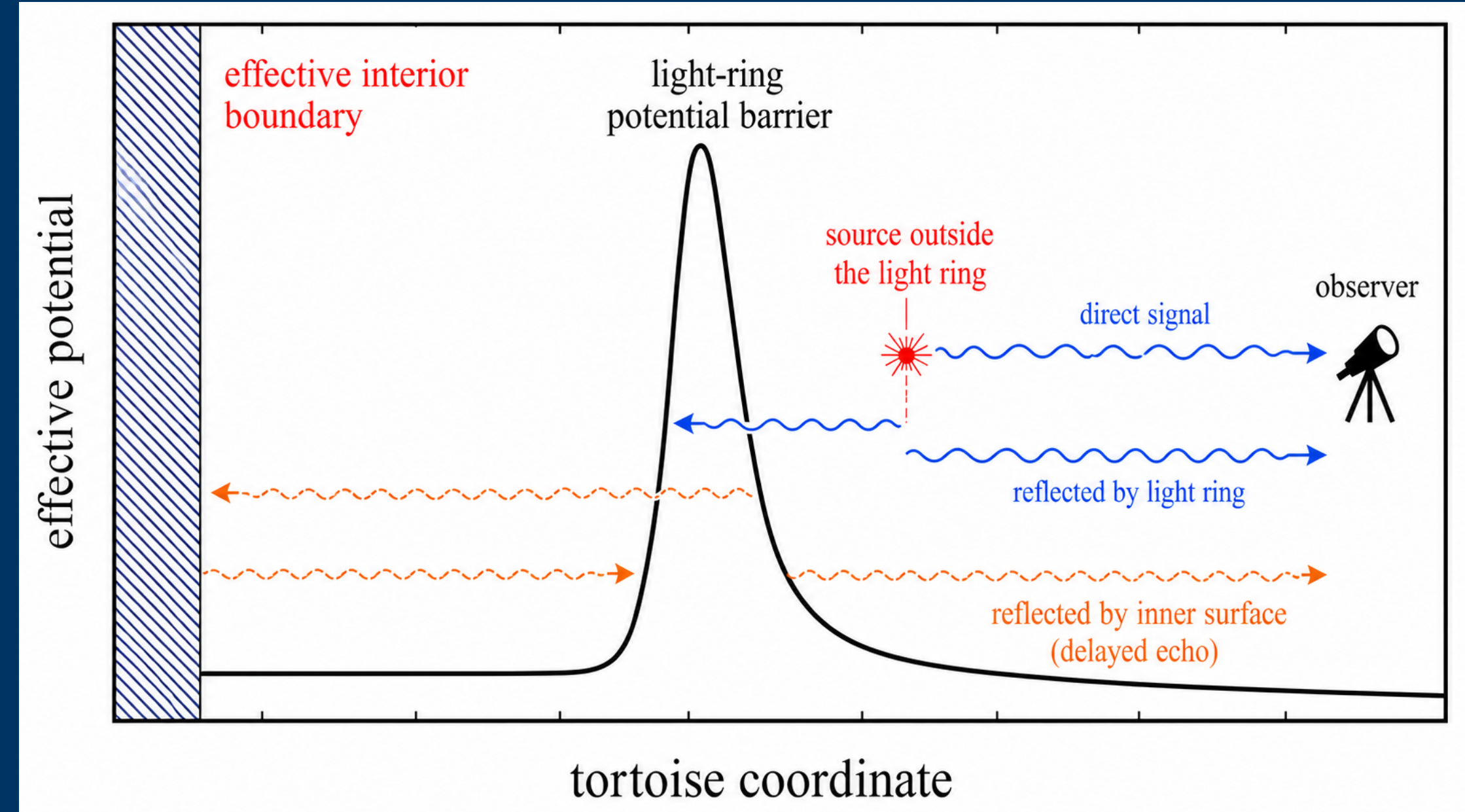
ZXL, J.Ren arxiv: [2607.26010](https://arxiv.org/abs/2607.26010)

$$\tilde{G}_{\text{UCO}}(\omega, x, x') \rightarrow \frac{1}{2i\omega} \left[e^{i\omega(x-x')} + \frac{A_{\text{out}}^{\text{UCO}}(\omega)}{A_{\text{in}}^{\text{UCO}}(\omega)} e^{i\omega(x+x')} \right].$$

★Causal Decomposition

$$\tilde{G}_{\text{UCO}}(\omega, x, x') = \tilde{G}_{\text{BH}}^{-}(\omega, x, x') + \tilde{G}_{\text{UCO}}^{+}(\omega, x, x')$$

$$\tilde{G}_{\text{UCO}}^{+}(\omega, x, x') = \frac{A_{\text{out}}^{\text{UCO}}}{A_{\text{in}}^{\text{UCO}}} \frac{X_{\text{BH}}^{+}(\omega, x) X_{\text{BH}}^{+}(\omega, x')}{2i\omega}$$



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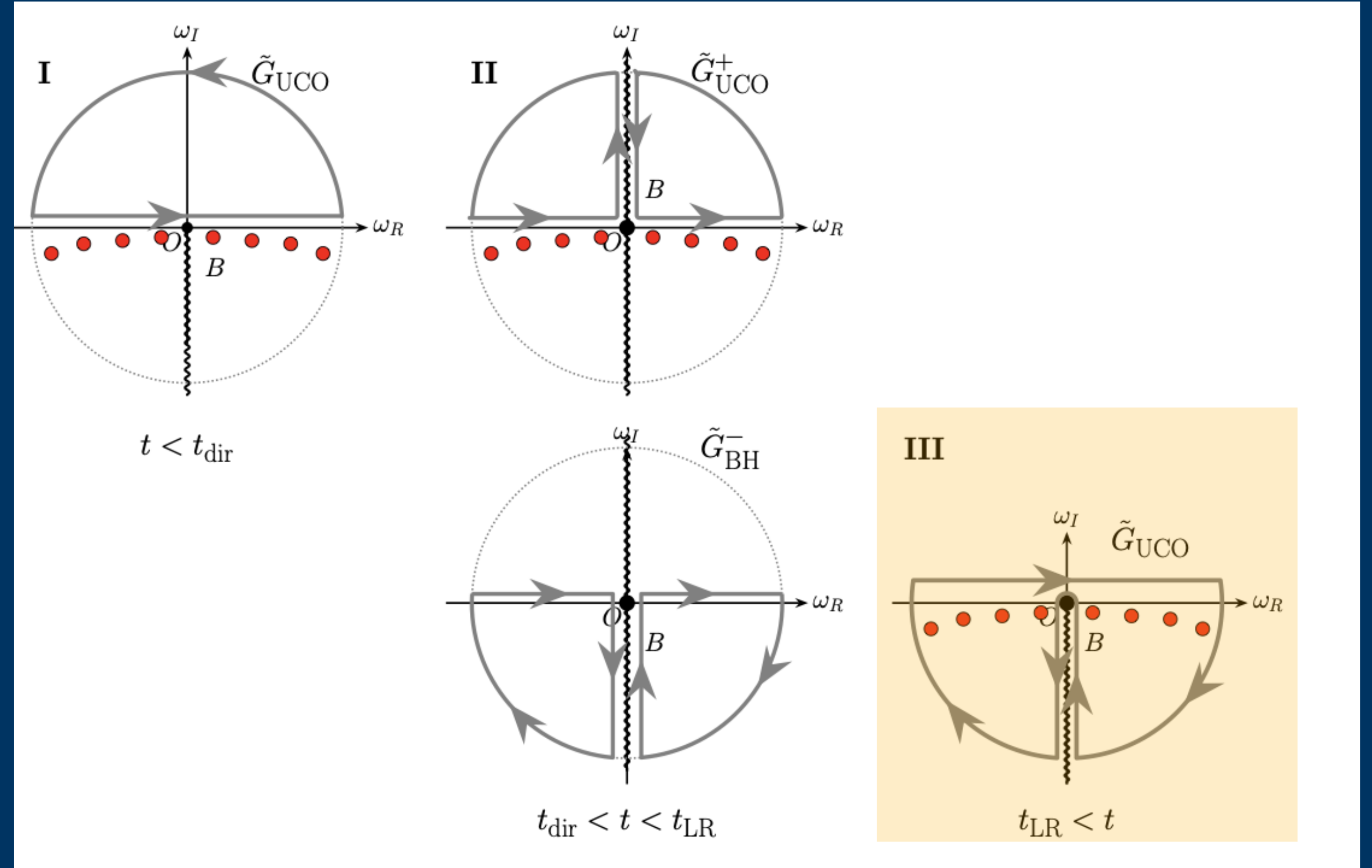
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Stage III ($t_{\text{LR}} < t$): QNMs, tail and echo from $\tilde{G}_{\text{UCO}}$



where $t_{\text{dir}} = x - x'$ $t_{\text{LR}} = x + x'$

•how can these UCO poles reproduce the black-hole prompt ringdown?

Causal decomposition for UCO Green function

(a). Source outside the light ring ($x' \gg x_{LR}$)

$$\tilde{G}_{\text{UCO}}(\omega, x, x') \rightarrow \frac{1}{2i\omega} \left[e^{i\omega(x-x')} + \frac{A_{\text{out}}^{\text{BH}}(\omega)}{A_{\text{in}}^{\text{BH}}(\omega)} e^{i\omega(x+x')} + \frac{T_{\text{BH}}^2 R_{\text{wall}} e^{-2i\omega x_0}}{1 - R_{\text{BH}} R_{\text{wall}} e^{i\omega t_d}} e^{i\omega(x+x'+t_d)} \right].$$

★Equivalent Decomposition

$$\tilde{G}_{\text{UCO}}^+(\omega, x, x') = \tilde{G}_{\text{BH}}^+(\omega, x, x') + \tilde{G}_{\text{echo}}^+(\omega, x, x')$$

$$\tilde{G}_{\text{echo}}^+(\omega, x, x') = \frac{T_{\text{BH}}^2 R_{\text{wall}} e^{-2i\omega x_0}}{1 - R_{\text{BH}} R_{\text{wall}} e^{i\omega t_d}} \frac{X_{\text{BH}}^+(\omega, x) X_{\text{BH}}^+(\omega, x')}{2i\omega}$$

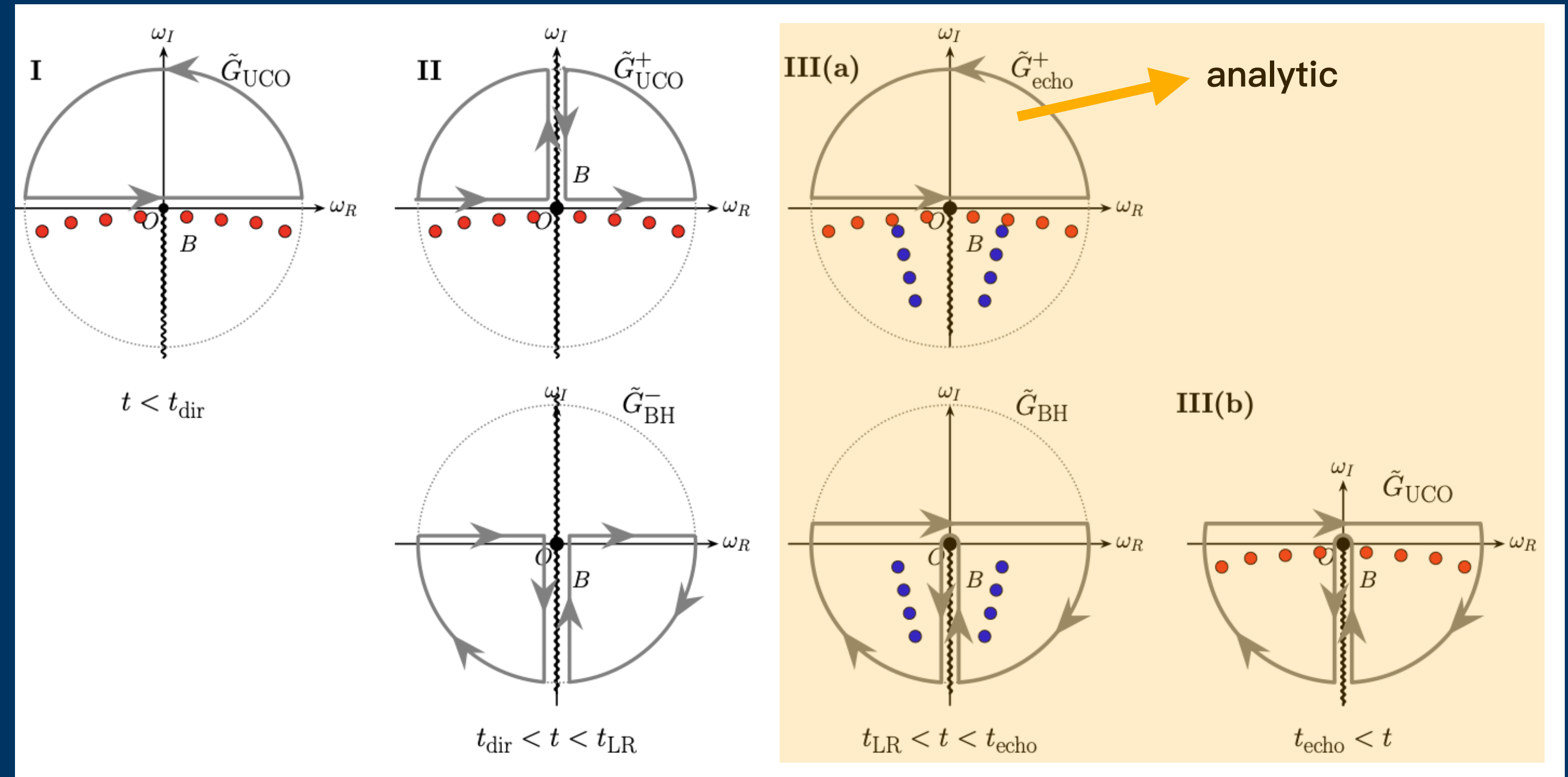
[Z.Mark et al.PRD 96(2017)]

Stage III(a) ($t_{\text{LR}} < t < t_{\text{echo}}$): QNMs, power law tail from $\tilde{G}_{\text{BH}}^+$

Stage III(b) ($t_{\text{echo}} < t$): Echos from $\tilde{G}_{\text{UCO}}$

★Answer:

•UCO-QNM sum is causally equivalent to the BH QNM+tail.



Causal decomposition for UCO Green function

(b). Source inside the light ring ($x' \ll x_{LR}$)

$$\tilde{G}_{\text{UCO}}(\omega, x, x') \rightarrow \frac{1}{2i\omega A_{\text{in}}^{\text{UCO}}} \left[e^{i\omega(x-x')} + R_{\text{wall}} e^{i\omega(x+x')} \right].$$

★Causal Decomposition

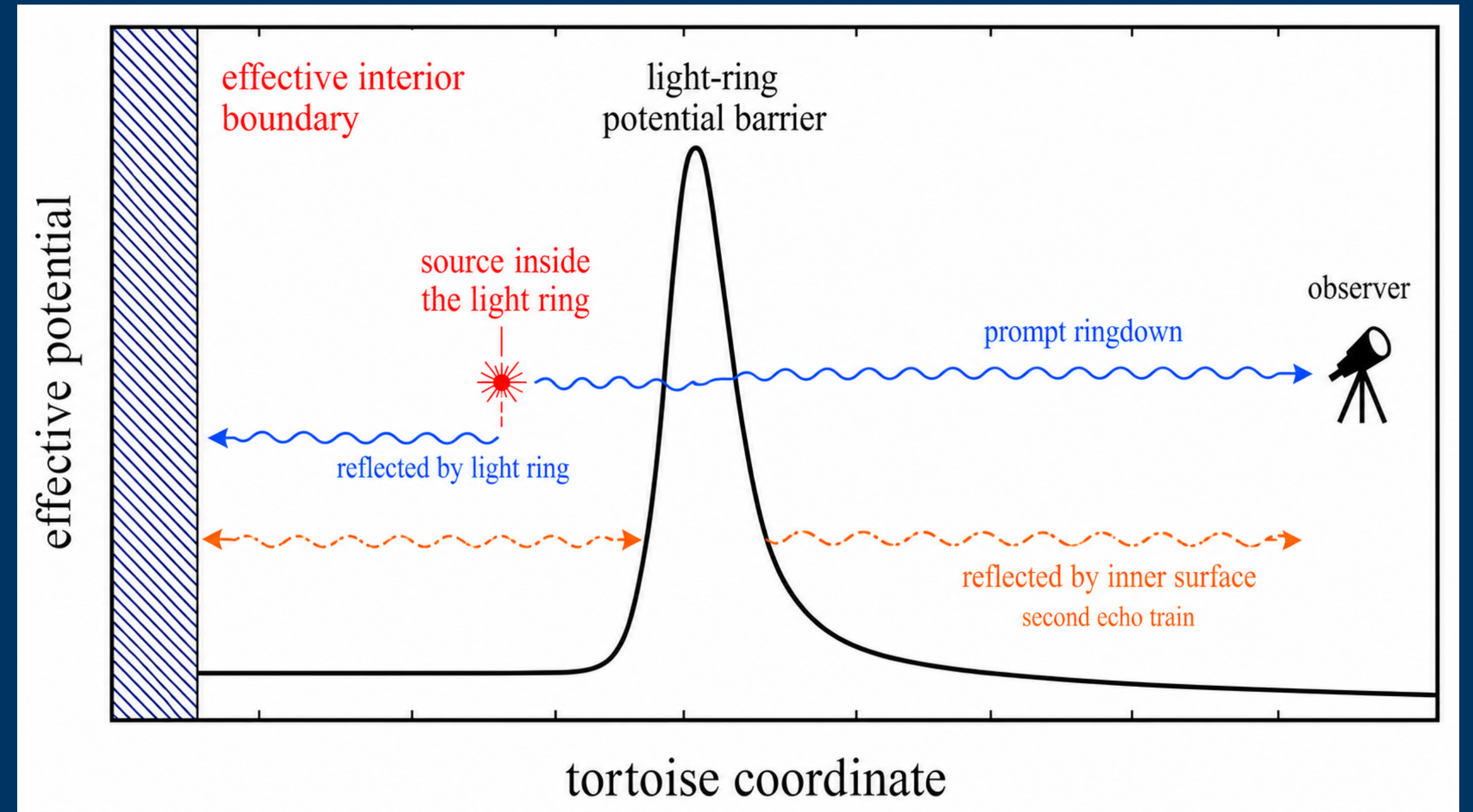
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Stage II ($t_{\text{LR}} < t < t_{\text{ref}}$): **BH QNMs** and **part echo** from $\tilde{G}_{\text{UCO}}^+$

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where $t_{\text{ref}} = x - x' + 2x_0$

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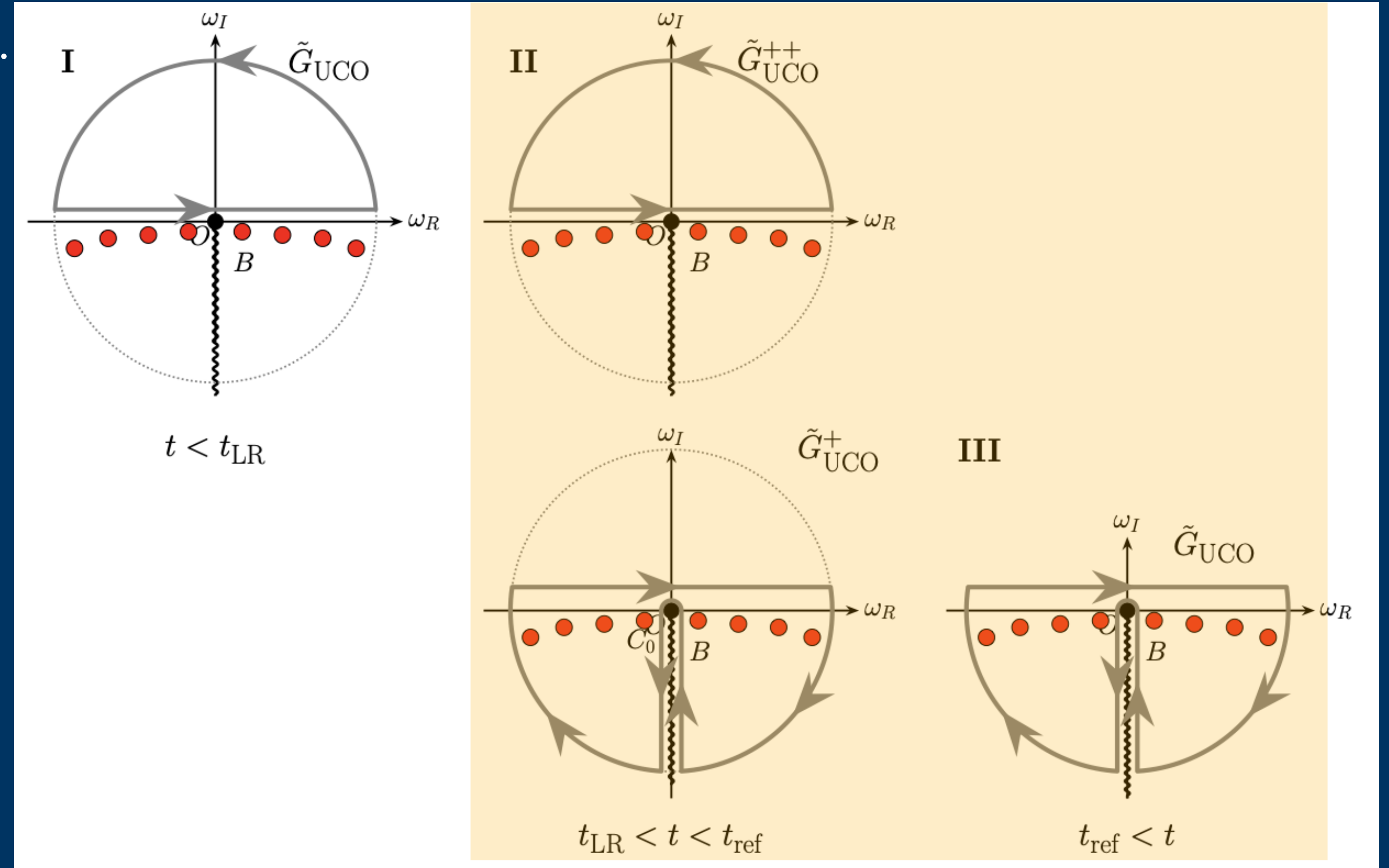
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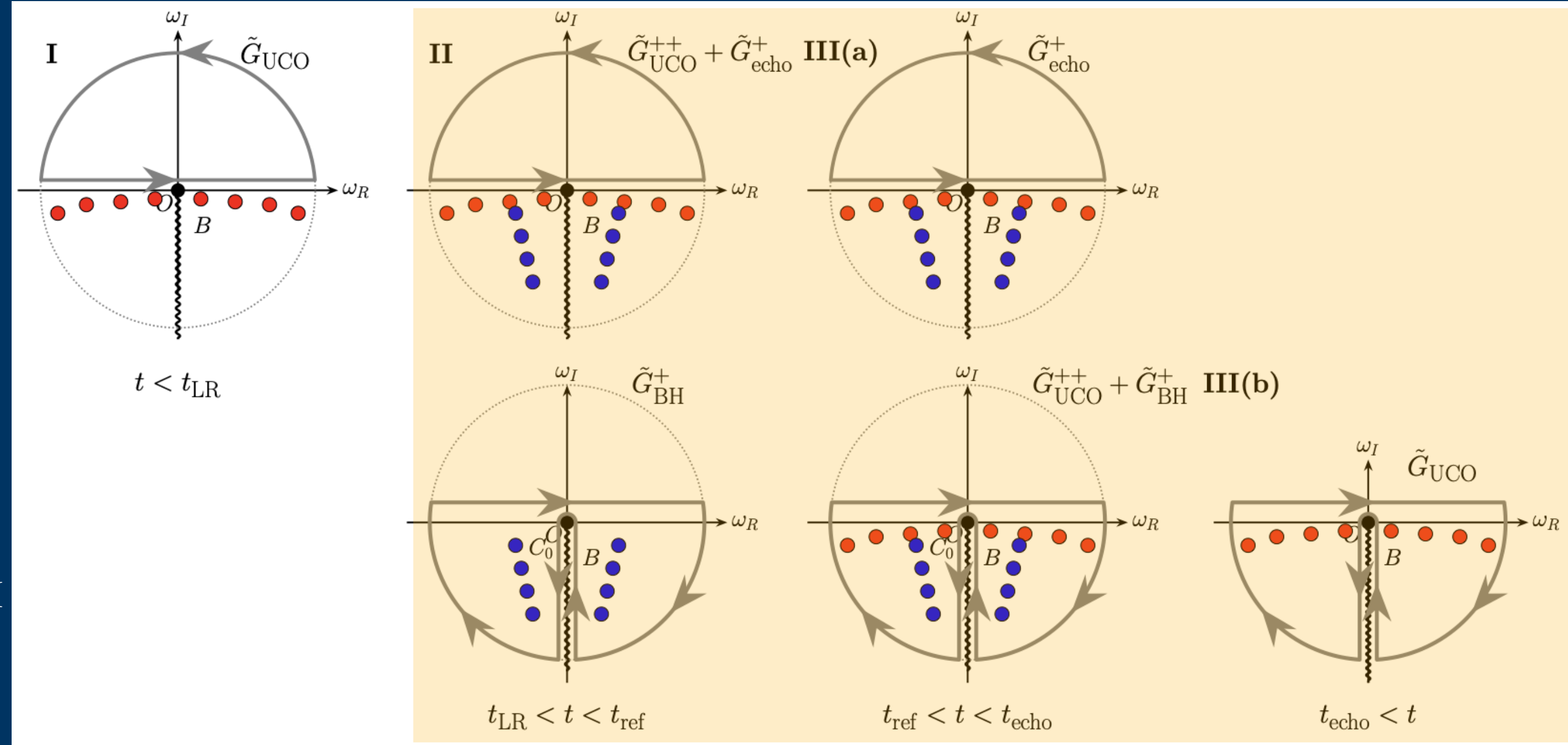
$$\tilde{G}_{\text{UCO}}^+(\omega, x, x') = \tilde{G}_{\text{BH}}^+ + \tilde{G}_{\text{echo}}^+$$

$$\tilde{G}_{\text{echo}}^+(\omega, x, x') = \frac{R_{\text{BH}} R_{\text{wall}} e^{i\omega t_d}}{A_{\text{in}}^{\text{UCO}}(\omega)} \frac{X_{\text{BH}}^+ X_{\text{BH}}^-}{2i\omega}$$

Stage II ($x - x' < t < x + x' + t_s$): **BH QNMs** from $\tilde{G}_{\text{BH}}^+$

Stage III(a) ($x + x' + t_s < t < x + x' + t_d$): **QNMs, tail** and **part echo** from $\tilde{G}_{\text{UCO}}^+$ and $\tilde{G}_{\text{BH}}^+$

Stage III(b) ($t > x + x' + t_d$): **Echo** from $\tilde{G}_{\text{UCO}}$



III. Numerical Validation

Time-domain Green's function for UCOs

Numerical Simulation (black line)

QNM Reconstruction (colored line)

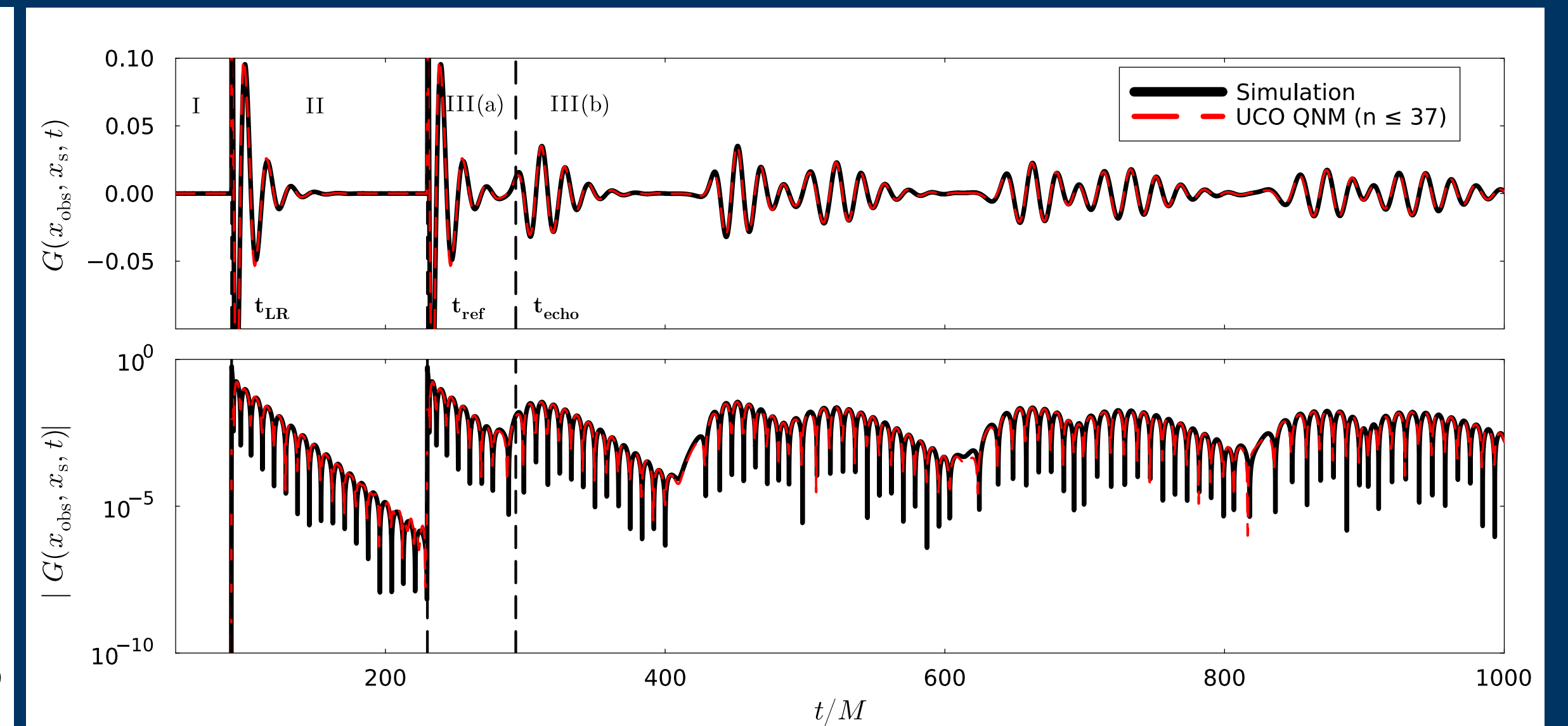
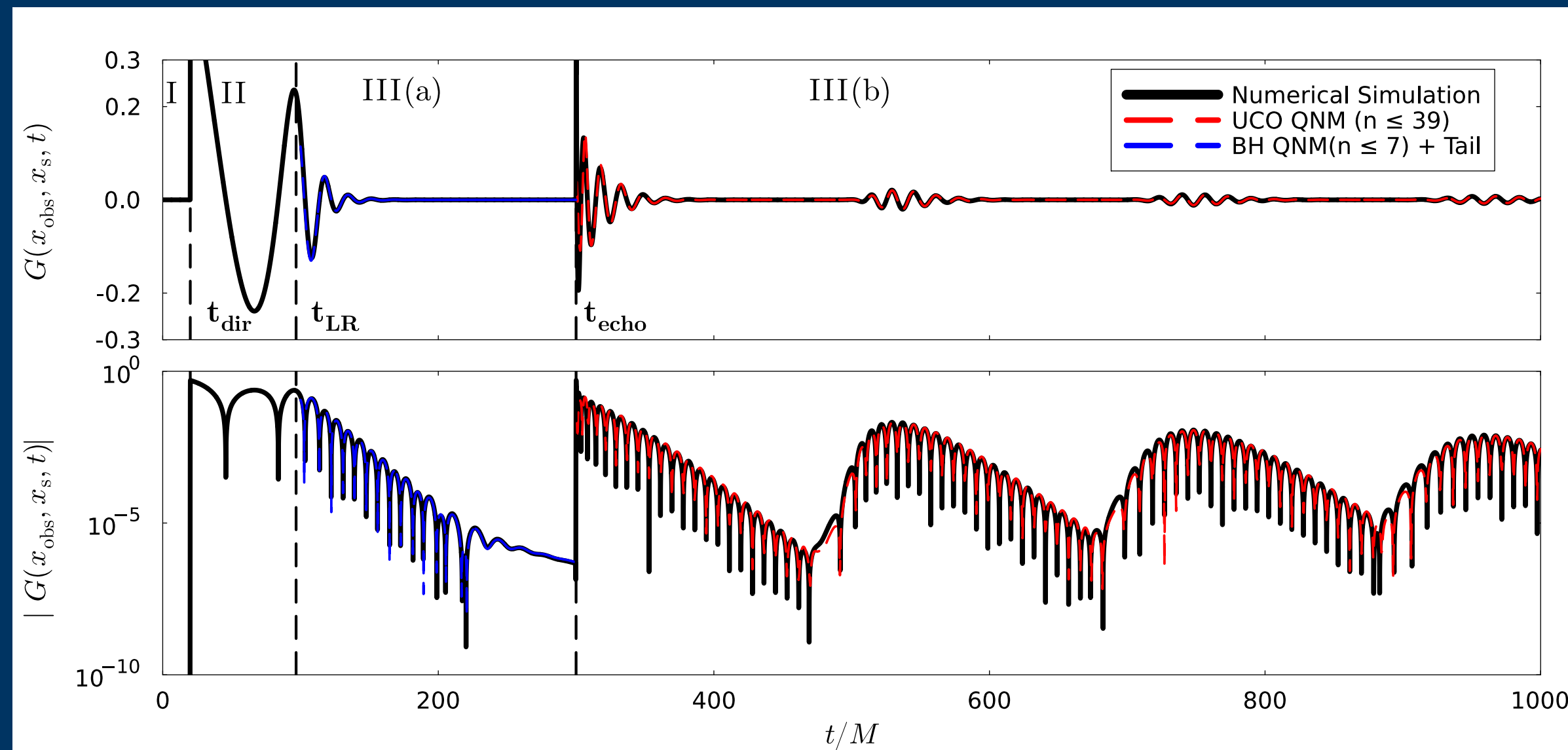
$$\left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial t^2} - V(r) \right) G(t, x) = \delta(x - x')$$

$$G(t, x, x') = 2\Re \sum_n B_n e^{-i\omega_n(t-t_{\text{ref}})}$$



(a). Source outside the light ring

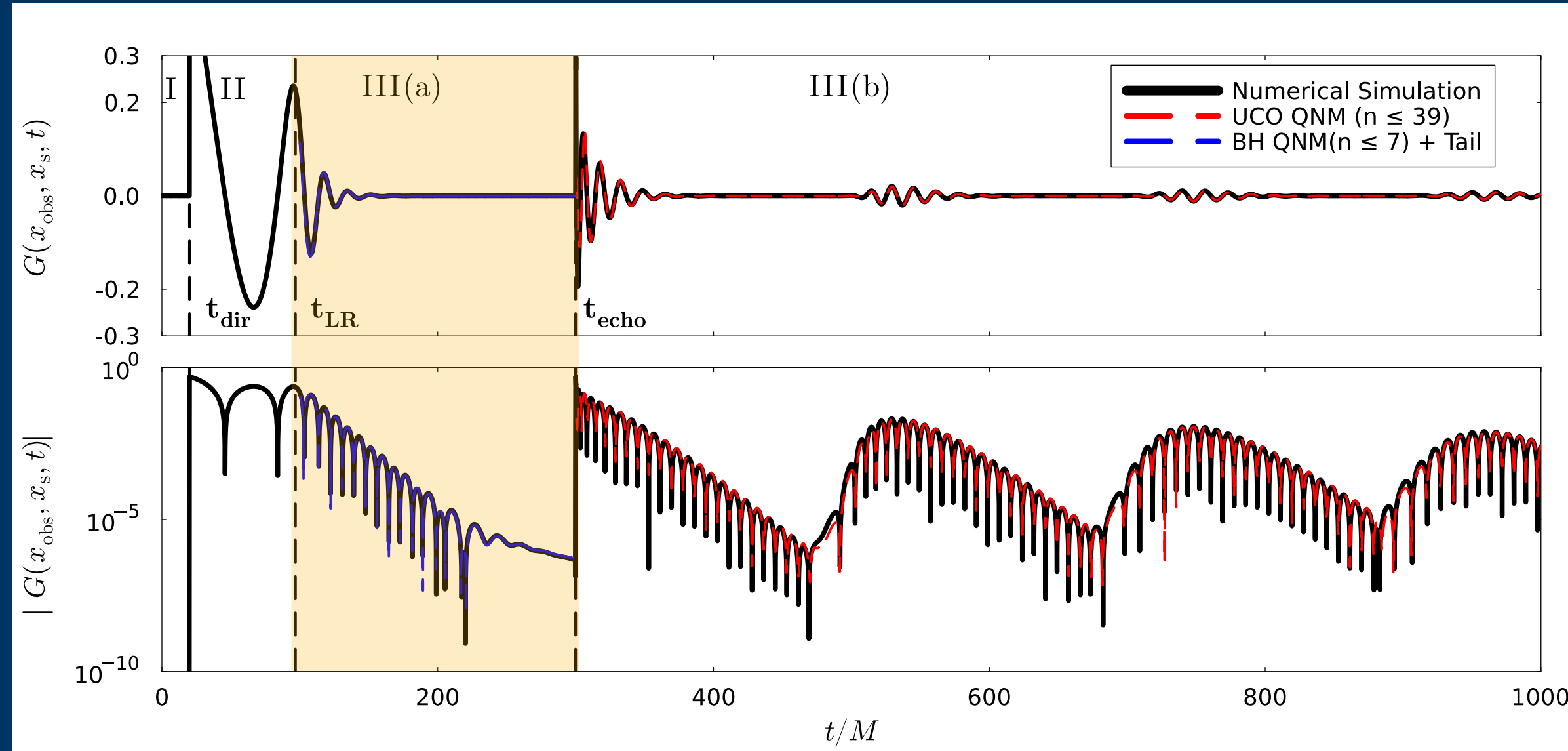
(b). Source inside the light ring



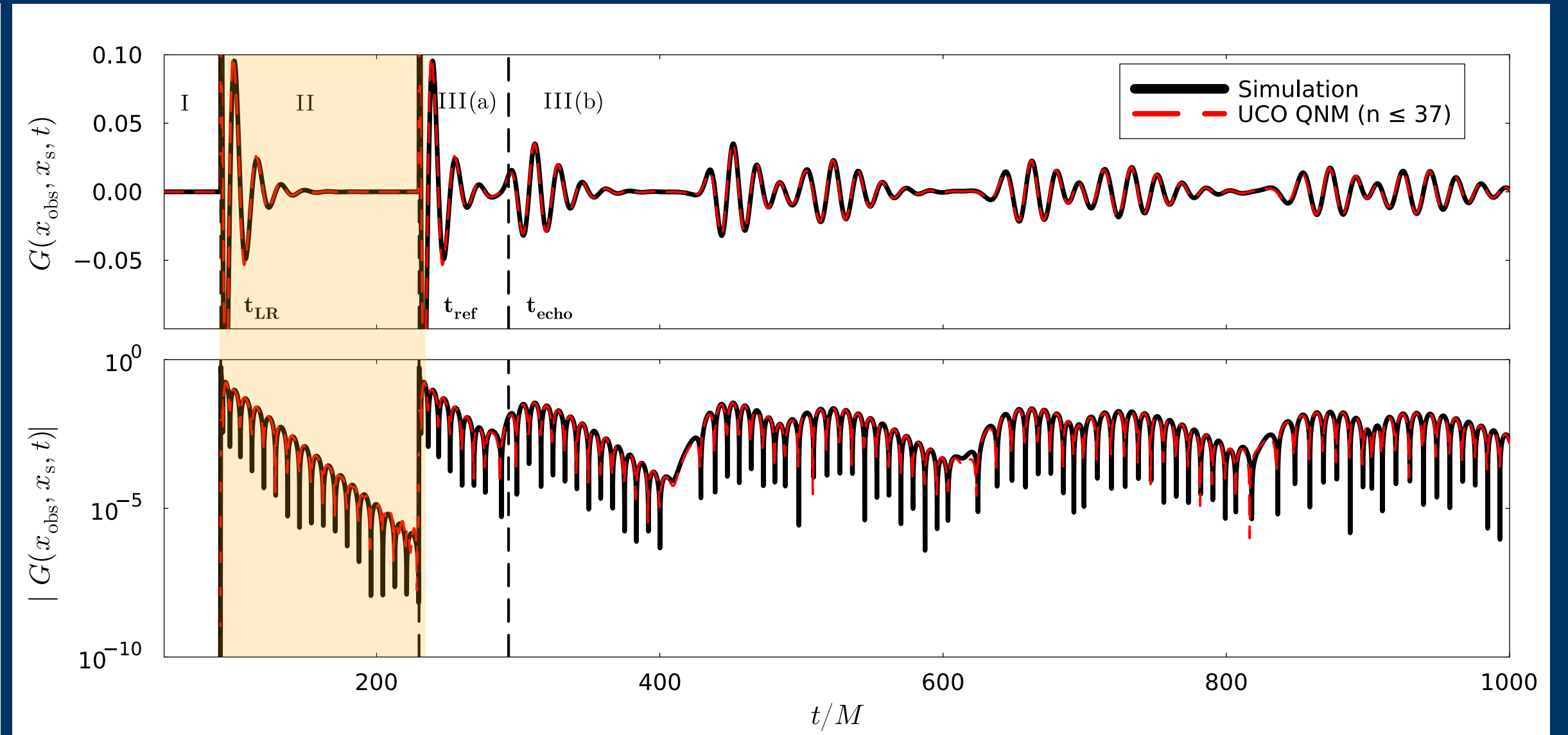
• QNM reconstruction **matches** the numerical simulation

Can UCO QNMs reproduce a prompt ringdown?

(a).Source outside the light ring (b).Source inside the light ring



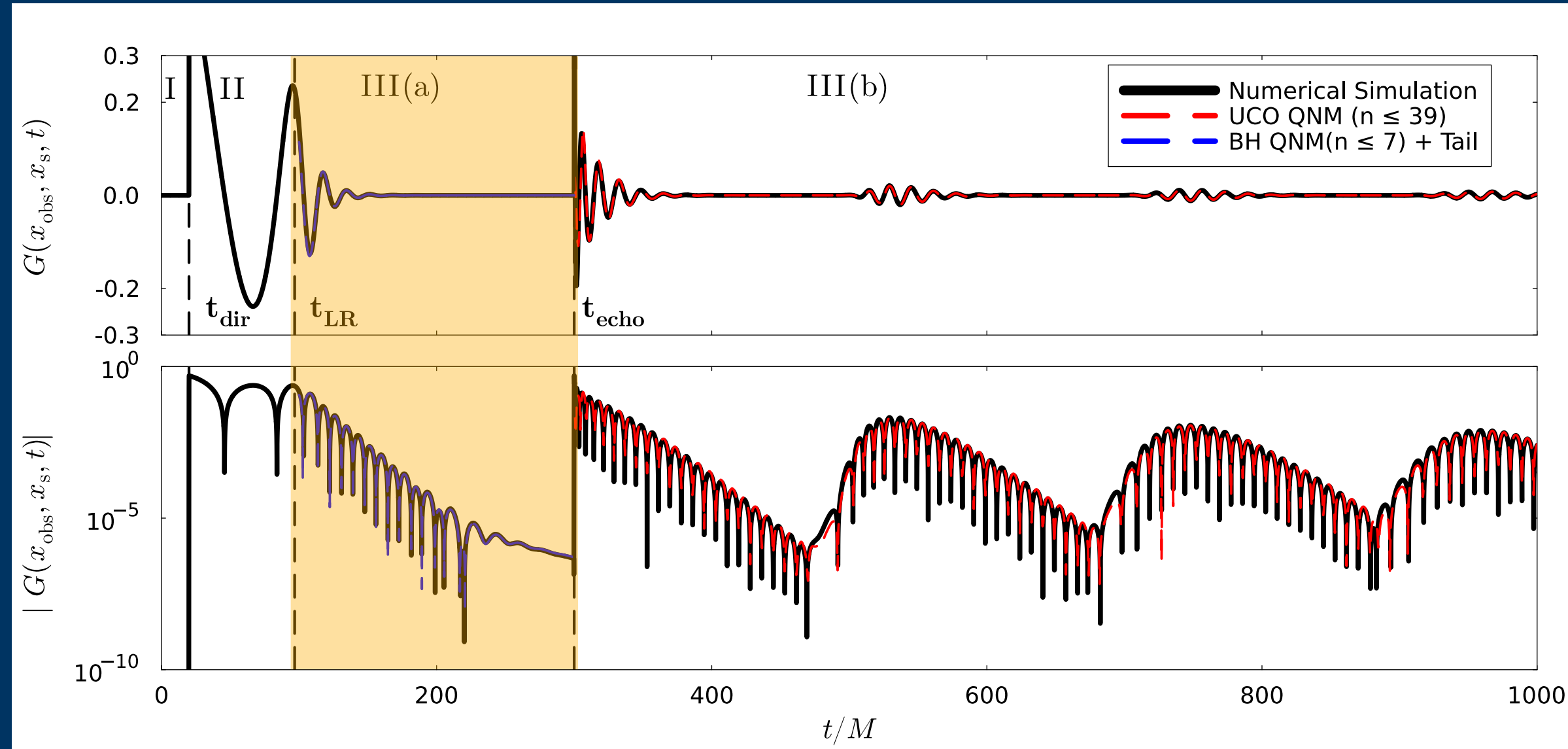
Not Yet



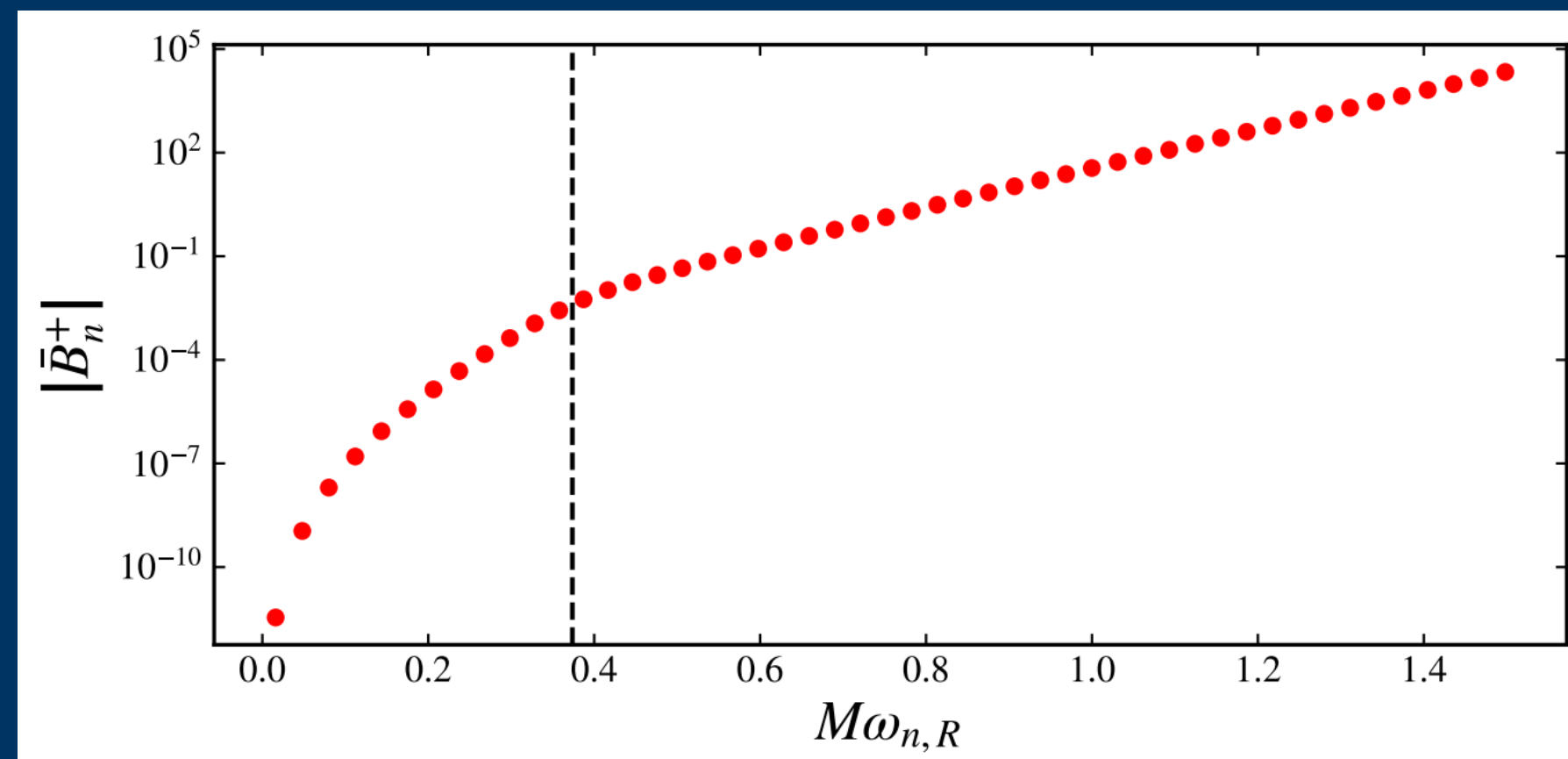
Successful

Can UCO QNMs reproduce a BH ringdown?

(a).Source outside the light ring

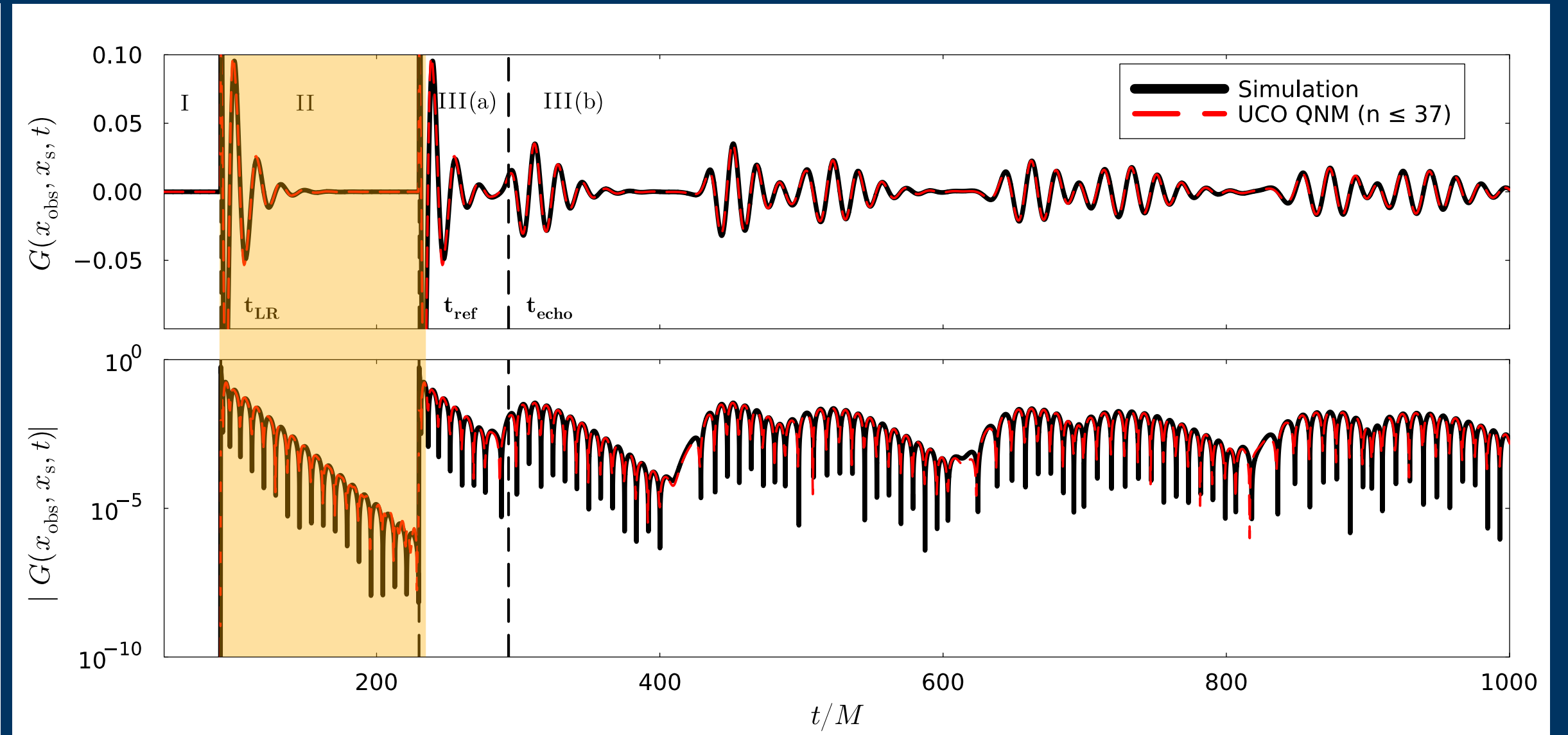


Not Yet

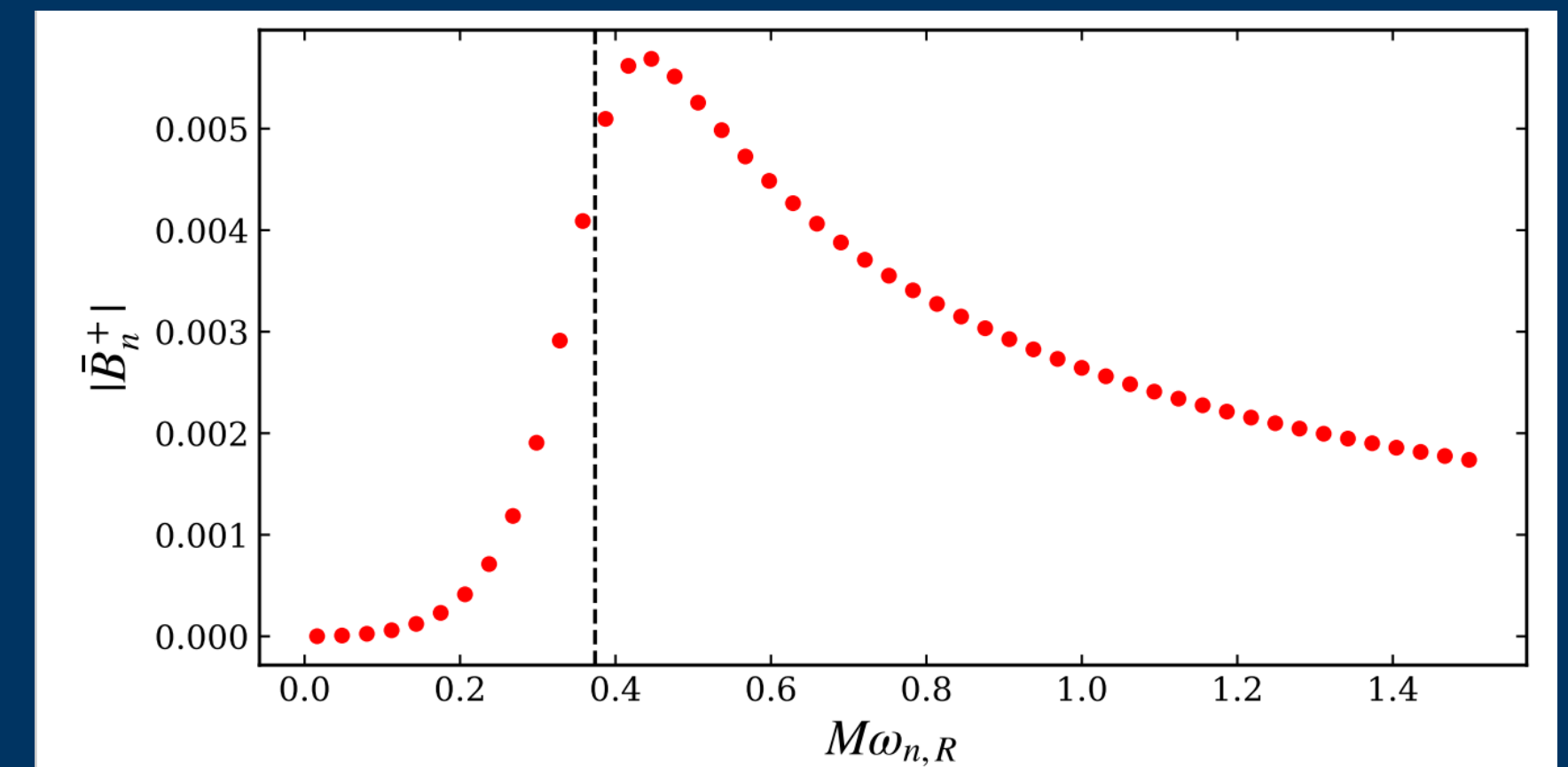


Exponentially growing EF

(b).Source inside the light ring



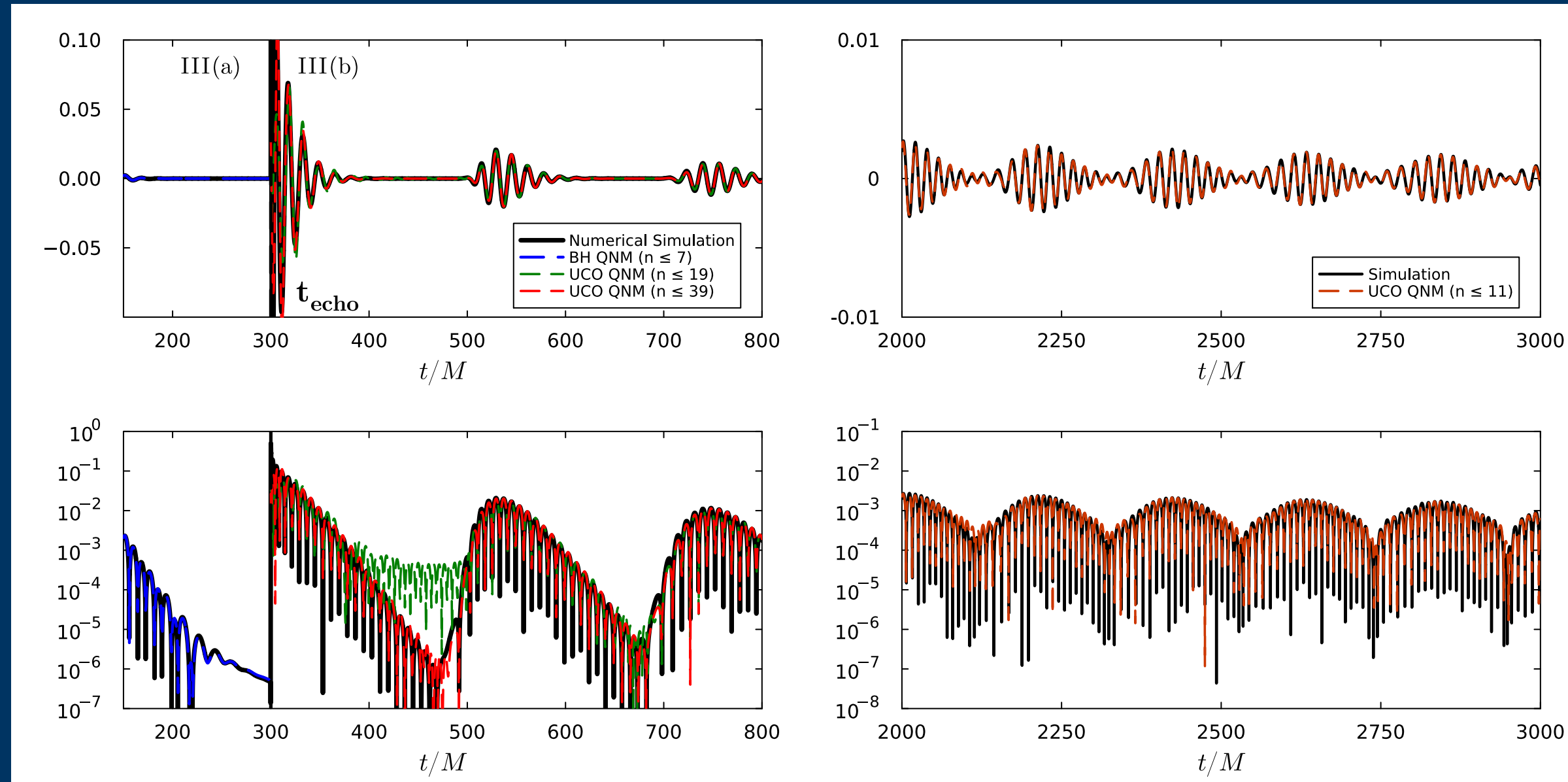
Successful



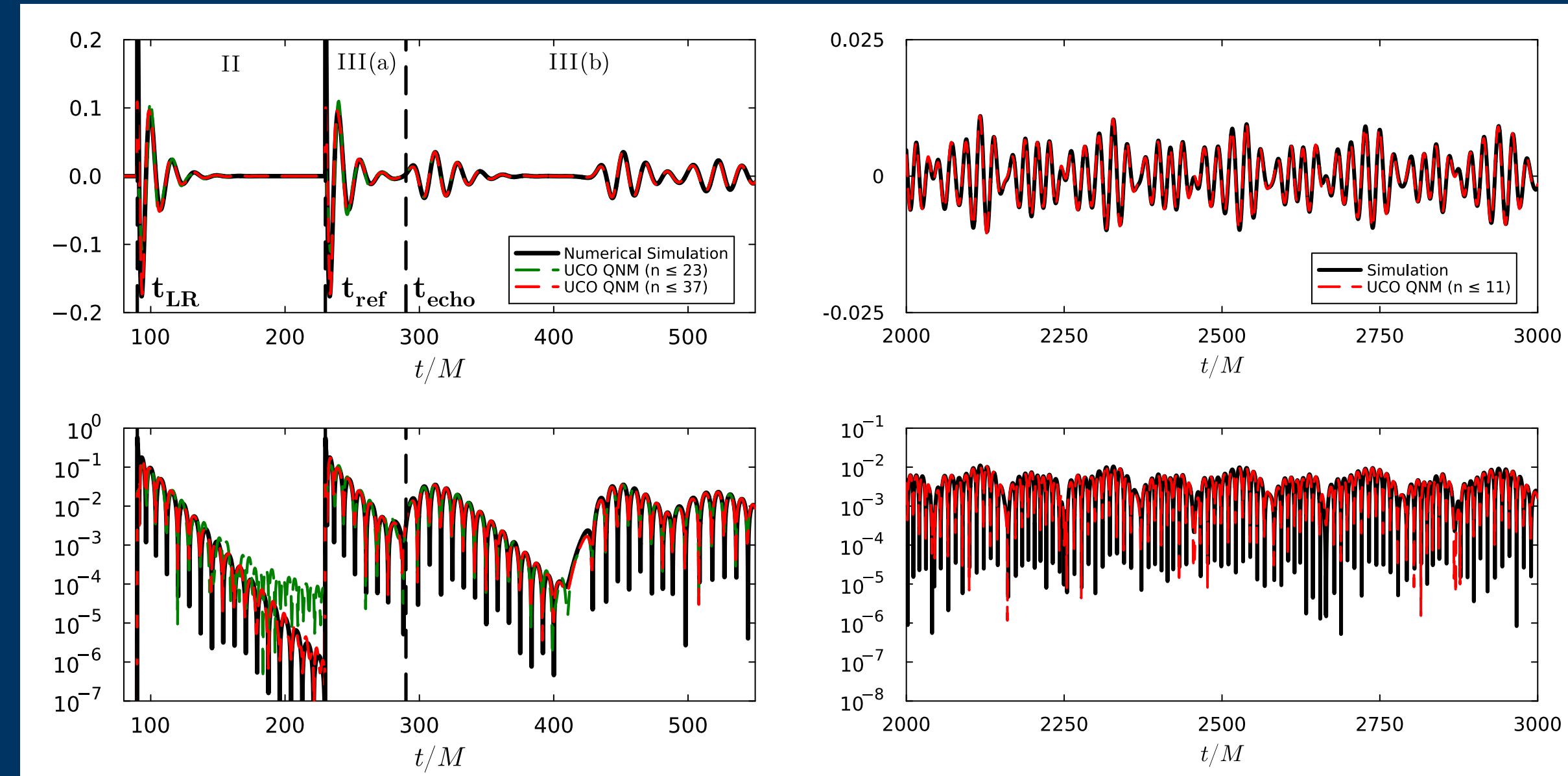
EF pressured under high-frequency

Comparison of High- and Low-Frequency Modes

(a). Source outside the light ring



(b). Source inside the light ring



- **High-frequency** modes are crucial for reproducing the sharp rise after the first echo and the deep dips between successive echoes
- Late-time echoes are dominated by long-lived **low-frequency mode**

IV.Summary

Summary

★Tension: Different spectrum obtain the same early waveform

★UCO QNMs reconstruct the waveform. At early times, the UCO QNM is formally equivalent to the BH QNM.

★The practical efficiency of the UCO-QNM basis depends strongly on the source location

★In the late-time echo regime, both cases are efficiently captured by few long-lived trapped modes.

Thanks For Listening