

Gravitational Atomic Spectra: Dynamics, Emission, and the Axion Potential

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Motivation

- The Target: Axions as dark matter have various **self-interaction**
- The limitation: Terrestrial labs: weak free fields, self-interaction is invisible
- A new window: Compact stellar objects as natural labs: dense strong fields with self-interaction

Background

Axion-like dark matter

- Ultralight axions (< 1 eV): **coherent classical waves** (high occupation number, long de Broglie wavelength)
- **Periodic potential with self-interaction**

$$V_{\cos} = \mu^2 f_a^2 \left(1 - \cos \frac{a}{f_a} \right)$$
$$V_{\text{chiral}} = -\frac{\mu^2 f_a^2}{C} \sqrt{1 - 4C \sin^2 \frac{a}{2f_a}}$$

Gravitational atom

- Ultralight boson: **hydrogen-like levels**, in $1/r$ gravitational potential
- **Field Amplification**: Macroscopic condensate with amplitude approaches f_a
- Growth:
 - Superradiance (**spinning black holes**)
 - **Dynamical relaxation** (**stars and black holes**), see Gilad Perez's talk on 25 Aug

Self-interaction driven relaxation

Scattering by a^4 interaction (Budker, Eby, Gorghetto, Jiang, Perez, 2023)

$$a_{\text{free}} + a_{\text{free}} \rightarrow a_{\text{bound}}(100) + a_{\text{free}}$$

Condensation & Key Features

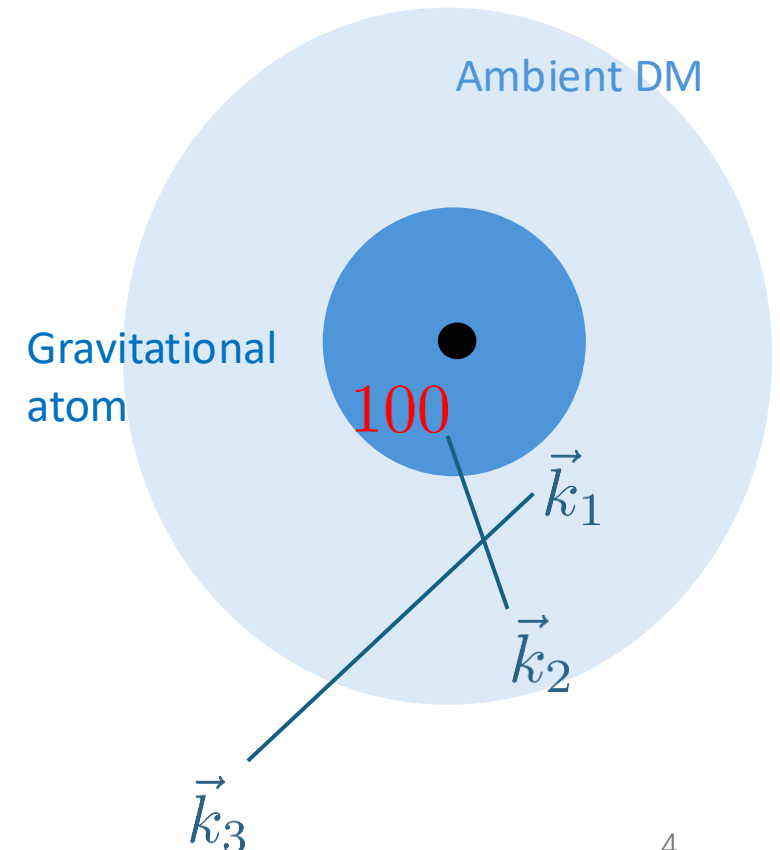
- Dominant mode: Ground 100 state (**spherical**)
- **No BH spin required** (cf. Superradiance)
- Relaxation (growth) rate

$$\frac{\dot{N}_0}{N_0} \propto \Gamma_{\text{rel}} \propto \frac{\lambda_4^2}{f_a^4} \frac{\rho_{\text{amb}}^2}{\mu^4 v_{\text{amb}}^2}$$

Ideal Target: Sgr A* (SMBH)

- **Large ρ_{amb} and small μ** enhance relaxation rate
- Axion mass $\sim 10^{-17}$ eV

$$V_{\text{int}} = \mu^2 f_a^2 \left[-\frac{\lambda_4}{4!} \left(\frac{a}{f_a} \right)^4 + \dots \right]$$



The growth of GA

Energy balance (variational approach)

- Kinetic (repulsive) vs gravity + self-interaction (attractive)

$$E_0 \approx \int \left(N_0 \frac{|\nabla \psi_0|^2}{2\mu} - N_0 \frac{\alpha |\psi_0|^2}{r} - N_0^2 \frac{\lambda_4 |\psi_0|^4}{16f_a^2} + \dots \right) d^3 \vec{r}$$

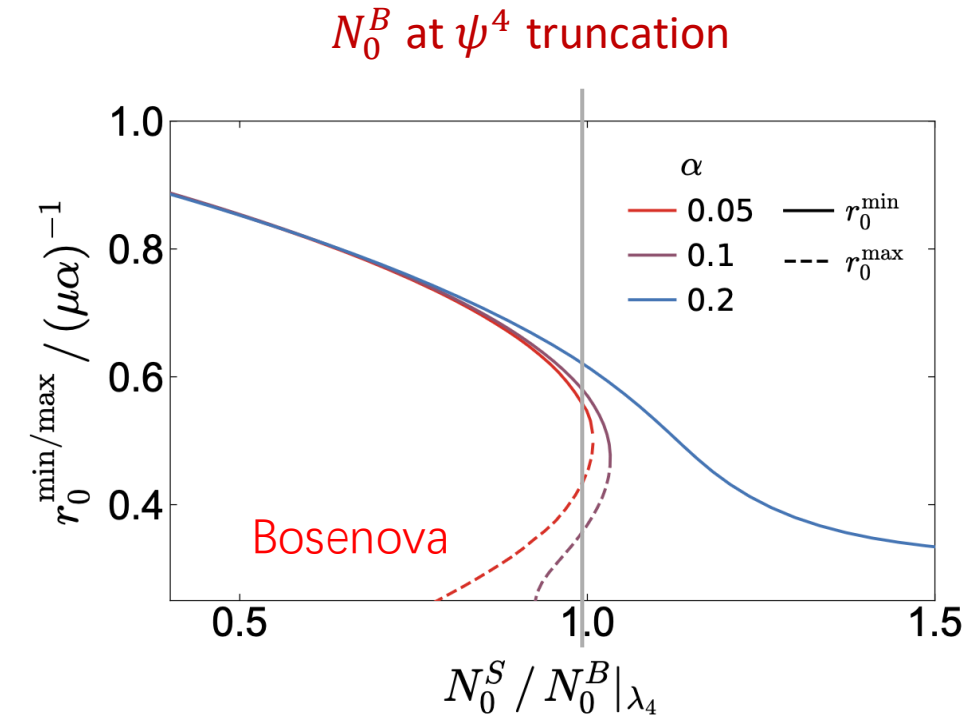
Cloud Shrinkage & Critical Instability

- Equilibrium state

$$\partial E_0 / \partial r_0 = 0, \quad r_0^{\min} = \frac{1}{2\mu\alpha} \left(1 + \sqrt{1 - \frac{3}{32\pi} \frac{\mu^2 \lambda_4}{f_a^2} \alpha N_0} + \dots \right)$$

- Collapse threshold: $r_0^{\min}$ vanishes

$$N_0^B|_{\lambda_4} = \frac{32\pi}{3\alpha} \frac{f_a^2}{\lambda_4 \mu^2}$$



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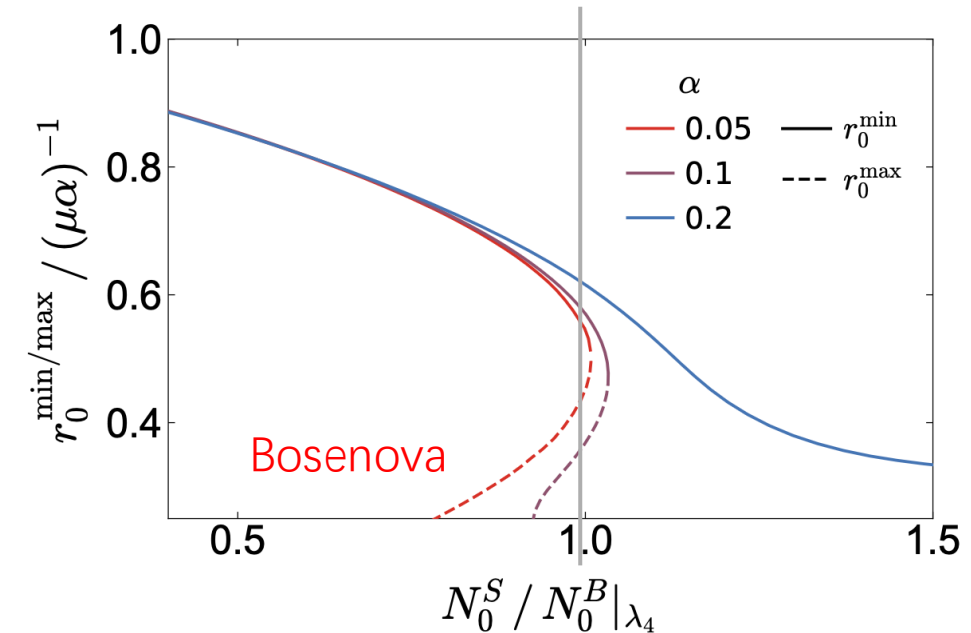
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N_0^B at ψ^4 truncation



Start from Bohr radius $(\mu\alpha)^{-1}$
Shrink when $N_0 \uparrow$

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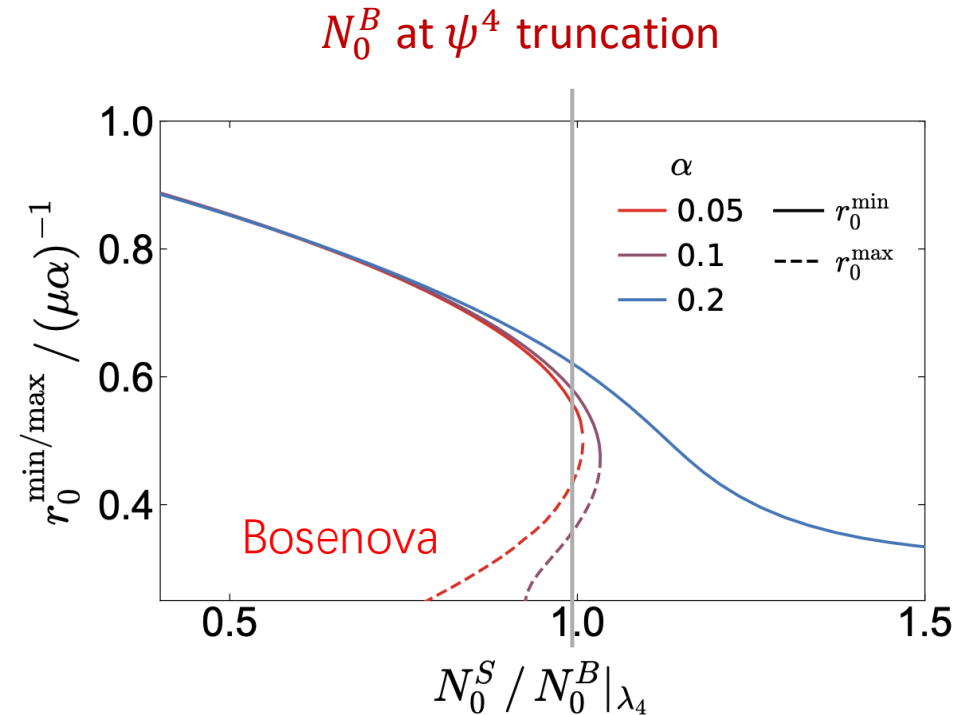
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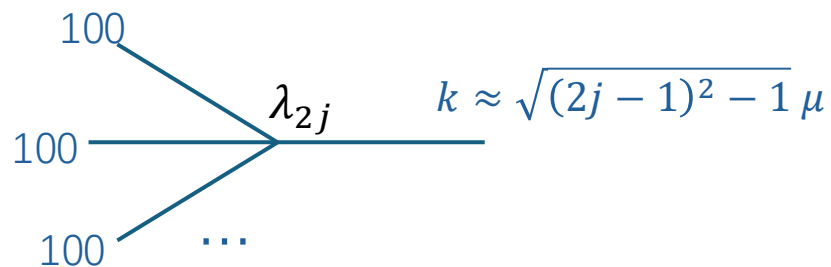


Start from Bohr radius $(\mu\alpha)^{-1}$
Shrink when $N_0 \uparrow$

What mechanism halts the growth in the relaxation regime?

Bosenova or saturation?

Self-interaction: $(2j-1)|100\rangle \rightarrow 1$ relativistic free particle
(Eby, Suranyi, Wijewardhana, 2015)

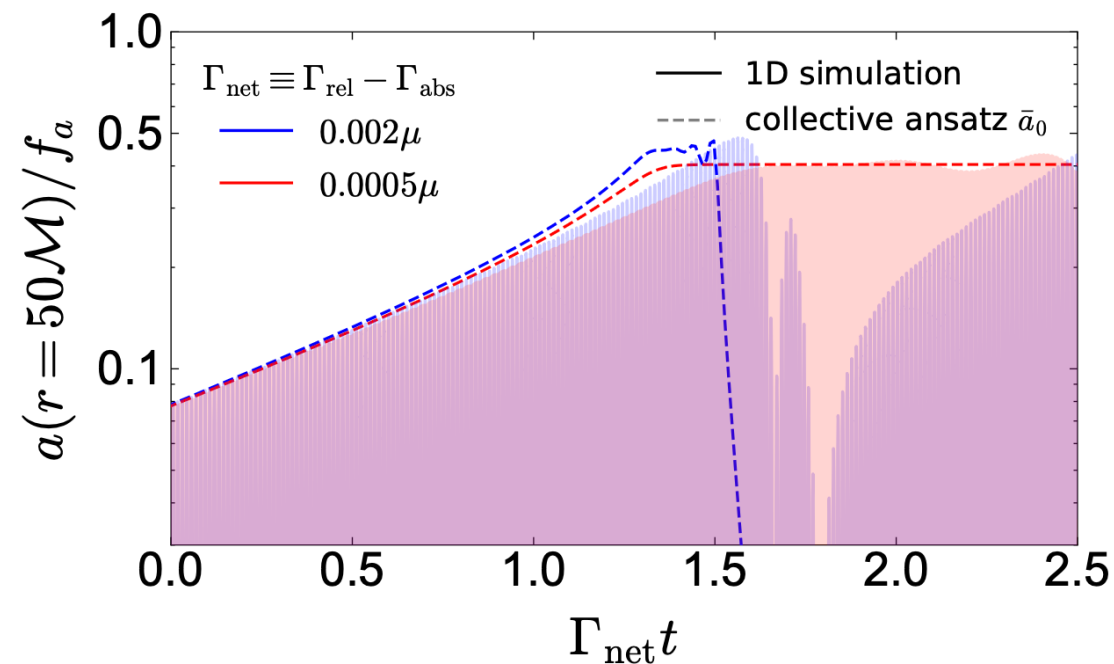


Growth (relaxation): $\left(\frac{\dot{N}_0}{N_0}\right)_{\text{in}} \propto \frac{\lambda_4^2}{f_a^4} \frac{\rho_{\text{amb}}^2}{\mu^4 v_{\text{amb}}^2}$

Loss (radiation): $\left(\frac{\dot{N}_0}{N_0}\right)_{\text{out}} \propto -\frac{N_0^2}{r_0^{11}}$

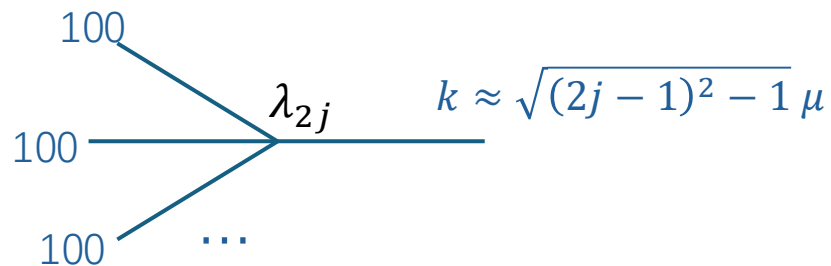
Saturation: Growth = Loss, at N_0^S

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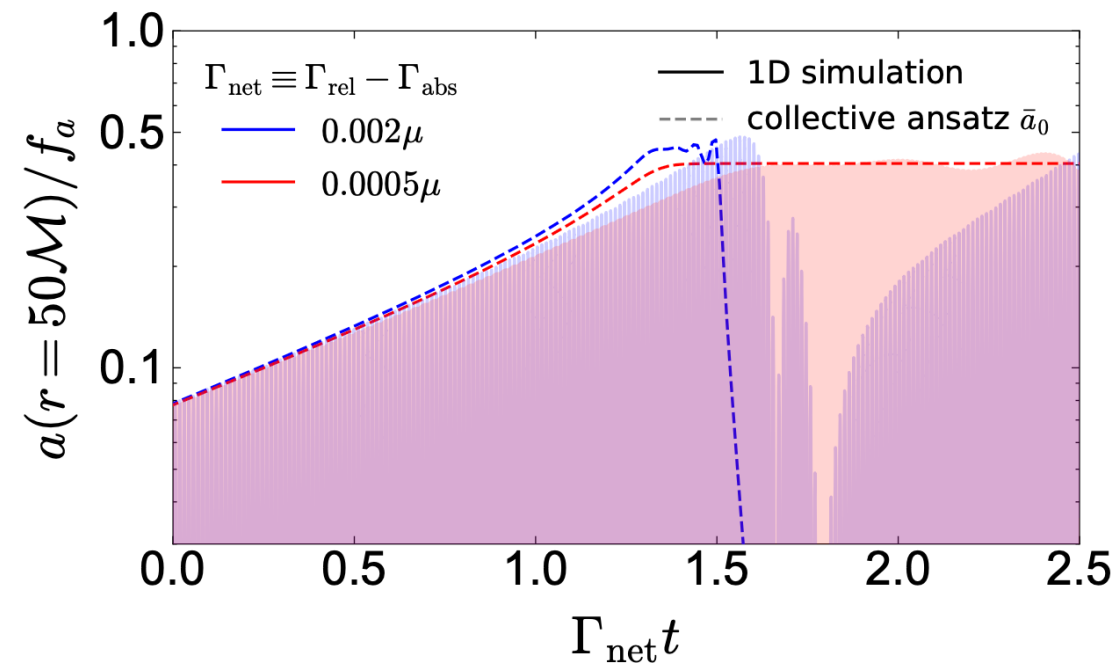
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N_0^S vs N_0^B ?

$N_0^S > N_0^B$: Bosenova collapse

$N_0^S < N_0^B$: Stable saturation, with continuous axion radiation



Axion Emission

Relativistic Emission Channel

- Multi-particle annihilation

$$(2j - 1) |100\rangle \rightarrow \psi_k, E_k = (2j - 1)\omega_0$$

- Primary monochromatic peak: **3→1**

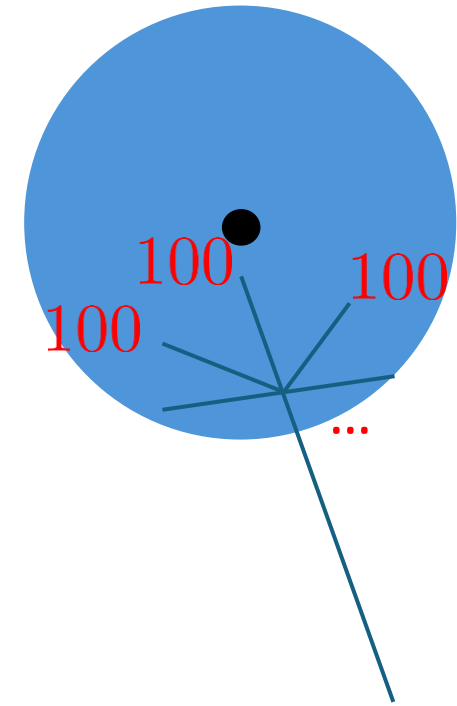
Earth Flux & Detectability

- Field amplitude

$$\bar{a}_{\text{rad}}^{3\omega_0} \propto \frac{\sqrt{\Gamma_{\text{rel}}/\mu \cdot N_0^S}}{r}$$

- Enhanced by large Γ_{rel}
- Suppressed by large r

- Flux from Sgr A*: $\sim 10^{-4}$ of local DM
- Axion wind: **10^3 enhancement** in g_{an} vs local DM



g_{an} : storage rings, magnetometers ...

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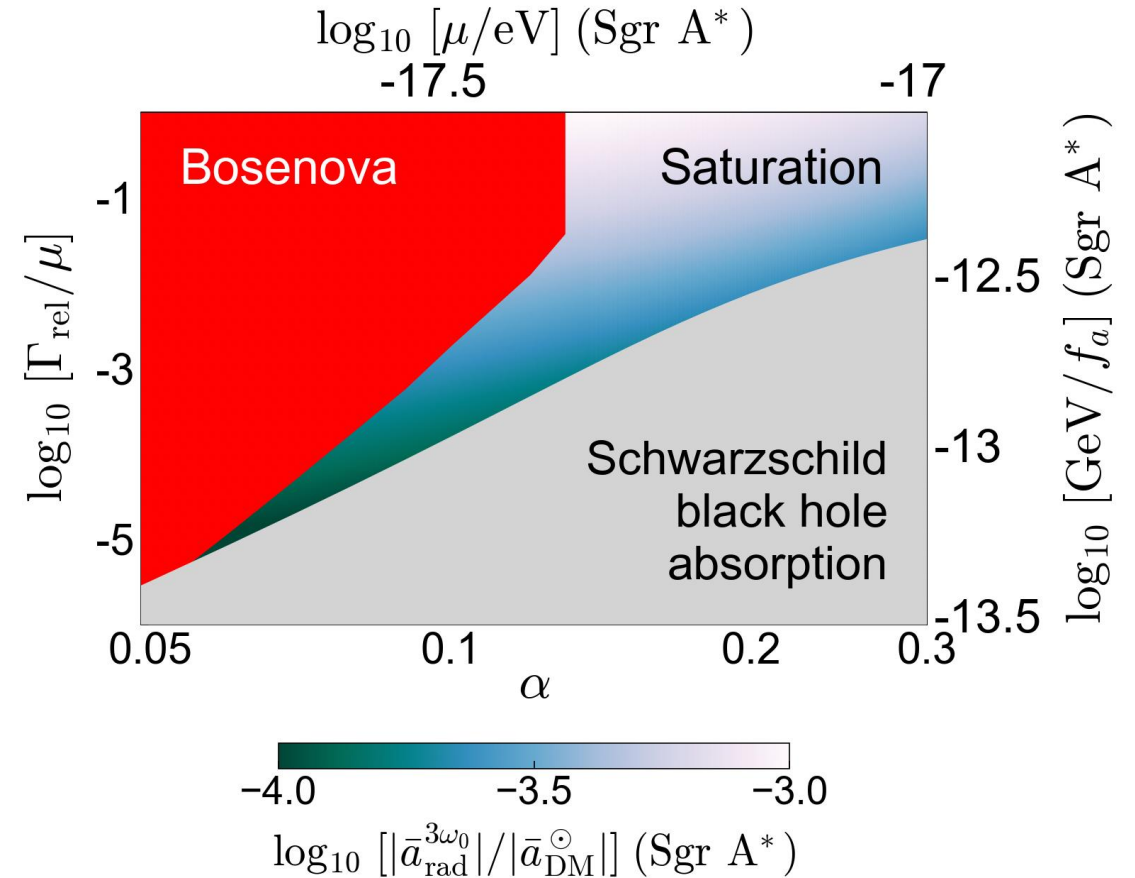
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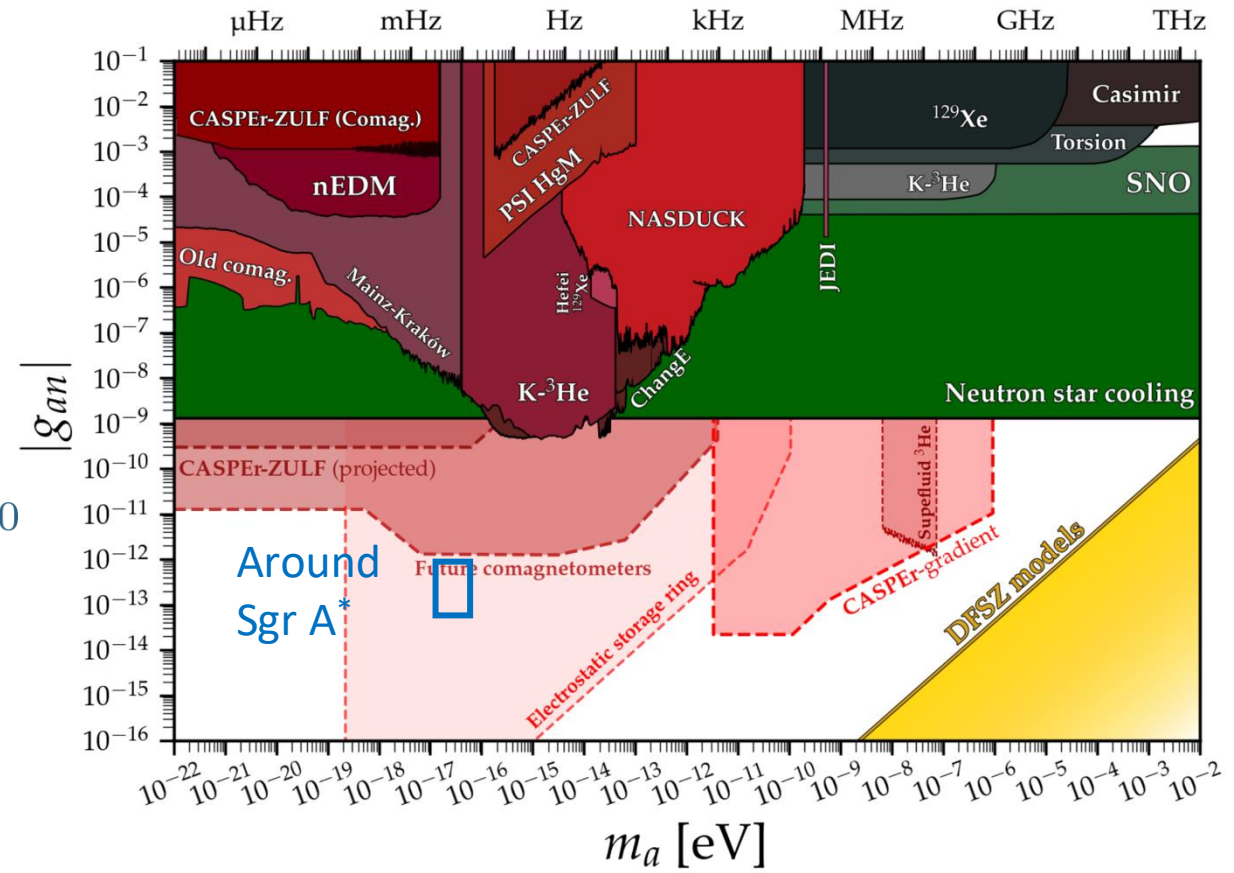
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Credit: cajohare.github.io/AxionLimits/

Reconstruct potential

Higher Harmonic Spectra

- Transition at odd harmonics
- Production rate for $(2j - 1)\omega_0$: $\propto |\lambda_{2j}|^2 a^{4j}$

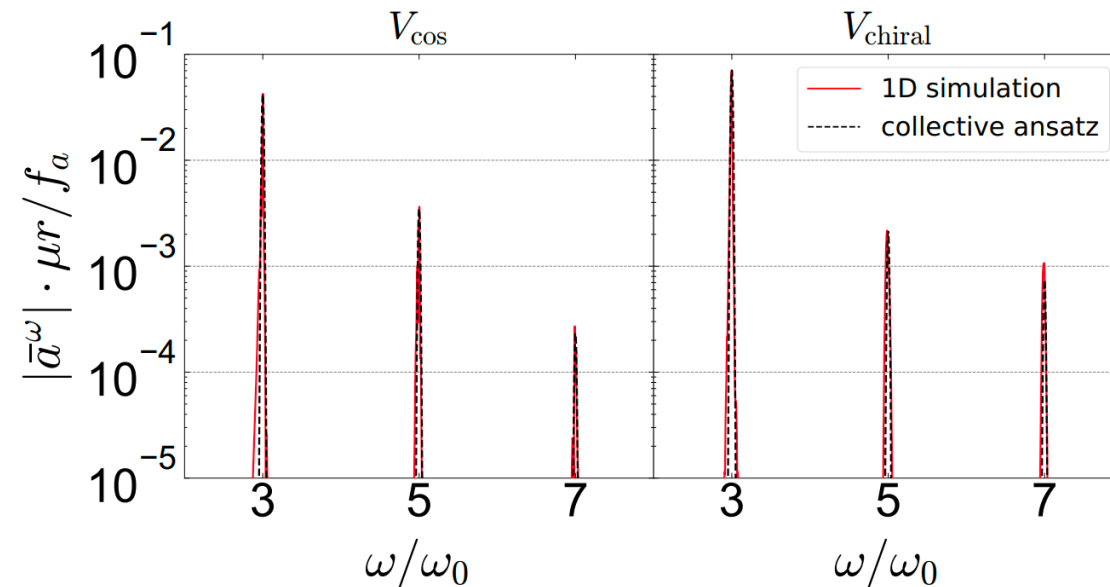
Reconstruction via Double Ratio

$$\frac{\bar{a}_{7\omega_0}^{\text{rad}} / \bar{a}_{5\omega_0}^{\text{rad}}}{\bar{a}_{5\omega_0}^{\text{rad}} / \bar{a}_{3\omega_0}^{\text{rad}}} \simeq 0.9 \frac{|\lambda_8 \lambda_4|}{|\lambda_6^2|}$$

Key advantage: Independent of field value a
Direct probe of self-interaction λ_{2j}

Model Discrimination

- Distinguishes non-perturbative shapes



$$V_{\text{int}} = \mu^2 f_a^2 \sum_{j=2} (-1)^{j+1} \frac{\lambda_{2j}}{(2j)!} \left(\frac{a}{f_a} \right)^{2j}$$

$$V_{\text{cos}} = \mu^2 f_a^2 \left(1 - \cos \frac{a}{f_a} \right)$$

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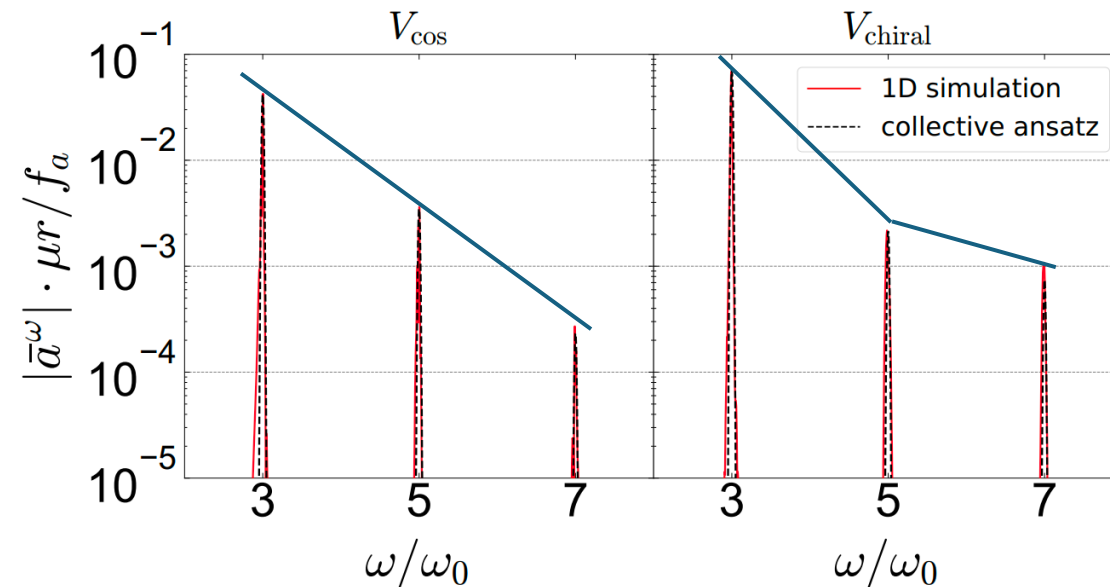
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Peak **decay slope** $\rightarrow$ self-interaction parameters



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Summary

- GA dynamics: Established the non-linear fate around compact stellar objects (**Bosenova vs. Stable Saturation**)
- Relativistic Emission: Saturated GA radiates **multi-harmonic axion waves** ($\omega = 3, 5, 7, \dots \omega_0$) into reach of terrestrial experiments
- Potential Reconstruction: Propose using axion spectra to directly **extract self-interaction parameters** λ_{2j} and distinguish axion potential models: V_{cos} vs V_{chiral}



Thank you for your attention.

Relaxation and emission rate

Relaxation growth ($2 \rightarrow 1+1$)

$$a_{\text{free}} + a_{\text{free}} \rightarrow a_{\text{bound}}(100) + a_{\text{free}}$$

Matrix element

$$\eta_{\text{rel}} \propto \int d^3\vec{r} e^{-r/r_0} \prod_{i=1}^3 \psi_{\vec{k}_i}$$

Key Physics

- When $v_{\text{amb}} \ll \alpha$, free states are flat across core region $r < r_0$
- **Insensitive** to ambient DM velocity distribution

Relativistic emission ($3 \rightarrow 1$)

$$3 \times a_{\text{bound}}(100) \rightarrow a_{\text{free}}$$

Matrix element

$$\eta_{3 \rightarrow 1} \propto \int d^3\vec{r} e^{-3r/r_0} \frac{\sin(kr)}{r}$$

Key Physics

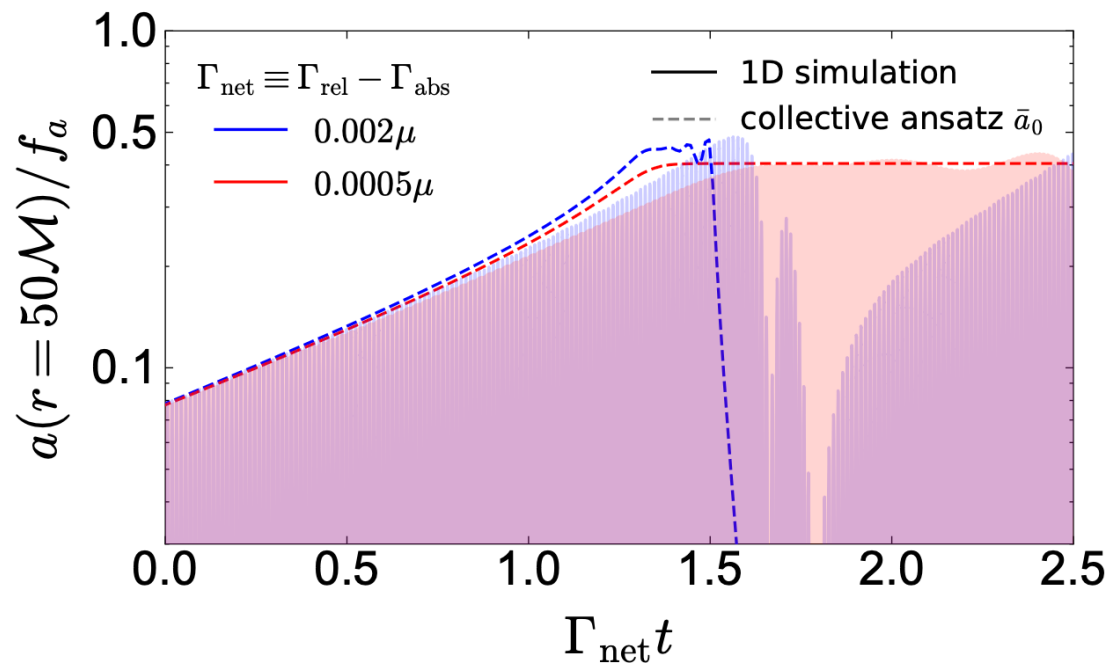
- Strong localization of three bound axions into a relativistic wave ($k \sim \sqrt{8}\mu$)
- Loss rate is **greatly amplified as cloud shrinks** ($P \propto N_0^3/r_0^{11}$)

Dynamical evolution

1D simulation

$$\square a = \frac{\partial V(a)}{\partial a} - 2\Gamma_{\text{net}} \frac{\partial a}{\partial t}$$

$$V(a) = \mu^2 f_a^2 \left(1 - \cos \frac{a}{f_a} \right)$$



Collective ansatz

$$a_0(t) = \frac{c_0}{\sqrt{2\mu}} \psi_0(\vec{r}, t) e^{-i\mu t} + \text{h.c.} \quad \text{Global phase}$$

$$\psi_0(\vec{r}, t) = \frac{1}{\sqrt{\pi r_0^3}} \exp \left(-\frac{r}{r_0} + i b r^2 - i \vartheta \right)$$

Hydrogen 100 Radial breathing Global phase

Integrate for total energy E_0

Euler-Lagrange equation of r_0, b, ϑ

$$\frac{\partial N_0}{\partial t} = 2\Gamma_{\text{net}} N_0 - \sum_{j=2}^{\infty} (2j-1) |\eta_{2j}|^2 N_0^{2j-1},$$

$$\frac{\partial}{\partial t} (3N_0 r_0^2) = \frac{\partial E_0}{\partial b},$$

$$6N_0 r_0 \frac{\partial b}{\partial t} = -\frac{\partial E_0}{\partial r_0},$$

$$\frac{\partial \vartheta}{\partial t} = \frac{\partial E_0}{\partial N_0} + 3r_0^2 \frac{\partial b}{\partial t},$$