

Impact of the Lens Galaxy on the Search for GW microlensing by compact objects

Uddeepta Deka

Shanghai Astronomical Observatory (SHAO - CAS)

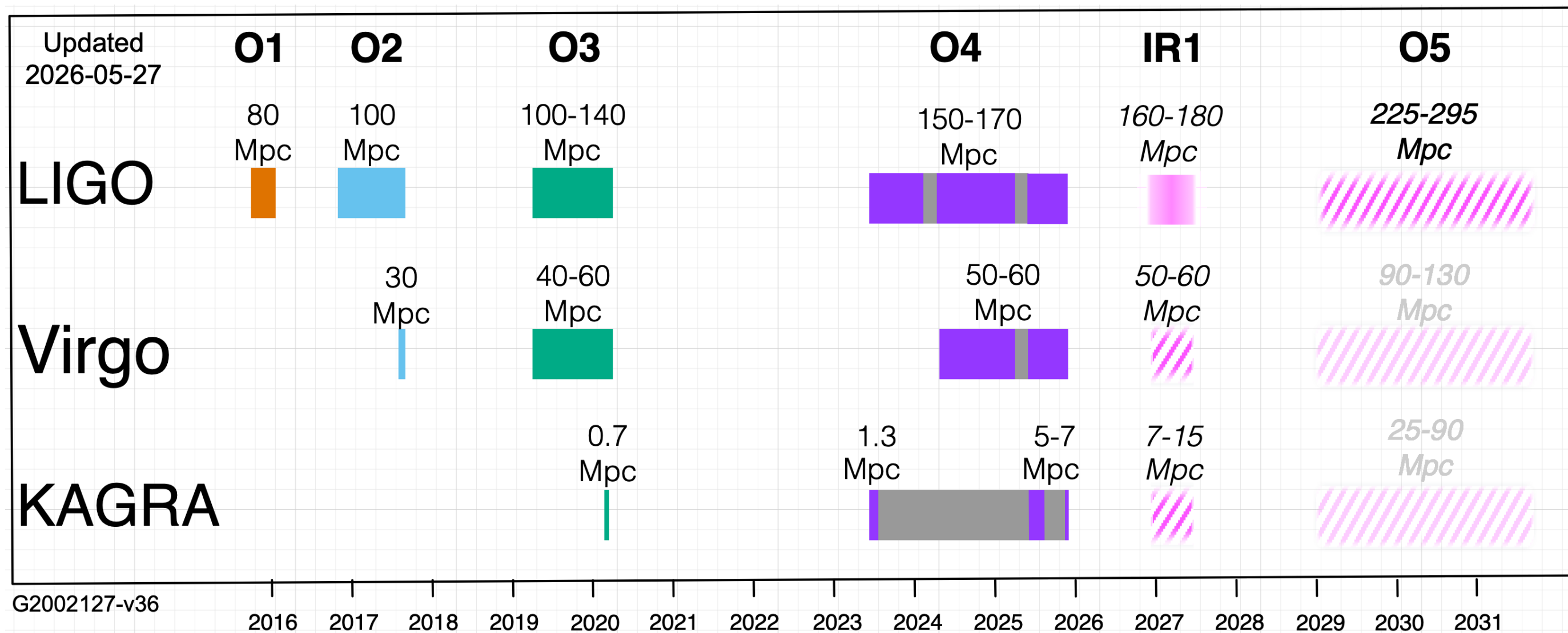
In collaboration with:

Apratim Ganguly (IUCAA, Pune),

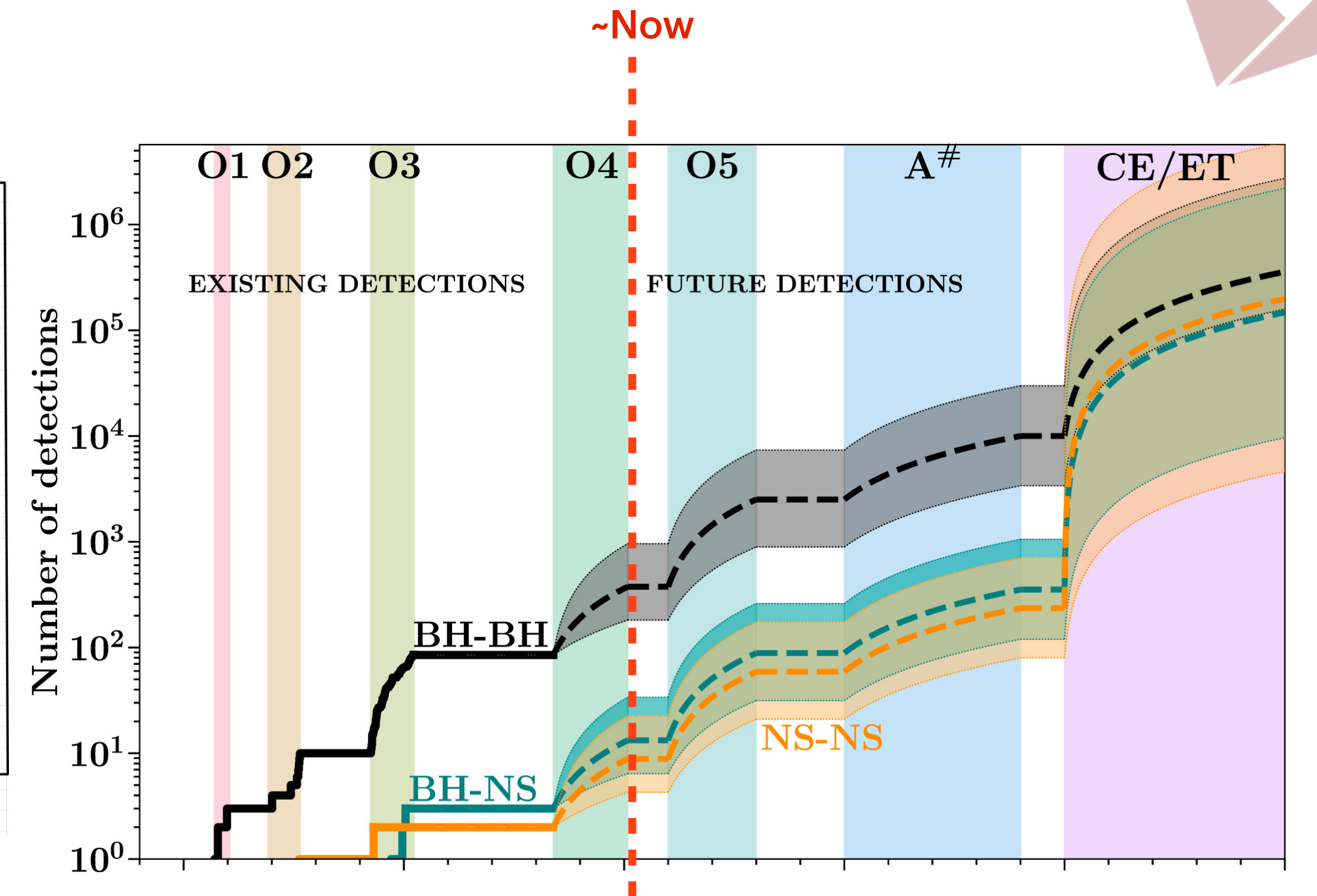
Soham Bhattacharyya (IIT Madras),

Ajith Parameswaran (ICTS, Bengaluru)

The future of ground-based gravitational-wave astronomy



Planned sensitivity improvement. O4 concluded. Credits: [LVK](#)



Current (~390 confirmed detections in GWTC-5) and projected number of detections. Credits: [Broekgaarden+,2024, ApJ 969 108](#)

Gravitational lensing of gravitational-waves

- Small fraction (0.1 – 0.5 %) of detectable BBH mergers in next generation detectors are expected to be lensed.
- **Bending of GWs** due to compact objects or mass distribution.
- No confident detection of lensed pairs / microlensing signatures till O4a [[LVK, arXiv:2512.16347, 2025](#)].
- Lensing of GWs will open up unexplored avenues of astrophysics, cosmology and fundamental physics.

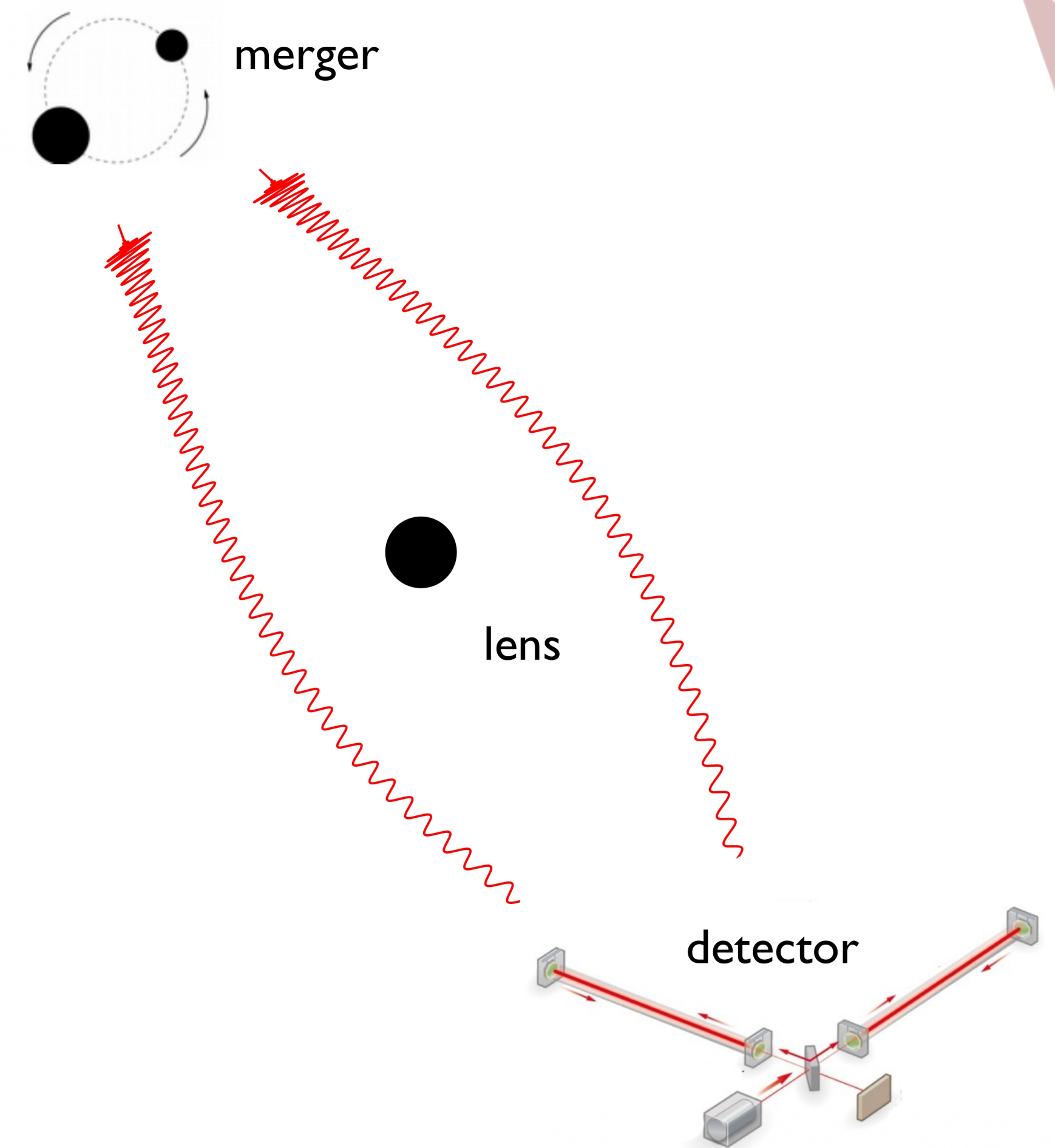
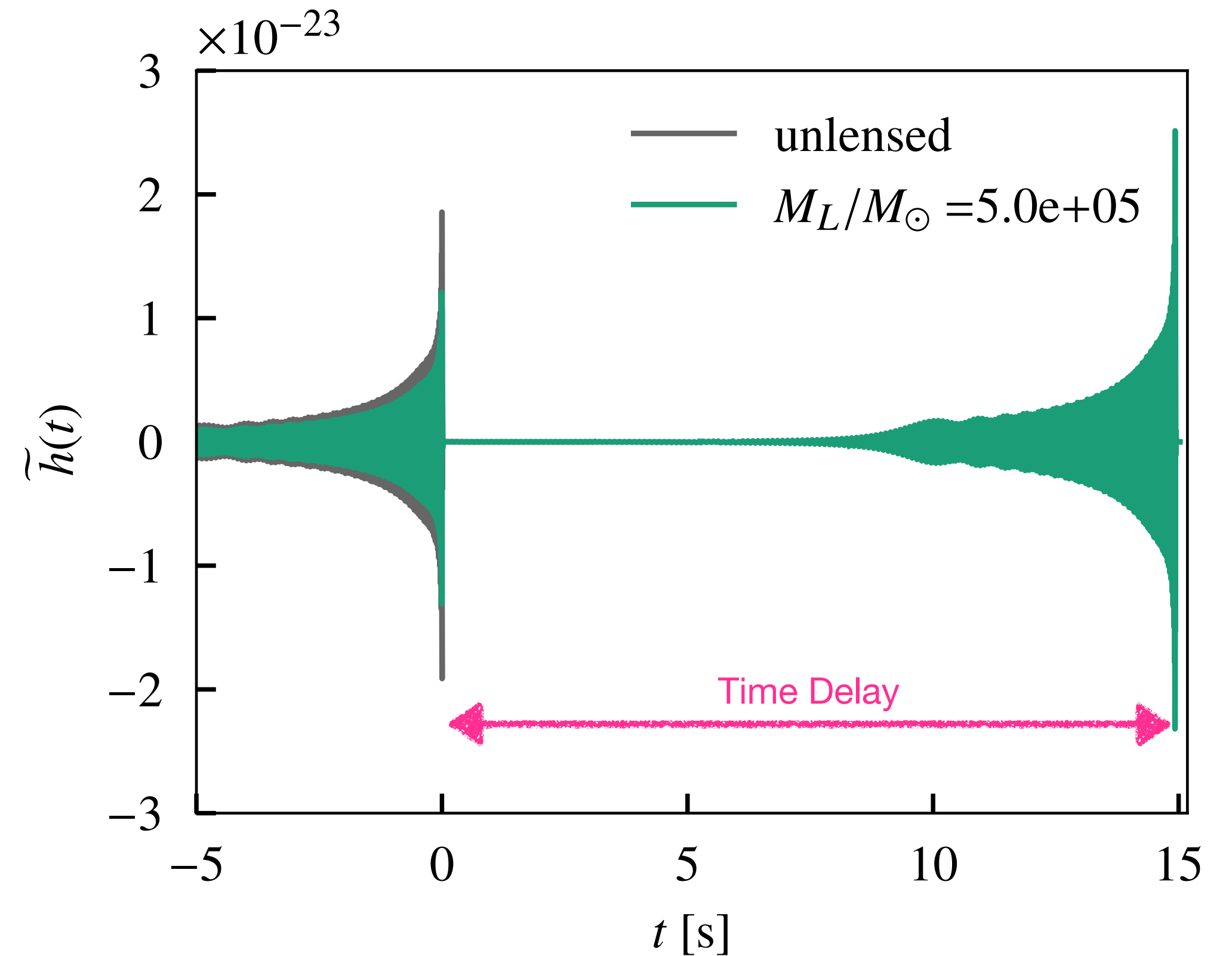


Illustration for gravitational lensing due to a compact object.
Credits: P. Ajith

Gravitational lensing of gravitational-waves

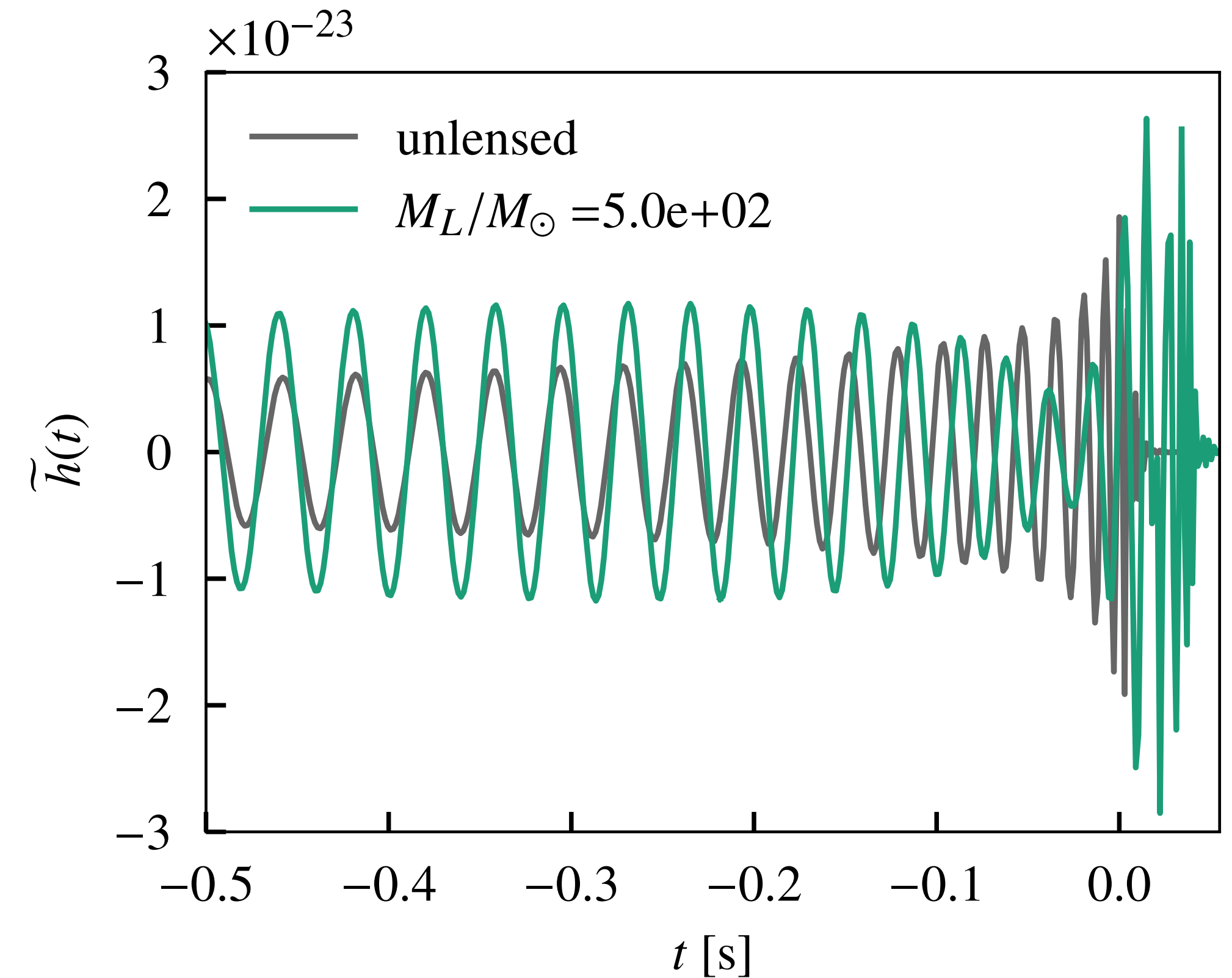
- Geometric optics: $\lambda_{\text{GW}} \ll M_L$
 - Lens: galaxy, cluster of galaxies, compact objects of mass $M_L \gtrsim 10^5 M_\odot$.
 - Lens sufficiently well aligned with source $\rightarrow$ multiple GW signals $\rightarrow$ strong lensing .
 - Frequency-independent relative magnification:
$$h_L^i(f) = \sqrt{\mu^i} h_{\text{UL}}(f) \times (\text{constant phase shift})$$
 - Wavefronts from different events arrive at different times $\rightarrow$ time delay from minutes to years.



Strong lensing of GWs due to a point mass lens

Gravitational lensing of gravitational-waves

- **Wave optics:** $\lambda_{\text{GW}} \sim M_L$
 - Lens: compact objects with mass $M_L \in [10^2 - 10^4]M_\odot$: intermediate mass black holes (IMBH), primordial black holes (PBHs), other exotic compact objects.
- **Microlensing:** Frequency-dependent modulation of the unlensed signal: $h_L(f) = \mathcal{F}(f) \times h_{\text{UL}}(f)$
- Exploration of scales beyond optical or radio telescopes: discovering previously unobserved populations.
- Challenging from waveforms point of view.



Microlensing of GWs due to a point mass lens

The Fermat potential and the magnification function

Lensed waveform $\rightarrow$ $h_L(f; \vec{\vartheta}, \vec{\Lambda}) = \mathcal{F}(f; \vec{\Lambda}) \times h_{UL}(f; \vec{\vartheta})$

Lensing parameters $\{M_L, \vec{y}, \dots\}$ $\rightarrow$ $\mathcal{F}(f; \vec{\Lambda})$

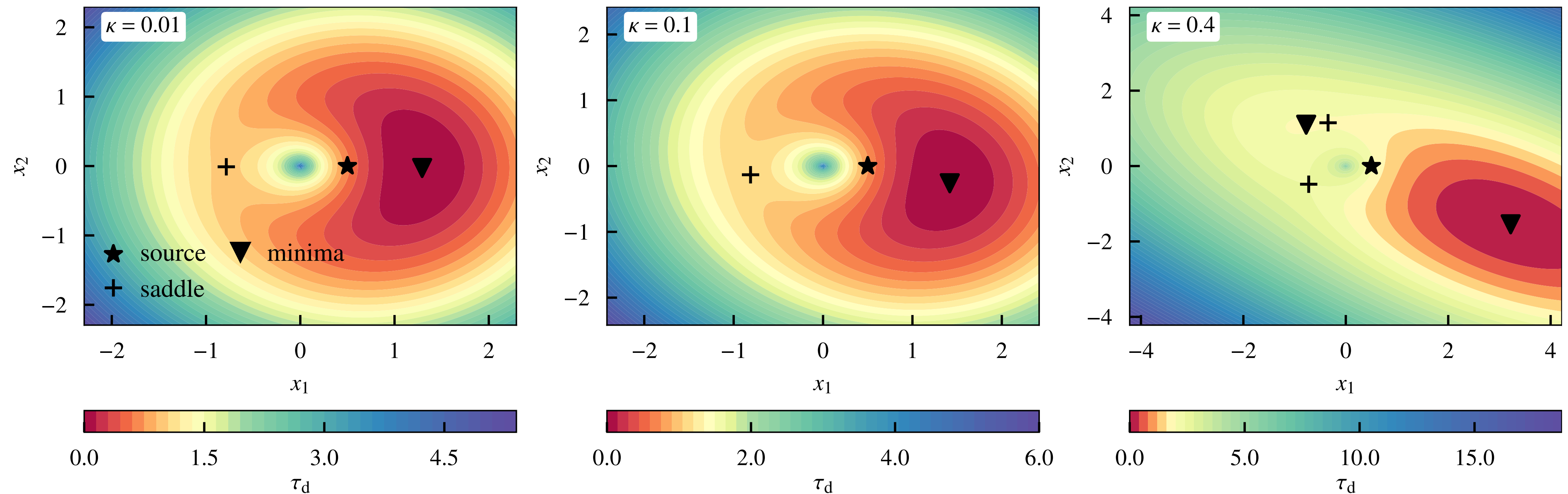
Binary parameters $\{m_1, m_2, \vec{S}_1, \vec{S}_2, \dots\}$ $\rightarrow$ $h_{UL}(f; \vec{\vartheta})$

Lens plane coordinates $\rightarrow$ $d^2\vec{x}$

Normalised frequency $\rightarrow$ $i\omega\tau_d(\vec{x})$

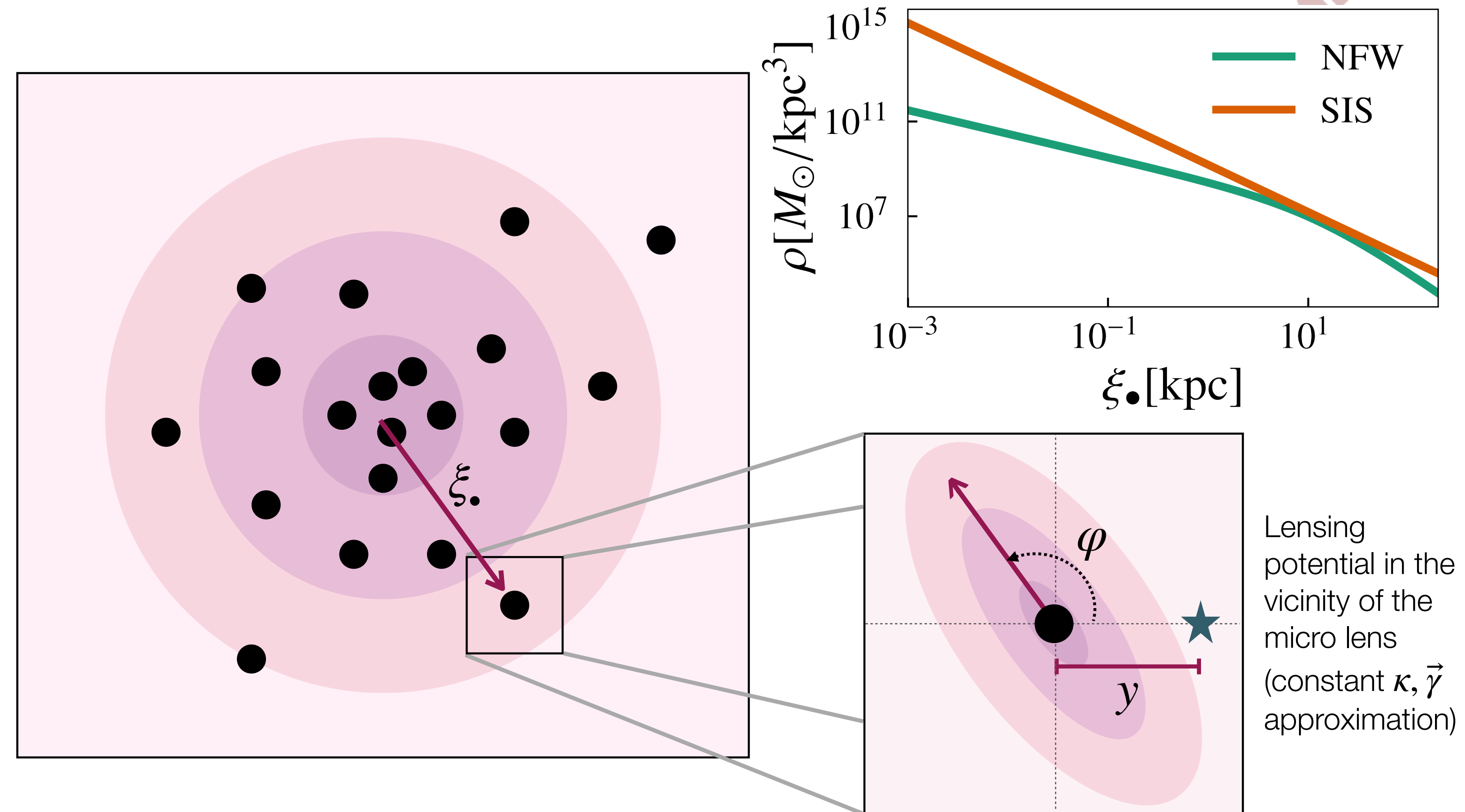
$$\mathcal{F}(w) = C(w) \int d^2\vec{x} \exp [i\omega\tau_d(\vec{x})]$$

Information about the lens and source encoded in the time delay function (Fermat potential): $\tau_d = \frac{|\vec{x} - \vec{y}|^2}{2} - \psi(\vec{x}, \vec{\lambda})$



Motivation and aim

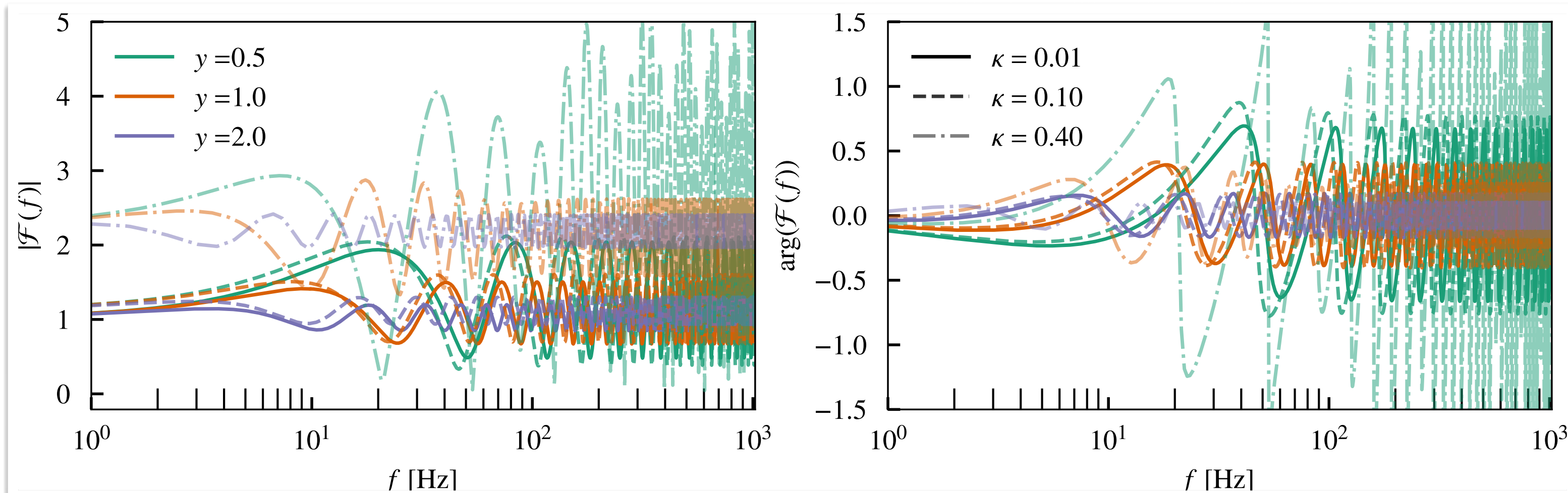
- Current microlensing searches assume **isolated point mass lens (PML)** model.
- Compact objects that produce microlensing effects are likely to be **part of a galaxy**.
- Quantify **impact of host galaxy** on the efficacy of current microlensing searches **statistically**.



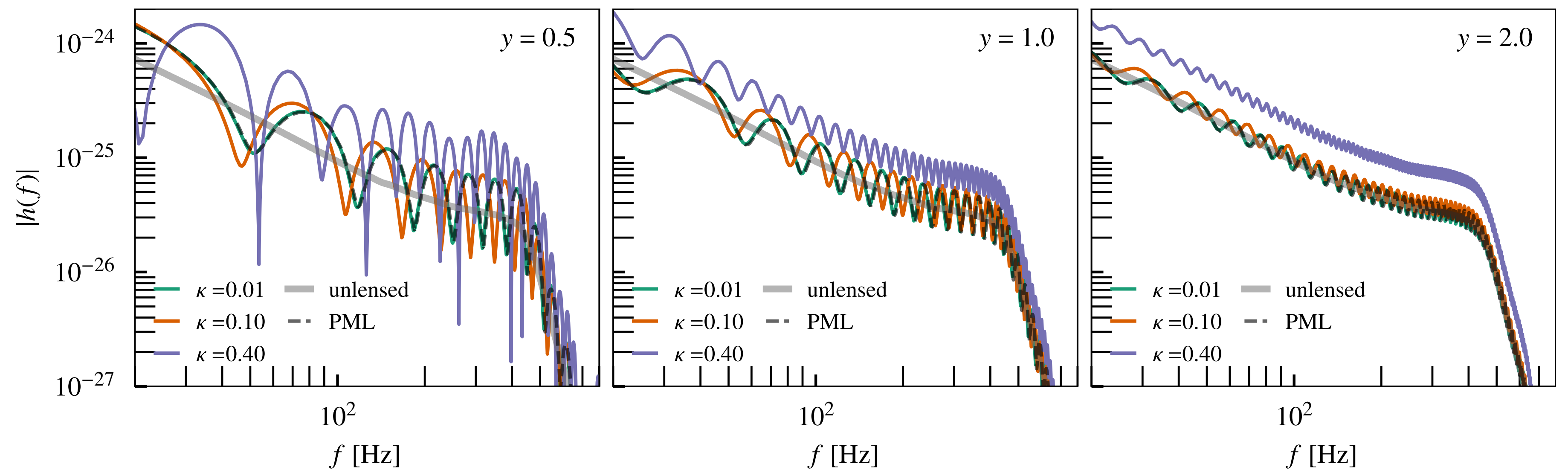
PML contribution Isotropic convergence Anisotropic shear

$$\psi = \ln |\vec{x}| + \frac{\kappa}{2} |\vec{x}|^2 + \frac{\gamma}{2} [\cos(2\varphi) (x_1^2 - x_2^2) + 2 \sin(2\varphi) x_1 x_2]$$

Microlensed waveforms due to a single PMX lens



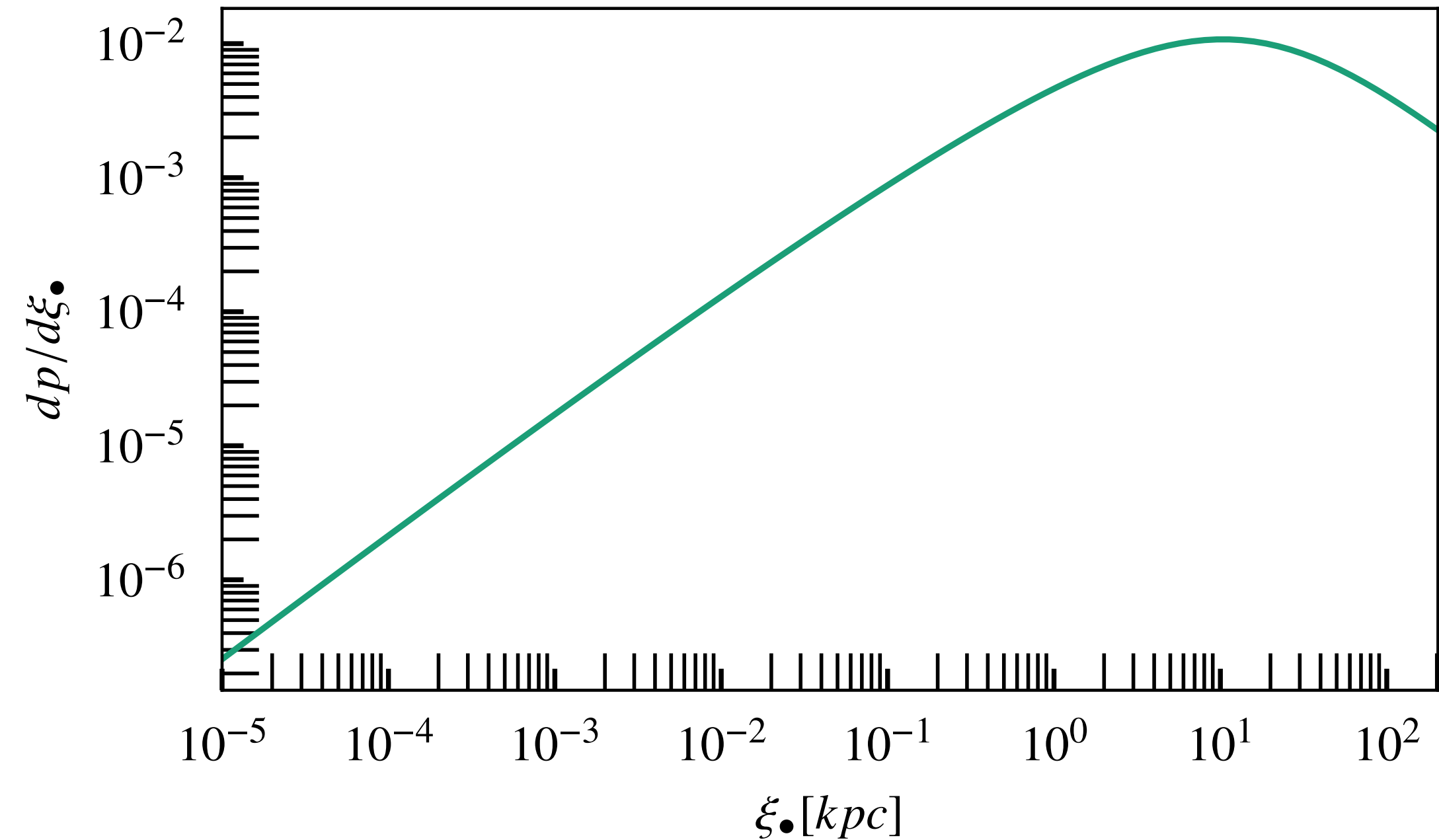
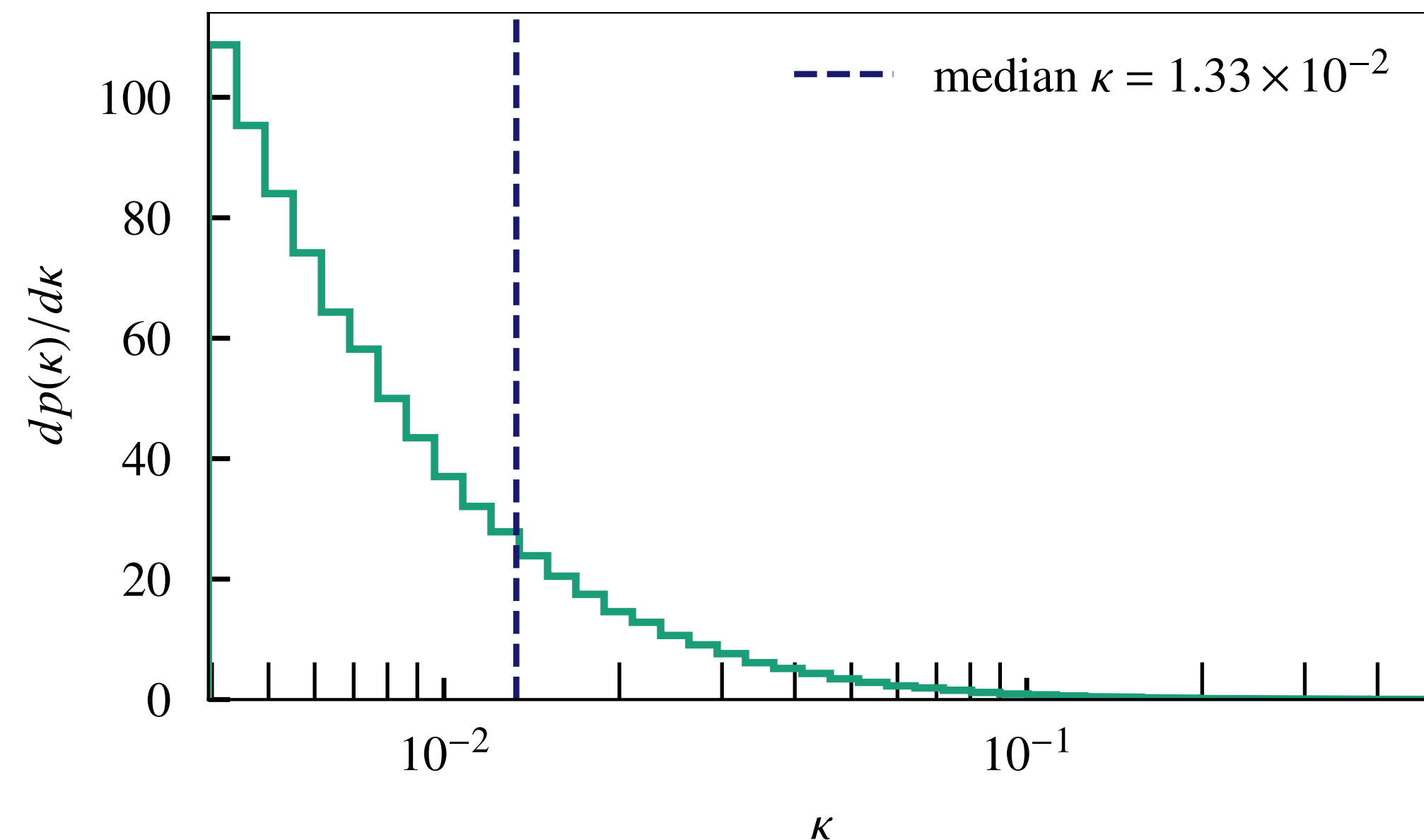
Incorporating external potential introduces distinct waveform features that significantly deviate from the isolated PML model.
Shear angle fixed at $\pi/3$.



Populating dark matter microlenses in a galaxy

- Sample lens parameters $\kappa, \gamma, \varphi, y$, assuming monochromatic mass function.
- Distribution of compact (dark matter) lenses follow projected NFW profile:

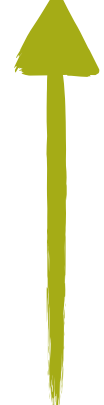
$$dp = \Sigma_{\text{NFW}}(\xi_{\bullet}) 2\pi\xi_{\bullet} d\xi_{\bullet}$$



- Convergence / shear of the host galaxy follows a SIS profile with $\kappa = \gamma$.
- Source location uniform in area: $p(y) \propto y$
- Isotropic in shear angle: $p(\varphi) = \mathcal{U}[0, 2\pi)$

Fitting factor analysis

- Statistic that distinguishes two models $\rightarrow$ Bayes factor: $\mathcal{B}_U^L \equiv \frac{p(d | \mathcal{H}_L)}{p(d | \mathcal{H}_U)}$



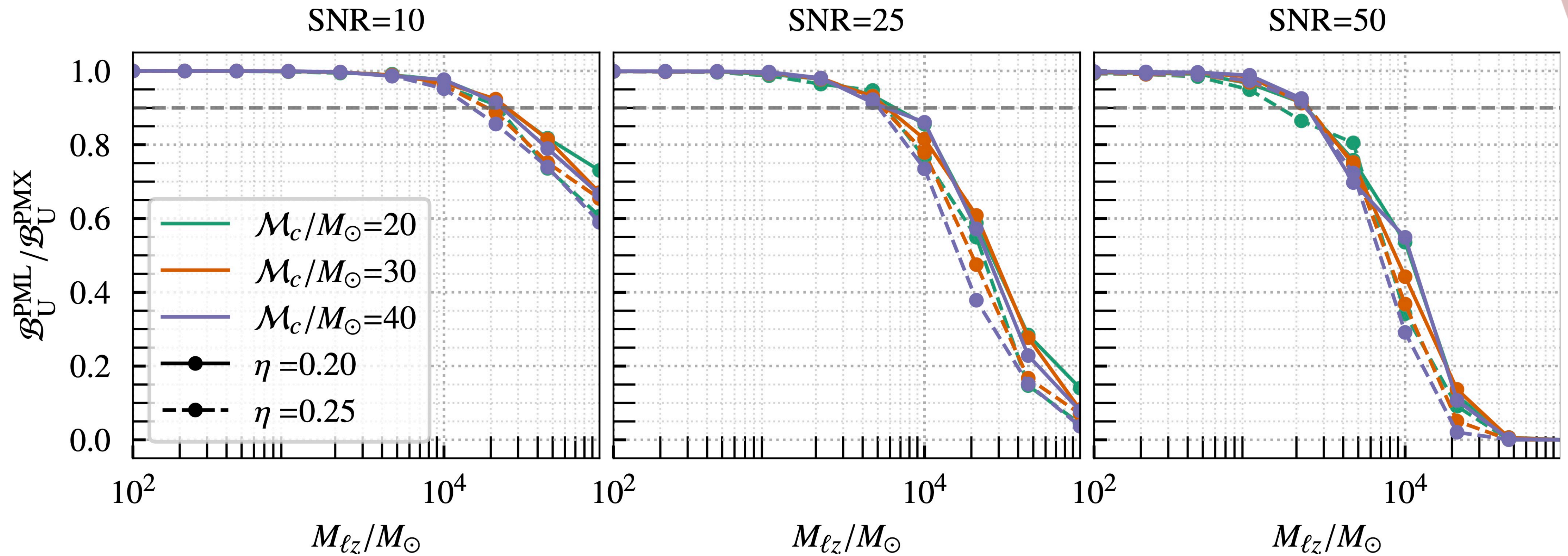
Lensing hyp preferred

Unlensed hyp preferred
- If the true signal is microlensed by a PMX lens with parameters $\vec{\theta}_{\text{tr}}$, but the inference is carried out using the simplified PML model, then the degradation of Bayes factor is quantified using

$$\frac{\mathcal{B}_U^{\text{PML}}}{\mathcal{B}_U^{\text{PMX}}} \simeq \exp \left[-\frac{\text{SNR}^2}{2} \left(1 - \langle \text{FF} \rangle_{\kappa, \varphi_\gamma, y}^2 \right) \right] \quad (\text{in large SNR limit})$$

$$\text{FF} \equiv \max_{\vec{\theta}_{\text{PML}}} \mathcal{M}(\vec{\theta}_{\text{tr}}, \vec{\theta}_{\text{PML}}) \quad \mathcal{M}(\vec{\theta}_{\text{tr}}, \vec{\theta}_{\text{PML}}) \equiv 4\mathcal{R} \int_{f_{\text{low}}}^{f_{\text{upp}}} \frac{h_{\text{PMX}}(f; \vec{\theta}_{\text{tr}}) h_{\text{PML}}^*(f; \vec{\theta}_{\text{PML}})}{S_n(f)} df$$

Loss in search sensitivity



Ratio of Bayes factors between the unlensed and lensed hypotheses (PML or PMX models) for true signals varying in chirp mass and symmetric mass ratio.

Conclusions

- We estimate the **reduction in Bayes factor** due to modeling the PMX lens as an isolated point mass.
- Neglecting host galaxy effects lead to systematic underestimation of the Bayes factor in the high $M_{\ell z}$ regime $\rightarrow$ **reduced sensitivity** to microlensing searches.
- Future searches may need to incorporate the additional effect of the lens galaxy, particularly those aimed at placing robust constraints on the abundance of compact dark matter.
- Limitations and future directions:
 - SIS does not fully capture the diversity of galactic potentials, e.g. elliptical profiles. Using **realistic models** for background galaxy.
 - high-SNR approximation may not hold for all observed events. Rigorous treatment requires assessing model evidences within a full Bayesian framework. Implement **fast waveform generation techniques** (such as surrogate models) for full Bayesian parameter estimation.