

Gravitational-Wave Echoes from Layered Compact Objects: A Double-Shell Model

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Based on our forthcoming work, arXiv:2608.XXXXX

- Layered structures are ubiquitous in realistic astrophysical objects. In black holes (BHs) formed through gravitational collapse, such structures persists during the whole range of time definition used by outside probes.
- Reference [1] identifies such layering as the microscopic-state of BHs and proves the area law formula or Bekenstein-Hawking entropy follows. Our question here is: such a layering structure measurable at all? through e.g. GW of binary merger process.
- Refs. [2,3] showed that the ring-down signal is insensitive to the structure inside the photon sphere. However, the late-time echoes carry information about such structures; the canonical example is a single thin-shell model.
- We extend the canonical single thin-shell model of Refs.[2,3] to two shells and study whether different mass distributions at fixed total mass produce distinguishable echo waveforms.

[1] D.-F. Zeng, *Eur. Phys. J. C* **84**, 370 (2024), arXiv:2311.11764.

[2] V. Cardoso, E. Franzin, and P. Pani, *Phys. Rev. Lett.* **116**, 171101 (2016), arXiv:1602.07309.

[3] V. Cardoso et al., *Phys. Rev. D* **94**, 084031 (2016), arXiv:1608.08637.

Double-Shell Model Setup

- We extend Cardoso's model to a two-shell configuration.

$$r_1 = (2 + \epsilon)m_1, \quad r_2 = (2 + \epsilon)M,$$

$$M = m_1 + m_2, \quad q = \frac{m_1}{M},$$

$$f(r) = 1 - \frac{2m(r)}{r}, \quad m(r) = \begin{cases} 0, & r < r_1, \\ m_1, & r_1 \leq r < r_2, \\ M, & r \geq r_2. \end{cases}$$

- The metric is obtained by direct matching of different regions.
- The tortoise coordinate is continuous across interfaces, and the waveform satisfies the junction conditions.
- Both perturbation sectors satisfy the master equation

$$[-\partial_t^2 + \partial_{r_*}^2 - V_\ell(r)] \Psi(t, r_*) = 0. \quad (1)$$

- We start with an initial Gaussian wave packet and solve the master equation.

- $q = 0$ and $q = 1$: the same single-shell baseline.
- $0 < q < 1$: genuine double-shell configurations.
- The inner region is regular, while the outer boundary condition is outgoing.
- The shell configuration is fixed; we focus on wave propagation on this background.

Introduction

Model

Effective Potential

Time Domain Waveform

Waveform Spectra

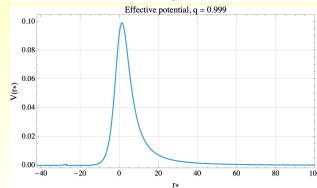
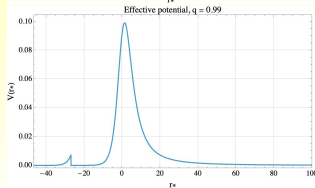
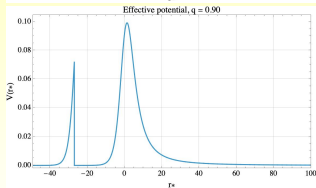
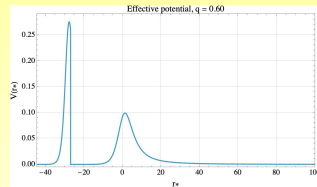
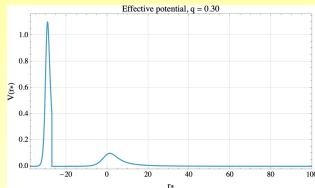
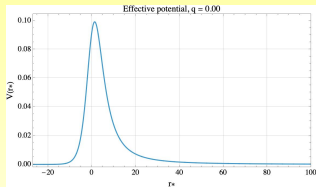
Robustness

Conclusion

Thanks

- The effective potentials of probe-scalars and Regge-Wheeler (RW) perturbations are

$$V_{\ell}^{\text{sca}} = f(r) \left(\frac{\ell(\ell+1)}{r^2} + \frac{f'(r)}{r} \right), \quad V_{\ell}^{\text{ten}} = f(r) \left(\frac{\ell(\ell+1)}{r^2} - \frac{6m(r)}{r^3} \right) \quad (2)$$



The Characteristic Propagation Lengths

- The double-shell geometry creates coupled propagation regions characterized by four length scales:

$$2L_1, \quad 2L_2, \quad L_1 + L_2, \quad L_2 - L_1. \quad (3)$$

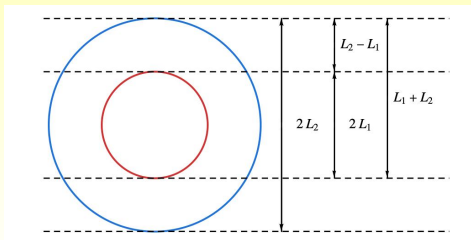
- Here, $L_i \equiv r_{*,i} - r_*(0)$ ($i = 1, 2$), with $L_1 < L_2$, denotes the tortoise-coordinate distance from the regular center to the maximum of the i th potential barrier, rather than to the mass shell.
- These propagation scales organize the resonance structure in the echo spectrum.

$2L_1$ Inner $\leftrightarrow$ inner resonant cavity.

$2L_2$ Outer $\leftrightarrow$ outer resonant cavity.

$L_1 + L_2$ Inner $\leftrightarrow$ outer & central-passing

$L_2 - L_1$ Inner $\leftrightarrow$ outer but no central-passing



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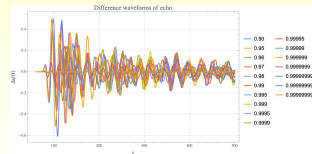
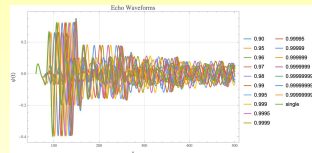
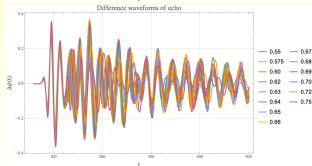
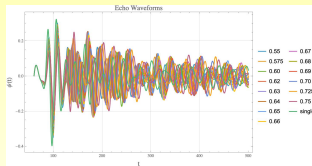
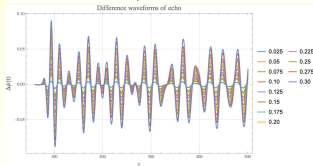
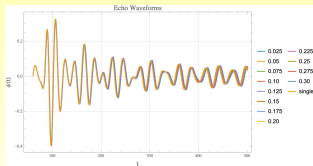
Waveform
Spectra

Robustness

Conclusion

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- For $q \lesssim 0.4$, the waveform mainly exhibits an overall time delay relative to the single-shell baseline, with no significant morphological deformation.
- As q increases further, the multiple propagation cavities become strongly coupled, producing substantial differences between the double-shell and single-shell waveforms.
- As $q \rightarrow 1$, the overall time delay decreases again, and the waveform eventually returns to the single-shell limit.



Amplitudes of the First Echo and Late Echoes

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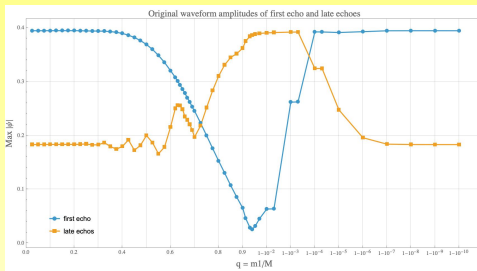
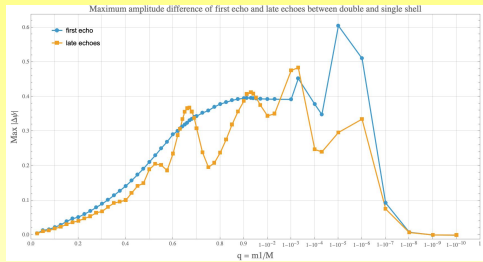
Time Domain
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- The left panel presents the maximum amplitude differences between the double-shell and single-shell waveforms for the first and late echoes, while the right panel shows their corresponding maximum amplitudes.
- For $q \lesssim 0.4$, the waveform mainly exhibits an overall time delay, with no significant morphological deformation.
- As q further increases, the double-shell structure produces increasingly significant differences from the single-shell waveform, resulting in suppression of the first echo and enhancement of the late echoes.

Amplitudes of the First Echo and Late Echoes

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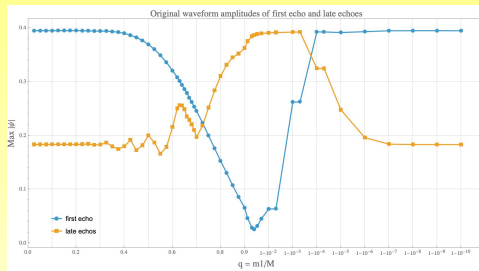
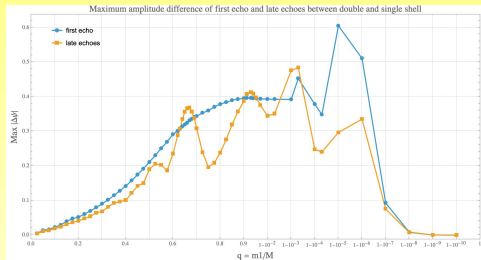
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- Around $q \sim 2/3$, a local maximum appears in the late echoes, corresponding to the point where the maximum of the inner effective-potential barrier begins to be truncated by the outer shell.
- Due to the stretching of the Schwarzschild tortoise coordinate near the Schwarzschild radius, the double-shell configuration rapidly approaches the single-shell limit only when q is extremely close to 1. Therefore, we adopt a logarithmic scale in $1 - q$ for $q > 0.99$.

Residual and Time-Aligned Waveform Differences

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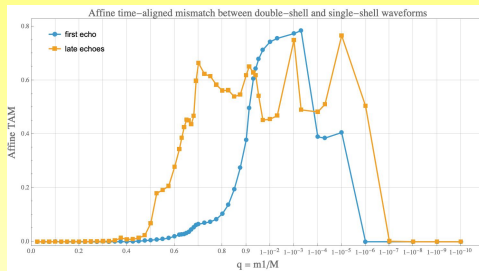
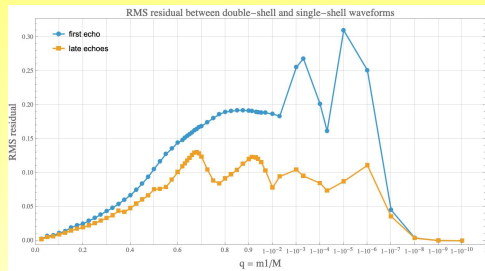
Time Domain
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- The left panel presents the Root Mean Square (RMS) differences between the double-shell and single-shell waveforms, while the right panel presents the affine Time-Aligned Mismatch (TAM).
- The RMS exhibits an overall trend similar to the amplitude difference.
- For $q \lesssim 0.4$, the waveform is characterized by an overall phase shift toward later times, with no significant change in its morphology.

Residual and Time-Aligned Waveform Differences

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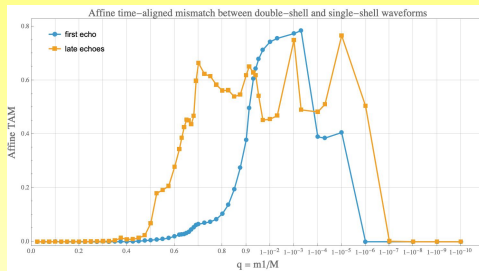
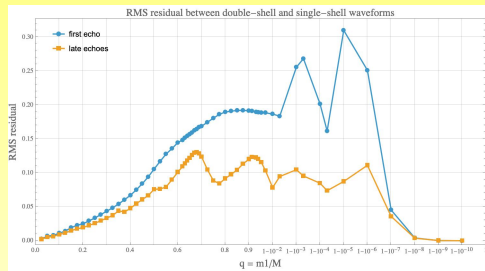
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- As q increases, the morphology of the first echo remains nearly unchanged, whereas the late echoes exhibit increasingly significant deviations from the single-shell waveform.
- For $q \sim 0.9$, the first echo also begins to undergo substantial deformation.
- When q becomes extremely close to 1, the entire waveform rapidly approaches the single-shell limit.

- We calculate echo spectra for double-shell configurations by scanning the mass-distribution parameter q from 0 to 1. This scan does not represent a dynamical evolution: each value of q labels a different stationary layered compact object with the same total mass but a different internal mass distribution.
- At both $q = 0$ and $q = 1$, the spectrum reduces to the same single-shell spectrum. However, continuous branch tracking shows that only the lowest-frequency branch returns to its original single-shell frequency, whereas the higher tracked branches do not return to their original ordered peak positions at $q = 1$, but are instead rearranged among the peaks of the final single-shell spectrum.
- We denote by $P_k^{(0)}$ and $P_k^{(1)}$ the k th ordered peaks of the single-shell spectra at $q = 0$ and $q = 1$, respectively. The symbol S_j denotes the j th branch entering the analyzed frequency band from the high-frequency side.
- Within the tracked frequency band, the endpoint branch mappings are:

$$P_1^{(0)} \rightarrow P_1^{(1)}, \quad P_2^{(0)} \rightarrow P_3^{(1)}, \quad P_3^{(0)} \rightarrow P_4^{(1)}, \quad P_4^{(0)} \rightarrow P_6^{(1)} \quad (4)$$

$$S_1 \rightarrow P_2^{(1)}, \quad S_2 \rightarrow P_5^{(1)}, \quad S_3 \rightarrow P_7^{(1)}. \quad (5)$$

The Spectral-Peak Queueing

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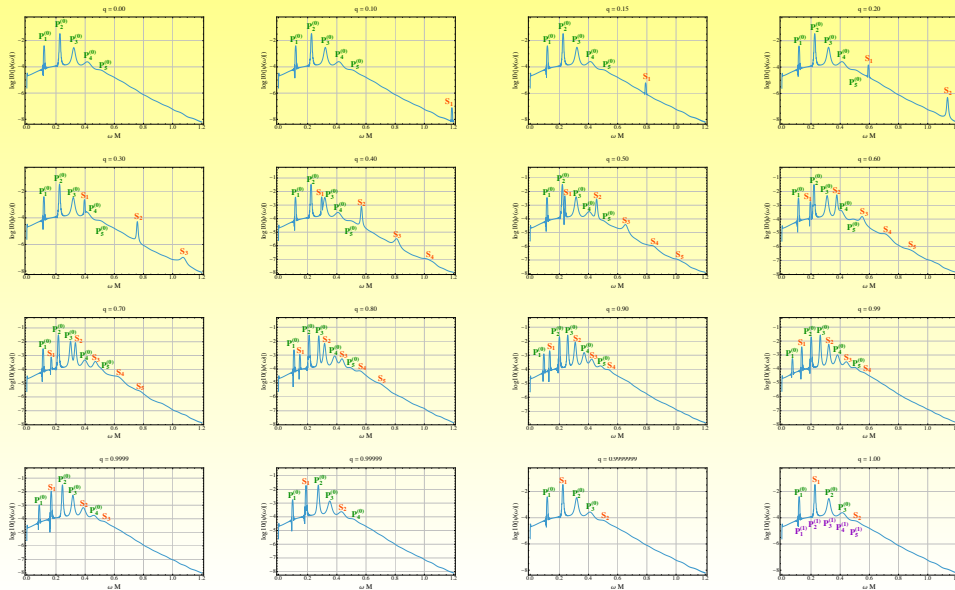
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The Spectral-Peak Queueing

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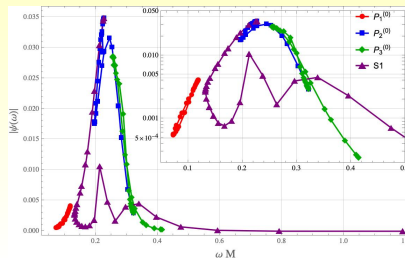
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- 1 **High-frequency branch entry:** new branches S_j enter the analyzed band from high frequencies. They are associated with the growing inner round-trip length $2L_1$ before strong barrier truncation.
 - 2 **Spectral-peak migration:** baseline-derived higher-frequency branches $P_k^{(0)}$ shift toward higher frequencies. This primarily follows from the decrease in the inter-barrier length $L_2 - L_1$.
 - 3 **First-peak return:** $P_1^{(0)}$ first shifts downward and then returns to $P_1^{(1)}$. This follows the nonmonotonic evolution of the global length $2L_2$.
- We refer to these three phenomena collectively as Spectral-peak Queueing (SQ).
 - Each curve tracks a peak's frequency and amplitude as q varies across stationary backgrounds.
 - **Colors:** red—first-peak return; blue/green—peak migration; purple— S_1 branch entry.



$\ell = 2$ Scalar Probe Perturbation

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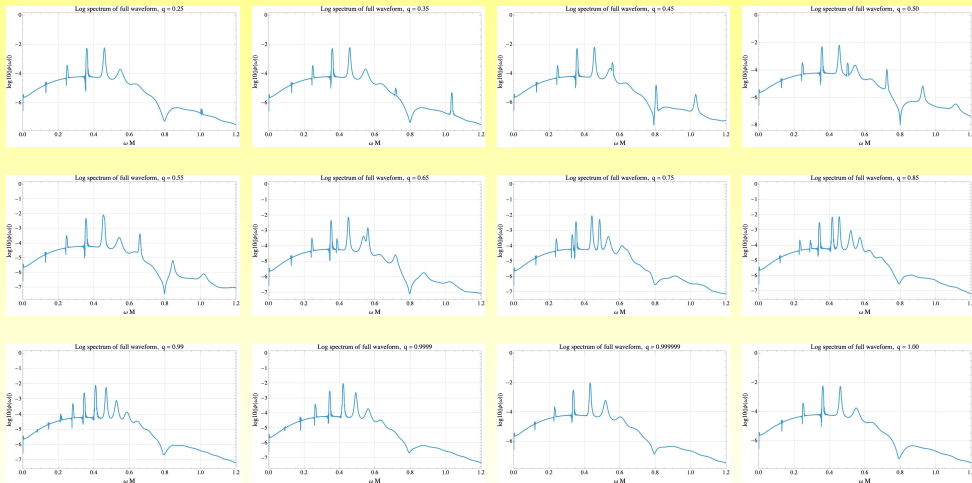
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$\ell = 2$ RW Perturbation

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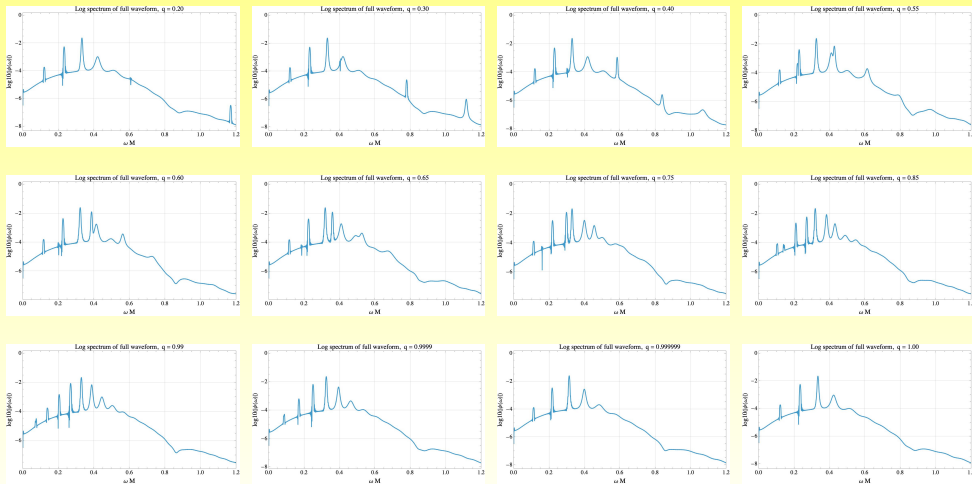
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SQ Persists Across the Tested Perturbation Channels

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- The three SQ components found for the fiducial scalar $\ell = 1$ channel also persist in scalar $\ell = 2$ and RW $\ell = 2$ perturbations.
- The scalar $\ell = 2$ spectrum contains higher-frequency, more readily resolved branches, yet retains the same three-component queueing pattern.
- Despite their different effective potentials, the main RW $\ell = 2$ peak frequencies remain close to their scalar $\ell = 1$ counterparts.
- The strongest channel dependence appears in peak amplitudes; the frequency structure is governed mainly by characteristic propagation lengths and coupling between the propagation regions.

Echo Spectra Carry Information about Internal Layering

- ① For genuine double-shell configurations ($0 < q < 1$), the second shell introduces an inner potential barrier and creates multiple coupled propagation regions.
- ② At small q , waveform differences are dominated by an overall delay, or equivalently a phase shift; genuine deformation appears first and more strongly in the late echoes, especially near $q \simeq 2/3$.
- ③ Although the $q = 0$ and $q = 1$ spectra are identical as frequency sets, continuous branch tracking reveals a nontrivial reordering through high-frequency branch entry, peak migration, and first-peak return.
- ④ Across the scalar and Regge-Wheeler channels tested here, the SQ frequency pattern is governed mainly by geometry, while peak amplitudes are more channel-sensitive.

Note: q labels distinct stationary backgrounds, not the time evolution of one source. After accounting for their overall mass scales, spectra from different objects may be compared to constrain internal layering.

Outlook: Can gravitational-wave spectroscopy distinguish different microscopic layering structures?

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Thanks for your attention!