

Relativistic treatment of EMRIs crossing accretion disks

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Physical picture

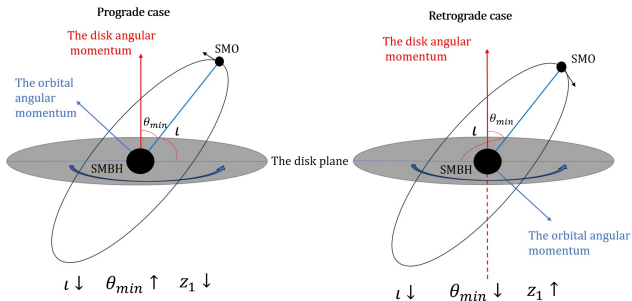


Figure 1: Schematic illustration of a wet EMRI scenario with SMO-disk collisions.
 $\cos^{-1}(\hat{L}_{\text{disk}} \cdot \hat{L}_{\text{orbit}}) = l$. $0 < l < 90^\circ$: prograde cases; $90^\circ < l < 180^\circ$: retrograde cases.
 $z_1 = \cos \theta_{\min} = \sin l$. SMO: a star or a stellar-mass black hole (sBH).

Theoretical framework

- Forced geodesic

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma_{\alpha\beta}^\mu \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = f^\mu, \quad (f^\mu u_\mu = 0) \quad (1)$$

- The osculating method under the Schwarzschild metric

$$\frac{\partial}{\partial I_0^a} \left(\frac{dx_G^\mu}{d\tau} \right) \frac{dI_0^a}{d\tau} = f^\mu \rightarrow \frac{dI_0}{d\tau} = C_r^{(I_0)} f^r + C_\theta^{(I_0)} f^\theta + C_\phi^{(I_0)} f^\phi. \quad (2)$$

- The aero-drag force and dynamical friction force models,

$$\mathbf{F}_{\text{aero}} = -\pi R_\star^2 \rho_g v_{\text{rel}} \mathbf{v}_{\text{rel}}, \quad \mathbf{F}_{\text{dyn}} = -\mathcal{J} \frac{4\pi (Gm_\bullet)^2 \rho_g}{v_{\text{rel}}^3} \mathbf{v}_{\text{rel}}, \quad (3)$$

$R_\star$: star radius; $m_\bullet$: sBH mass; ρ_g : disk gas density.

Comparison between the full-orbit and adiabatic approximation

The evolution of $a \equiv p/(1 - e^2)$, e and ι , comparison between the full forced geodesic and the adiabatic approximation.

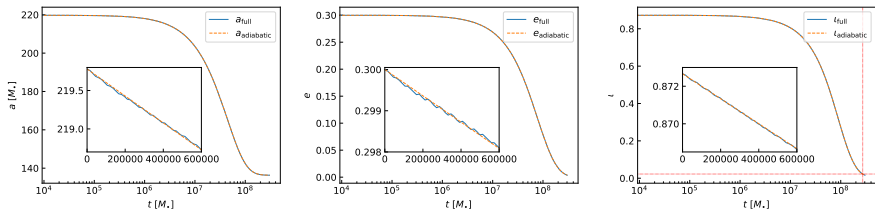


Figure 2: Aero-drag force model, damping coefficient $\gamma_0 = 2 \times 10^{-6} M_\bullet^{-1}$, disk height $H_{\text{disk}} = 1.5 M_\bullet$. Initial conditions: $p_{\text{ini}} = 200 M_\bullet$, $e_{\text{ini}} = 0.3$, $\iota_{\text{ini}} = 50^\circ$, the red dotted lines in the right panel: $t_{\text{crit}} = 3H_{\text{disk}}/p_{\text{ini}}$ (horizontal) and $t_{\text{crit}} = 2.7 \times 10^8 M_\bullet$ (vertical).

Evolution trends of eccentricity e

Averaged eccentricity evolution rate $\left\langle \frac{de}{d\tau} \right\rangle$ versus inclination $i \in (0^\circ, 180^\circ)$.

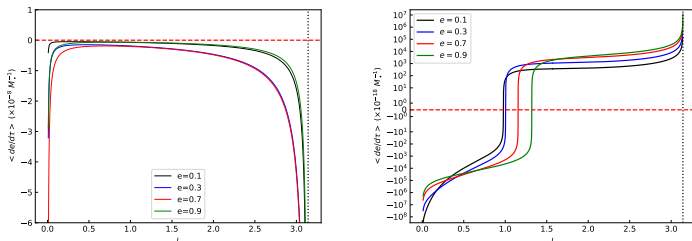


Figure 3: Left: aero-drag force model, $\gamma_0 = 2 \times 10^{-6} M_\bullet^{-1}$; Right: dynamical friction model, $\gamma_0 = 4 \times 10^{-13} M_\bullet^{-1}$. For both models $H_{\text{disk}} = 1.5 M_\bullet$, $p = 300 M_\bullet$.



Evolution results of the aero-drag force model

The evolution of a , e and t under the aero-drag force model, with $\gamma_0 = 2 \times 10^{-6} M_\bullet^{-1}$, $H_{\text{disk}} = 1.5 M_\bullet$.

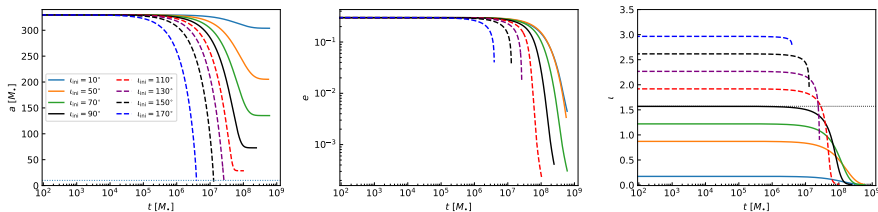


Figure 4: Initial conditions: $p_{\text{ini}} = 300 M_\bullet$, $e_{\text{ini}} = 0.3$, different i_{ini} values. Solid lines for $i_{\text{ini}} < 90^\circ$ (prograde), dashed lines for $i_{\text{ini}} > 90^\circ$ (retrograde). The blue dotted horizontal line in the left panel marks $a_{\text{crit}} = 10 M_\bullet$.

Evolution results of the dynamical friction model

The evolution of a , e and i under the dynamical friction model, with $\gamma_0 = 4 \times 10^{-13} M_\bullet^{-1}$, $H_{\text{disk}} = 1.5 M_\bullet$.

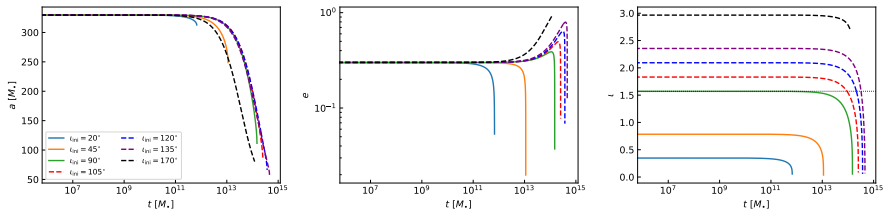


Figure 5: Initial conditions: $p_{\text{ini}} = 300 M_\bullet$, $e_{\text{ini}} = 0.3$, different t_{ini} values. Solid lines for $t_{\text{ini}} < 90^\circ$ (prograde), dashed lines for $t_{\text{ini}} > 90^\circ$ (retrograde).

Capture timescale estimation: SG disk model

Capture timescales versus t_{ini} , with the scaling relations for two models.

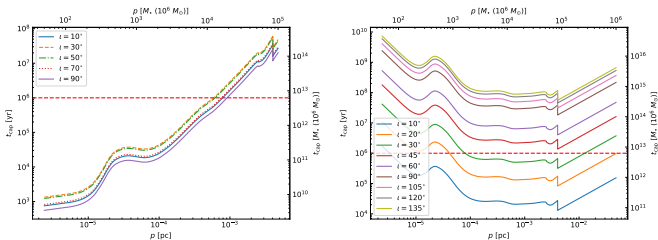


Figure 6: Left / Right: aero-drag force model / dynamical friction model. Σ_g from the SG disk model ($\alpha = 0.1$, $M_\bullet = 10^6 M_\odot$) via `pagn` code, `Sirko` module.

¹D. Gangardt, A. A. Trani, C. Bonnerot, and D. Gerosa, *Monthly Notices of the Royal Astronomical Society* 530, 3689-3705 (2024).

²Y. Wang, Z. Zhu, and D. N. C. Lin, *Monthly Notices of the Royal Astronomical Society* 528, 4958 (2024).



Conclusions

- a and t decrease monotonically for both prograde and retrograde orbits under both forces, indicating orbital-energy dissipation and angular-momentum alignment with the disk;
- Aero-drag: e decreases monotonically;
- Dynamical friction: e is excited at high inclinations for some prograde and all retrograde orbits;
- SMO capture by the disk or orbital shrinkage drives near-circularization;
- High-inclination stars, stars closer to the SMBH, and low-inclination sBHs are more readily captured.

Thank You!